Adoption Costs, Age of Capital and Technological Substitution*

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Abstract

In this paper we introduce adoption costs in a vintage capital model. We assume that the incorporation of technological innovations into the production sector requires an extra labor cost during a fixed period. First, we show how adoption crucially matters in the shape of short run and asymptotic dynamics. Then, we analyze the consequences of adoption costs in technological substitution extending the model in two ways: we let adoption costs depend on the technical growth rate, and we endogenize them, depending on the technological gap. When adoption costs depend on the technical growth rate, the effect of growth on optimal lifetime of machines is indeterminate; the creative destruction effect can be compensated by the adoption effect, and faster growth rates delay the technological substitution. Finally, when adoption costs are endogenous, we recover the typical obsolescence effect in vintage capital models and show that technological progress has a negative effect on the technological gap.

Keywords: Machine replacement, Optimal scrapping, Economic fluctuations, Technology adoption.

Journal of Economic Literature: E22, E32, O40, C63

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1 Introduction

In the neoclassical theory technological progress is assumed to increase the productivity of all existing capital. However, the embodied nature of technological progress has been invoked in some recent empirical contributions. In particular, Greenwood, Hercowitz and Krusell (1997) estimate that the investment specific technological progress accounts for 63% of output growth in the US economy. The key assumption in investment specific technology is that technological progress is embodied in new machines, which gives rise to an endogenous process of creative destruction, that is, the process of replacing old machines with new and more productive ones[1]. Some theoretical papers[2] have stressed the suitability of vintage capital models for the analysis of investment volatility as investment takes the form of a replacement activity.

A number of empirical contributions have emphasized the importance of adoption costs (which takes the form of learning, experience, skill requirements or technological barriers) in the process of implementing a new technology. Bahk and Gort (1993) find that a plant's productivity increases by 15% over the first 14 years of its lifetime due to learning effects. Using cross-country data, Flug and Hercowitz (1997) point out that an increase in equipment investment leads to a rise in the demand of skilled labor. Alder an Clark (1991) reinforces the previous idea and show that the opening of a plant is followed by a temporary increase in the use of skilled labor whose job is to get the production process "up to speed". Quantitatively, adoption costs have been estimated in Jovanovic (1995) as 10% of GDP.

The process of implementing new technologies is uncertain in nature and the best way to model it is from a Bayesian perspective (Jovanovic and Nyarko, 1995). However trying to do this in a general equilibrium framework is a daunting task, and general equilibrium models deal with this problem assuming a mechanistic process. Parente and Prescott (1994) introduce exogenous institutional and external constraints to adoption. Greenwood and Yorukoglu (1997) stress the difficulty of modelling the adoption process in an endogenous manner and assume that as the rate of technological increases, the more costly it becomes to adopt a new technology since enterprises will

[1]As shown by Benhabid and Rustichini (1993), endogenous replacement can be generated with any production function with complementary factors.

[2]See Boucekkine, Germain and Licandro (1997); Boucekkine et *al.* (1998).

be less familiar with it.

The adoption of a new technology may carry a large forgone output cost incurred during the learning period, as it takes some time for a new technology to operate at its theoretical level (Parente, 1994). On the other hand, an increase in the production costs (or a rise in the labor devoted to implementing new technologies) during a temporary period, can help the new capital to reach its potential productivity. Magnac and Verdier (1993) analyze in a duopolist framework the process of adoption when the new technology is characterized by adaptation costs and learning effects. They consider that technological implementation shows two phases; in the first phase, the immature stage of the new technology, the firm must incur larger costs since it has to adapt the new technology it is using for the first time; in the second period, the mature stage, the firm has already learned how to use the new technology and it is able to produce at a lower cost. This means that adopting the new technology generates learning and consequently decreases costs. In fact, the previous theories are equivalent in that they cause a decline in the firms' net revenues; in the first case this is introduced directly through a decrease in output, while in the latter case it is assumed that the decrease in productivity during the adoption phase is compensated by extra production costs or by hiring an extra amount of labor. In this paper we consider a Ramsey growth vintage capital model with linear utility and Leontieff technology, previously studied by Boucekkine, Germain and Licandro (1997), and we relax the zero adoption cost they assume. We model adoption costs, following the Magnac and Verdier setup of two adoption phases, as an additional labor cost to be paid over a fixed interval of time. The modelling of adoption through additional labor requirements is standard. In some recent contributions (as in Greenwood and Yorukoglu, 1997) adoption requires skilled labor. In others, as in Easterly et *al.* (1994), adoption costs are simply taken proportional to the labor force. Following Greenwood and Yorukoglu (1997) we let adoption labor requirements to depend positively on the rate of technical progress. The objective of the paper is twofold. First, we study how the short and long run dynamics are affected by the costly technological implementation process. In contrast with the model considered by Boucekkine et *al.* (1997), in which the dynamics of investment is purely periodic in the short and in the long run, the inclusion of adoption costs gives rise some "irregular patterns" in the short run and convergence to the steady state in the long run. Moreover, we analyze the consequences of costly adoption on the machine replacement process. We prove that the presence of adoption costs delays the replacement of the oldest machines and increases the lifetime of capital goods. Hence, costly adoption can destroy the inherent obsolescence effect obtained in vintage capital models; when adoption costs depend on the technical growth rate, the effect of growth on the optimal lifetime of machines in indeterminate; the creative destruction effect can be compensated by the adoption effect and faster rate of technical progress may imply the use of older technologies by firms or countries facing up to higher adoption costs.

The problem of costly adoption is more crucial for developing countries which typically lack capital and skills and suffer from institutional and external barriers to adoption. As a result, technology upgrading tends to be less frequent and they often invest in dominated technologies. An extension of the model let us to obtain the optimal technological decisions taking into account the existing trade-off between investing in newer technologies and the consequent bigger adoption costs. Since it is assumed that implementation of dominated technologies are cheaper, the optimal investment decisions lead to a positive technological delay which is increasing in the adoption cost size.

This paper is organized as follows. Section 2 describes and shows the centralized equilibrium of the model. Section 3 develops the dynamics, in particular, we study the asymptotic stability of the model and the short run behavior. Section 4 characterizes the balanced growth paths of this economy, gives the comparative statics and shows the effect of costly adoption on the machine replacement process. In section 5 we analyze the effect of adoption costs on the technological delay when they depend on the adopted technology. Section 6 concludes.

2 The model

We consider an economy with an unique good produced by a representative firm. The production technology is the usual Leontieff vintage capital technology with exogenous (Harrod neutral) labor augmenting technical progress. Technical progress is continuously embodied in the new capital goods, which yields an endogenous process of creation and destruction through the replacement of the old obsolescent machines by the new and more productive capital goods. We assume that the productivity of new machines grows at fixed exponential rate γ, such that labor productivity grows exogenously at rate γ.

Our economy does not innovate, it simply adopts technologies already invented; however, it must exert an effort to adopt new technologies. In fact, we assume that adoption is costly and that it requires an extra amount of labor during a fixed period of time, say D. The modelling of adoption through additional labor requirements is standard. In some recent contributions (as in Greenwood and Yorukoglu, 1997) adoption requires skilled labor. In others, as in Easterly *et al.* (1994), adoption costs are simply taken proportional to the labor force. In our setting, although we consider that adoption and production are two distinct activities, we do not introduce any skill differential in the model: labor is homogeneous and hence the labor market is not segmented. More precisely, production and adoption are linked as follows: to form a production unit, a firm should combine one unit of capital, one unit of labor devoted to production (which is needed along the lifetime of the production unit), and α units of labor for D periods of time devoted to adoption. Our specification can be interpreted as follows. There are two distinct phases in the lifetime of a production unit: for the first D periods an extra labor effort is needed to operate the capital goods which incorporate the latest technological advances. Once this phase is finished, the firm is assumed to have enough expertise to produce the same quantity of output with a lower amount of labor. It is worth pointing out that because of the underlying Leontieff structure, our setting is equivalent to assuming a decrease in labor productivity for the first D periods, which the firm compensates by hiring an extra amount of labor which is a standard specification in the recent literature of the field.

The Leontieff technology described below states that a production unit created at t requires a capital unit of vintage t, an unit of productive labor along the lifetime of the production unit, α units of adoption labor during D periods and produces $\exp\{\gamma t\}$ units of output. If we denote $T(t)$ the age of the oldest operating machines at time t, aggregate output $Y(t)$ and aggregate employment $L(t)$ are given by:

$$
\begin{aligned}
Y(t) &= \int_{t-T(t)}^{t} H(s)e^{\gamma s}ds \\
L(t) &= \int_{t-T(t)}^{t} H(s)ds + \int_{t-D}^{t} \alpha H(s)ds
\end{aligned}
$$

where $H(t)$ denotes the production units creation rate. Under zero adoption costs, $H(t)$ also denotes the job creation rate at period t; in our model the job creation rate is equal to $(1+\alpha)H(t)$.

As it will be clear later, the lifetime of production units, $T(t)$, is determined endogenously in the model in contrast to the adoption period D which is taken as constant. The assumptions underlying our specifications are obviously consistent with $T(t) > D$ for every t: the adoption phase cannot exceed the lifetime of the production units. Instead, in the spirit of the model, the lifetime of the production units should be significantly greater than the adoption time. We will return to this issue later.

Investment costs, that is, the costs of creating a production unit are lineal and grow at the exogenous rate γ. To close the model, we specify the consumer side: the economy comprises a continuum of agents, indexed from 0 to 1. All individuals share the same linear preferences over lifetime consumption:

$$\int_0^\infty C(t)e^{-rt}dt$$

where $r > 0$ is the subjective rate of time preference, and $C(t)$ is the individual's consumption at time t; there is no disutility of labor, hence the labor supply is exogenous and equal to one.

2.1 The Central Planner Problem

The central planner solves the following problem:

$$\max \int_0^\infty C(t)e^{-rt}dt$$

subject to

$$\begin{aligned} Y(t) &= \int_{t-T(t)}^{t} H(s)e^{\gamma s}ds && (1)\\ L(t) &= \int_{t-T(t)}^{t} H(s)ds + \int_{t-D}^{t} \alpha H(s)ds && (2)\\ i(t) &= H(t)e^{\gamma t} \\ C(t) &= Y(t) - i(t) \\ 0 &\leq i(t) \leq Y(t) \end{aligned}$$

given $H_0(t)$ for all $t < 0$.

In order to solve this control problem we maximize the associated Lagrangian function; then, after changing the order the integration (following Malcomson (1975)), and some algebra, the problem can be rewritten as:

$$\begin{aligned} L(t) &= \int_0^\infty \left[Y(t) - H(t)e^{\gamma t} - \phi(t)Y(t) + \omega(t)\right] e^{-rt}dt + \\ &\int_0^\infty H(t)[\int_t^{t+J(t)} [e^{\gamma t}\phi(s) - \omega(s)]e^{-r(s-t)}ds - \alpha \int_t^{t+D} \omega(s)e^{-r(s-t)}ds]e^{-rt}dt + \\ &\int_{-T(0)}^0 H(t)\left[\int_0^{t+J(t)} [e^{\gamma t}\phi(s) - \omega(s)]e^{-r(s-t)}ds - \alpha \int_0^{t+D} \omega(s)e^{-r(s-t)}ds\right] e^{-rt}dt \end{aligned}$$

$$J(t) = T(t + J(t)) \quad (3)$$

$\phi(t)$and $\omega(t)$ are the Lagrangian multipliers associated with constraints (1) and (2) respectively. Equation (3) is just a definition: the expected lifetime of the new capital goods $J(t)$ is equal to the age of the oldest capital goods at time $t + J(t)$

The interior solution of this optimization problem is characterized by the following first order conditions:

$$\begin{aligned} \phi(t) &= 1 \qquad \forall t \\ e^{\gamma t} &= \omega(t + J(t)) \qquad (4) \\ e^{\gamma t} &= \int_t^{t+J(t)} \left[\phi(s)e^{\gamma t} - \omega(s)\right] e^{-r(s-t)}ds - \alpha(\gamma) \int_t^{t+D} \omega(s)e^{-r(s-t)}ds \qquad (5) \end{aligned}$$

Equation (4) is an exit condition which states that a production unit of vintage t will be replaced at $t + J(t)$ when its productivity does not cover the worker's reservation wage. Equation (5) is an entry condition which corresponds to the optimal investment rule. It equalizes the marginal cost of investment, on the left hand side, to the expected marginal revenue over its planned lifetime $J(t)$; the marginal revenue is determined by the production benefits minus the production labor costs, minus the cost of implementing a new and more productive technology. Whereas production labor costs depend on the future scrapping time, adoption costs depend on the adoption period. It is important to emphasize that although the adoption parameters (α, D) are exogenously given, adoption costs are endogenously determined; since they are measured in labor units, they depend on the reservation wage and on the creation units.

We are now able to define an equilibrium for our economy.

Definition 1 *Given the adoption time $D \geq 0$, and given the initial conditions $H_0(t)$, $\forall t < 0$, an equilibrium for our economy is a path for $T(t)$, $J(t)$,*

$H(t)$ and $Y(t)$, $\forall t \geq 0$, such that $T(t)$ and $J(t)$ are strictly greater than $D \geq 0$, and satisfies the following system of equations:

$$1 = \int_t^{t+J(t)} \left[1 - e^{-\gamma(t-s+T(s))}\right] e^{-r(s-t)} ds - \alpha \int_t^{t+D} e^{-\gamma(t-s+T(s))} e^{-r(s-t)} ds \quad (6)$$

$$J(t) = T(t + J(t)) \quad (7)$$

$$1 = \int_{t-T(t)}^{t} H(s)ds + \int_{t-D}^{t} \alpha H(s)ds$$

$$Y(t) = \int_{t-T(t)}^{t} H(s)e^{\gamma s} ds$$

2.2 Optimal Scrapping and Adoption Time

Our equilibrium conditions show a clear recursive forward-looking sub-block, namely the sub-block formed by equations (6)-(7). This sub-block allows to solve for the timing variables $T(t)$ and $J(t)$ independently of the other endogenous variables. By differentiating (6), using (7) and rearranging terms, we find the following functional relation

$$\begin{aligned} T(t) &= F(T(t+D), J(t)) \\ &= -\frac{1}{\gamma} \ln \left[\frac{1}{1+\alpha(\gamma)} \left(1 - (r-\gamma) - \frac{\gamma}{r}(1 - e^{-rJ(t)}) + \alpha(\gamma) e^{-\gamma T(t+D)} e^{-(r-\gamma)D}\right)\right] \end{aligned}$$

In order for function $F(.,.)$ to map from $R_+ * R_+$ into R_{++}, we need the following assumption:

Assumption 1 *The parameters of the model must satisfy the following conditions: $0 < \gamma < r < 1$.*

This assumption on the parameters is a standard condition for the existence of solutions in exogenous growth models; it is not difficult to prove that under assumption 1, (i) function $F(.,.)$ is increasing with respect to each of its arguments, and (ii) function $G(x) = F(x,x)$ has a unique strictly positive fixed-point. The following existence-uniqueness result generalizes Boucekkine, Germain and Licandro's Proposition 2 (1997).

Proposition 1 *Under assumption 1, for any value of the adoption time $D \geq 0$, the unique equilibrium paths for $T(t)$ and $J(t)$, $t \geq 0$, are constant and equal to the fixed-point T^* of the function $G(x) = F(x,x)$.*

The proof consists in constructing a sequence of upper bounds and lower bounds for $T(t)$, $\forall t$, and to show that these two sequences converge to the fixed-point of function $G(x)$. Recall that

$$\begin{aligned} T(t) &= -\frac{1}{\gamma}\ln\left[A + Be^{-rJ(t)} + Ce^{-\gamma T(t+D)}\right] \\ A &= \frac{r-\gamma}{1+\alpha}\left(\frac{1}{r}-1\right) > 0 \\ B &= \frac{\gamma}{r(1+\alpha)} > 0 \\ C &= \frac{\alpha}{1+\alpha}e^{-(r-\gamma)D} > 0 \end{aligned}$$

An obvious lower bound for $T(t)$ is $F(0,0) = -\frac{1}{\gamma}\ln[A+B+C]$, and a simple upper bound is $F(\infty,\infty) = -\frac{1}{\gamma}\ln[A]$. We get:

$$-\frac{1}{\gamma}\ln[A+B+C] \leq T(t) \leq -\frac{1}{\gamma}\ln[A]$$

for all $t \leq 0$. Since the previous inequalities hold for all t, they hold at $t+D$ and at $t+J(t)$. As $T(t+J(t)) = J(t)$, we can find another lower bound and another upper bound for $T(t)$ using the fact that function $F(.,.)$ is increasing in each of its arguments:

$$-\frac{1}{\gamma}\ln\left[A + Be^{\frac{r}{\gamma}\ln[A+B+C]} + Ce^{\ln[A+B+C]}\right] \leq T(t) \leq -\frac{1}{\gamma}\ln\left[A + Be^{\frac{r}{\gamma}\ln A} + Ce^{\ln[A]}\right]$$

We can keep on generating successive lower bounds (a_n) and upper bounds (b_n) for $T(t)$ in this way. For lower bounds we get the sequence $a_0 = -\frac{1}{\gamma}\ln[A+B+C]$ and $a_n = -\frac{1}{\gamma}\ln[A+Be^{ra_{n-1}}+Ce^{\gamma a_{n-1}}] = G(a_{n-1})$, for all $n \geq 1$. For upper bounds we get the sequence $b_0 = -\frac{1}{\gamma}\ln[A]$ and $b_n = -\frac{1}{\gamma}\ln\left[A+Be^{rb_{n-1}}+Ce^{\gamma b_{n-1}}\right] = G(b_{n-1})$. The sequence a_n (b_n) is trivially increasing (decreasing) and bounded. Thus, both sequences converge, they converge to the fixed point of function $G(.)$ by construction. Proposition 1 follows immediately.

Note that the constancy of the optimal lifetime of production units is obtained even in the zero adoption case, namely if $D = 0$. This property is useful for comparison purposes. A first interesting economic insight can be gained from the study of optimal lifetime when adoption time varies.

Proposition 2 *Under assumption 1, the optimal lifetime of production units T^* is an increasing but concave function of the adoption time D.*

See the proof in the appendix.

This proposition shows two interesting economic properties of the model. First, we analytically obtain that adoption costs increase the lifetime of production units and delay the technological substitution process. This is a very good property of the model since adoption costs have been repeatedly invoked to explain technological sclerosis and the higher age of capital in developing countries. Secondly, the growth rate of the production's units lifetime is decreasing with respect to the adoption time. This is rather an expected property having in mind equilibrium equation (6). An increase in the adoption time will increase the associated labor cost, which tends to increase the lifetime variable $J(t)$, as it requires time to recover the additional labor costs. However, an increase in the lifetime variable $J(t)$ decreases the "shadow" wages by equation (4), so it will reduce ex-post the labor costs. As a result, the optimal lifetime tends to growth less than the adoption time.

The previous proposition implies that it can be possible, in equilibrium, to scrap the oldest machine before finishing the adoption period; it would modify the structure of the problem as it has no sense to incur in implementation costs once the production unit have been removed from the production process. Indeed, the later issue as a whole is not that important in our simple framework since the adoption time is exogenous and can be fixed to a convenient value consistently with the view underlying our specification of adoption. Recall that, by definition of equilibrium, the optimal lifetime of production units should be lower than the adoption time. The concavity result above implies that the latter desired property may not be obtained for large values of D. We can prevent such an undesirable configuration by restricting the values of adoption time. We can establish the following existence result.

Proposition 3 *Under assumption 1, there exists* $D_0 > 0$ *such that* $T^* > D$ *if and only if* $0 \leq D \leq D_0$

The previous proposition restricts the value of the adoption time under which $T(t) = T^*$ is an equilibrium for all $t \geq 0$. We think of adoption time as a short transition period compared to the whole lifetime of production units. In this context, proposition 3 works extremely well.[3]

[3]For example, if $r = 0.05$, $\gamma = 0.03$, $\alpha = 0.2$ and $D = 3$, T^* equals to 11.25 years, while D_0 is 16.7 years.

3 Dynamics and Asymptotic Properties of Investment and Production

In this section we study the short and run dynamics and the asymptotic stability of investment and production. To derive the investment dynamics, we differentiate the labor market equilibrium equation. Production dynamics are obtained differentiating equation (1) and, taking into account equation (8):

$$H(t) = aH(t-D) + (1-a)H(t-T^*) \tag{8}$$

$$Y(t) = aY(t-D)e^{\gamma D} + (1-a)Y(t-T^*)e^{\gamma T^*} \tag{9}$$

where $a = \frac{\alpha}{1+\alpha}$.

With zero adoption costs ($\alpha, D = 0$), as in the RVCM studied by Boucekkine et al (1997), the dynamics of investment and production are purely periodic in the short and in the long run, and are given by:

$$\begin{aligned} H(t) &= H(t-T^*) \\ Y(t) &= Y(t-T^*)e^{\gamma T^*} \end{aligned}$$

The main implication of adoption costs is the appearance of a second delay in the investment dynamics. This second delay is obtained under the assumption that the two activities have not identical timing: the introduction of adoption costs will distort the equilibrium dynamics as long as they involve a different timing with respect to the main creation and destruction decisions. As the dynamics of $Y(t)$ are identical to those of $H(t)$, we will focus on the dynamics of $H(t)$.

Proposition 4 *All the nonzero roots of (8) are stable*

The characteristic function associated to equation (8) is

$$g(\lambda) = 1 - ae^{-\lambda D} - (1-a)e^{-\lambda T^*} = 0 \tag{10}$$

Let $\lambda = x + iy$, then $g(\lambda) = 0$ implies that the real and imaginary part of (2.10) must be zero.

$$1 - ae^{-xD}\cos(xD) - (1-a)e^{-xT^*}\cos(xT^*) = 0 \tag{11}$$

$$ae^{-xD}\sin(yD) + (1-a)e^{-xT^*}\sin(yT^*) = 0 \tag{12}$$

First, we prove that $g(\lambda)$ has a non-positive real part; we then check that if the real part is zero, the imaginary part is also zero. It is easy to see that $x > 0$ is impossible since it implies $ae^{-xD}\cos(xD)+(1-a)e^{-xT^*}\cos(xT^*) < 1$, ($e^{-z}$ and $\cos t \leq 1$, for any t, and for any $z > 0$). On the other hand, if $x = 0$, equations (11) and (12) can be written as

$$\begin{aligned} 1 - a\cos(xD) - (1-a)\cos(xT^*) &= 0 \\ a\sin(yD) + (1-a)\sin(yT^*) &= 0 \end{aligned}$$

and it is trivial to check that the only value of y that satisfies the previous equations is $y = 0$. So, the unique real root is the trivial root $\lambda = 0$.

The previous proposition gives us the following result for the short run and asymptotic behavior:

Corollary 1 *(i) Unless the initial condition function $H_0(t)$ is constant and equal to H, the investment dynamics have a cyclical, but "asymmetrical" behavior in the short run;*

(ii) For all $H_0(t)$,

$$\lim_{t\to\infty} H(t) = H = \frac{1}{T^* + \alpha D}$$

The previous corollary is a direct result of equation (8) and proposition 4. Given that the characteristic function associated to equation (8) has only $\lambda = 0$ as a real root, the investment dynamics are asymptotically stable and $H(t)$ converges to its steady value.

In contrast with the zero adoption costs model we have obtained two main results. First, when the adoption costs are positive, fluctuations vanish in the long run. Second, convergence to the steady state is cyclical, but shows some irregular patterns; this asymmetric behavior is a consequence of the existence of a second delay, that is, the adoption time, and depends on the size of the adoption costs.

Equation (8) lets us to analyze the role that α and D play in the short run dynamics. The parameter a is an increasing function of α. Given D, the adoption labor requirement measures the weight of the adoption delay in $H(t)$; if α is large (small), the investment behavior is driven mainly by the investment carried out D (T^*) periods before. On the other hand, given α, the adoption time gives the frequency of the irregular patterns. The irregular behavior of $H(t)$ depends crucially on the initial conditions. If we establish an increasing (decreasing) initial conditions, the irregular patterns are bigger than if we assume cyclical initial conditions. Since investment at period t is a linear combination of the previous investment D and T periods before, the irregular and convergence behavior is emphasized by the gap between $H(t-D)$ and $H(t-T)$. We report, in the appendix, some examples of the short run dynamics behavior, assuming different initial conditions.

4 Stationary equilibrium and Comparative Statics

This section characterizes the long run equilibrium and gives some comparative statics results. The balanced growth path of this economy is a situation in which the rate of job creation, detrended production and the optimal lifetime of the capital goods are constants, *ie.*, $H(t) = H$, $Y(t) = Ye^{\gamma t}$, $T(t) = T(t+D) = J(t) = T^*$. It is characterized by the following equations:

$$1 = \frac{1-e^{-rT^*}}{r} - \frac{e^{-\gamma T^*} - e^{-rT^*}}{r-\gamma} - \frac{\alpha e^{-\gamma T^*}(1-e^{-(r-\gamma)D})}{r-\gamma} \tag{13}$$

$$H = \frac{1}{T^* + \alpha D} \tag{14}$$

$$Y = \frac{H(1-e^{-\gamma T^*})}{\gamma} \tag{15}$$

Using the equations above, we can establish the following comparative statics results:

(i) $\dfrac{\partial T^*}{\partial D} > 0;\quad \dfrac{\partial T^*}{\partial \alpha} > 0;\quad \dfrac{\partial T^*}{\partial \gamma} < 0$

As we have proved in proposition 2, an increment in the adoption period will increase the associated labor costs; then, the optimal lifetime of machines tends to increase in order to recoup the additional costs. A raise in the amount of labor required to implement a new technology also rises the adoption costs and the lifetime of machines.

The inverse relation between T^* and γ is standard in vintage capital models; it is the typical obsolescence effect, reflecting that an increment in the rate of technological progress makes replacement more profitable. Since new technology is embodied in new capital and investment is irreversible, faster rate of technical progress encourages the machine replacement process in order to get rid of old capital and replace it with new and more productive kinds. Adoption costs do not take a relevant role on the obsolescence effect: as $(\alpha,\ D)$ are exogenous and constants, they are considered by the planner as an increment on the total labor costs.

(ii) $\dfrac{\partial H}{\partial D} < 0; \quad \dfrac{\partial H}{\partial \alpha} < 0, \quad \dfrac{\partial H}{\partial \gamma} > 0$

A rise in D (α) has two negative effects on the detrended investment. First, there is a direct effect: adoption costs increase labor costs and discourage investment. Second, there is an indirect effect which reinforces the direct one: adoption costs increase labor costs and augment the optimal lifetime T^*, then job creation should diminish to check the labor market equilibrium condition, and then investment should decrease as well.

An increase in the rate of technical progress speeds up the job creation process: faster technical progress decreases the optimal age of the machines and consequently, in equilibrium, job creation must increase.

(iii) $\dfrac{\partial Y^*}{\partial D} < 0; \quad \dfrac{\partial T^*}{\partial \alpha} < 0; \quad \dfrac{\partial Y}{\partial \gamma} < 0$

An increment in D (α) has two opposing effects on detrended output. It increases the optimal lifetime and tends to raise detrended output. Moreover, there is a negative effect working through a reduction of the detrended investment, pushing toward an increase in detrended output. As we show in the appendix, the total effect is unambiguously negative and adoption costs decrease the output level.

An increase in γ has three competing effects on detrended output:

$$\frac{\partial Y}{\partial \gamma} = \frac{1 - e^{-\gamma T^*}}{\gamma}\frac{\partial H}{\partial \gamma} + He^{-\gamma T^*}\frac{\partial T^*}{\partial \gamma} + \frac{H(\gamma T^* e^{-\gamma T^*} - 1 + e^{-\gamma T^*})}{\gamma^2}$$

First, there is a positive effect which works in the direction of increasing the equilibrium job creation rate, and hence augmenting detrended output. Second, a rise in the rate of technical progress decreases the optimal lifetime and has a negative effect on detrended output. Finally, given H and T^*, an increment in γ reduces the long run output. The positive effect, due to a rise in the creation of production units, is totally compensated by the two negative effects and faster technical progress rate has a negative effect on detrended output.

4.1 Adoption costs and technological progress

The process of implementing new technologies is uncertain in nature and the best way to model it is from a Bayesian perspective (Jovanovic and Niarko, 1995). However, trying to do this in a general equilibrium framework is a daunting task, and general equilibrium models deal with this problem assuming a mechanistic process. Greenwood and Yorukoglu (1997) stress the difficulty of modelling the adoption process in an endogenous manner, and assume that as the rate of technological process increases, the more costly it becomes to adopt the new technology since enterprises will be less familiar with it.

We consider the same approach as Greenwood and Yorukoglu (1997) and assume that adoption costs depend positively on the rate of technical progress. We think in an economy which does not innovate, it simply adopt technologies invented abroad, and better technologies are more difficult to implement and lead to higher adoption costs. We have pointed out that in vintage capital models with constant adoption parameters, as the technical growth rate increases, the obsolescence effect makes the replacement of the oldest machine profitable. We study the robustness of this result when the efforts required to adopt a new technology are positively correlated with the rate of technical progress. As we see, technological progress can hold up machine replacement if the adoption costs are sufficiently large.

We consider the following adoption cost function, $C(\alpha(\gamma), D)$; it implies that the adoption period is constant whereas the adoption labor requirement depends on the rate of technical progress[4].

Assumption 2 $\alpha'(\gamma) > 0, \;\; \alpha''(\gamma) > 0$

Assumption 2 states that faster technical progress makes the technological implementation process more difficult. The convexity assumption is required to reflect that faster technical growth rates lead to larger differences between

[4]The same results are obtained if $C(\alpha, D(\gamma))$.

technologies of different vintages, and so, implementation of the latest technologies is costly.

Proposition 5 *1) If the technical progress rate γ, the additional labor requirement α, or the duration of the adoption phase D, are sufficiently small, then $\frac{\partial T^*}{\partial \gamma}$ becomes strictly negative.*

2) If the adoption process is sufficiently costly, $\exists\ \gamma'$ so that $\frac{\partial T^*}{\partial \gamma} > 0\ \forall \gamma > \gamma'$. Besides, γ' is a decreasing function in the adoption period, D.

We differentiate equation (13) taking into account that α depends on the rate of technical progress:

$$\frac{\partial T^*}{\partial \gamma} = \frac{\partial T^*}{\partial \gamma_{|C}} + \frac{\partial T^*}{\partial \alpha(\gamma)} \frac{\partial \alpha(\gamma)}{\partial \gamma} \tag{16}$$

Technological progress also affects the optimal lifetime through the adoption cost function. As we can see, equation (16) clearly shows two opposing effects.

First, there is a *direct destruction creative effect* which decreases the age of capital. Given the adoption costs, an increase in the technical growth rate implies faster obsolescence and a decrease in the age of capital. The direct destruction effect decreases with γ since T^* is a decreasing and convex function of γ.

Second, there is an *adoption effect*, which increases the lifetime of capital: an increment in the growth rate of technological progress raises the additional labor requirement and increases T^* as it requires time to recoup the larger adoption costs.

The adoption effect is given by:

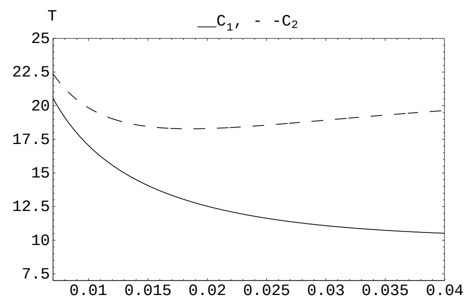
$$\frac{\partial T^*}{\partial \alpha(\gamma)} \frac{\partial \alpha(\gamma)}{\partial \gamma} = \frac{(r-\gamma)\alpha'(\gamma)(1-e^{-(r-\gamma)D})}{(r-\gamma)\gamma[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}]}$$

and it depends mainly on the adoption cost function; the adoption period and the adoption labor requirements reinforces the adoption effect. The total effect is ambiguous and depend mainly on the adoption costs function.

Proposition 5 shows that technological progress can delay the machine replacement process when adoption costs are sufficiently large. Since adoption costs and technical progress are treated jointly, the increment in adoption costs pushes for a longer lifetime of capital in order to recoup the additional costs. The inherent creative destruction effect is totally or partially compensated by the adoption effect, and faster technical progress may discourage the substitution of the older technologies for the leader one.

If we think that different adoption costs across countries reflect different skills or different institutional barriers, the previous proposition rises up an interesting property of the model: developed countries, with a free-fluid foreign trade and skill intensive labor, can compensate the negative adoption effect and faster growth accelerates technological substitution. However, in developing countries which confront larger adoption efforts, the predominant effect is the adoption one, and technical progress discourages machine replacement leading to technological sclerosis.

We deal with a numerical example to compare two economies with different adoption cost functions $(C_i(\alpha_i(\gamma), D_i)$, $i = 1, 2$ to depict a developed $(i = 1)$ and a developing economy $(i = 2)$. According to Greenwood and Yorukoglu (1997), adoption costs may amount about 10% GDP in developed countries. We set the following adoption labor requirement functions $\alpha_i(\gamma) = a_i\gamma^b$, with $a_1 = 300$, $a_2 = 700$, $b = 2$, $D_1 = 2$ years, $D_2 = 5$ years. With $r = 0.05$ and $\gamma = 0.03$, the ratio adoption costs over GDP is about 6% (12%) in the developed (developing) economy. The following Figure shows the relationship between γ and the optimal lifetime of machines. In the developed economy, the optimal lifetime of the machines is a decreasing function of the technical progress rate. The obsolescence effect totally compensates the adoption effect regardless of the value of γ. In contrast, in the developing economy the obsolescence effect predominates for low values of γ; but, as the technical growth rate increases, the predominant effect is the adoption one and the machine replacement process is delayed.



Note that the relationship between the rate of technical progress and the lifetime of capital depends mainly on the size of the first derivative of the adoption cost function (the adoption effect tends to zero when $\alpha'(\gamma)$ tends to zero). Further numerical examples show that the bigger $\alpha'(\gamma)$, the smaller is the adoption period and the technical growth rate required to break down the obsolescence effect.

5 Adoption of dominated technologies

In the previous sections we have assumed that adoption and investment costs are independent of the adopted technology and grow at rate γ. As a consequence the model predicts that all economies invest in the latest vintages whenever investments are made, and countries facing up to higher adoption costs differ only in the frequency of adoptions. A deficiency of these kinds of models is that there are not consistent with the stylized fact that developing countries do not operate with the latest technologies used in the developed ones.

There are many reasons whereby poor countries invest in older technologies. Regulatory and legal constraints, bribes and import tariffs, which have the effect of increasing the cost of technology adoption, can affect to the optimal investment decisions. Hence, the literature of vintage technology emphasizes the role of technology specific skills; different technologies may require completely different skills (Evenson and Westphal, 1994; Keller, 1994), and the skill factor may constraint the choice between technologies as far as new machines embody an increasing level of technological sophistication. As a result, the different investment-specific decisions taken by different countries give rise to a technological gap, recently studied in the empirical literature.

In this section we continue exploring the role of adoption costs on technological substitution, but we take into account the existing trade-off between investing in newer technologies and the costly consequences of these investment decisions. We also assume that there is a frontier technology at each date that increases exogenously at rate γ, but we introduce two important differences with respect to the theoretical framework studied in section 2. First, the investment cost is proportional to the productivity of the acquired technology. Second, the cost of technology adoption depends on how close the adopted technology is to the technological frontier, *ie.* adoption costs depend on the difference between the leader and the adopted technology. We assume that machines embodying older technologies have lower adoption costs; the underlying idea is that the older a technology is, the more public information available about its use, the less an economy must spend in implementing and learning about it.

We denote by $B(t)$ the technological delay in t, *ie.* the machine's relation to the technological frontier at the time of adoption. Then in period t, the economy adopts the capital goods created in $t - B(t)$, with an associated productivity $e^{\gamma(t-B(t))}$. We assume that the adoption period D is constant, whereas the labor requirement devoted to adoption depends on the technological delay, $\alpha(B(t))$; since we assume that older technologies have lower adoption costs, $\alpha'(B(t)) < 0$. Following Barro and Sala-i-Martin (1997), we define the technological gap at period t as the fraction between the productivity of the leader technology and the adopted one, $TG(t) = \frac{e^{\gamma t}}{e^{\gamma(t-B(t))}} - 1 = [e^{\gamma B(t)} - 1]$, which is fully determined by $B(t)$; unless the economy adopt the latest technology whenever it carries out the technological replacement process, the technological gap is positive.

Now, the central planner solves the following problem:

$$\max_{\{Y(t),H(t),B(t),T(t),J(t)\}} \int_{t=0}^{\infty} [Y(t) - i(t)]e^{-rt} dt$$

subject to

$$\begin{aligned} Y(t) &= \int_{t-T(t)}^{t} H(s)e^{\gamma(s-B(s))} ds && (17) \\ 1 &= \int_{t-T(t)}^{t} H(s)ds + \int_{t-D}^{t} \alpha(B(s))H(s)ds && (18) \\ i(t) &= H(t)e^{\gamma(t-B(t))} \\ 0 &\leq i(t) \leq Y(t) \\ B(t) &\geq 0 && (19) \end{aligned}$$

given $H_0(t)$ for all $t < 0$. This problem is equivalent to that solved in section 2.1, apart from the additional endogenous variable, $B(t)$: each period the planner chooses, among the invented ones, which technology to adopt, taking into account the adoption and the investment costs. Since the economy only can adopt technologies already invented, $B(t) \geq 0$. Note that an implicit requirement of the model is that the adopted technology must be superior to the scrapped one; this issue is addressed later on.

The Lagrangian function associated to the planner's problem is given in the appendix. We focus on the first order condition which characterize the interior solution:

$$\phi(t) = 1 \quad \forall t$$

$$w(t + J(t)) = e^{\gamma(t-B(t))} \tag{20}$$

$$e^{\gamma(t-B(t))} = \int_t^{t+J(t)} \left[\phi(s)e^{\gamma(t-B(t)} - w(s)\right] e^{-r(s-t)}ds - \alpha \int_t^{t+D} w(s)e^{-r(s-t)}ds \tag{21}$$

$$\gamma \int_t^{t+J(t)} \phi(s)e^{\gamma(t-B(t))}e^{-r(s-t)}ds = \gamma e^{\gamma(t-B(t))} - \alpha' \int_t^{t+D} w(s)e^{-r(s-t)}ds \tag{22}$$

Equation (20) is an exit condition with states that a production units adopted at period t, with an associated productivity of $e^{\gamma(t-B(t))}$, will be replaced when its productivity does not cover the worker's reservation wage, $w(t + J(t))$. Equation (21) is the entry condition which corresponds to the optimal investment rule, and equation (22) is the optimal condition for the technological delay: it requires that the marginal revenue of decreasing the adoption lag equalizes the marginal cost of investment and the additional adoption costs.

We restrict our analysis to the stationary equilibrium of our economy.

Definition 2 *Given the adoption time $D \geq 0$, and given the initial conditions $H_0(t)$, $\forall t < 0$, a stationary equilibrium for our economy is a situation in which the rate of job creation, detrended output, the optimal lifetime of capital, the technological delay and the technological gap are constants, ie. $H(t) = H$, $Y(t) = Ye^{\gamma t}$, $T(t) = J(t) = T$, $B(t) = B$, $TG(t) = TG$.*

The stationary equilibrium is given by the following equations:

$$1 = \frac{1 - e^{-rT}}{r} - \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma} - \frac{\alpha(B)e^{-\gamma T}[1 - e^{-(r-\gamma)D}]}{r - \gamma} \tag{23}$$

$$1 = \frac{1 - e^{-rT}}{r} + \frac{\alpha'(B)e^{-\gamma T}[1 - e^{-(r-\gamma)D}]}{\gamma(r - \gamma)} \tag{24}$$

$$Y = \frac{He^{-\gamma B}[1 - e^{-\gamma T}]}{\gamma}$$

$$1 = HT + \alpha(B)HD$$

$$TG = e^{\gamma B} - 1$$

It is straightforward to verify that although the economy incurs in a technological delay in the long run, it always adopt more productive technology than the scrapped one. Notice that the optimal lifetime of the capital goods and technological delay remain constants in the long run, then, $e^{-\gamma B} > e^{-\gamma(T+B)}$.

We next study the existence of a balanced growth path. This problem is reduced to check the existence of a pair $(T,\, B)$, which solves equations (23) and (24). Equation (23) is obtained from the free entry condition taking into account the shadow price of labor. It defines a negative relation between optimal lifetime of machines and the technological delay; as B increases, the embodied productivity of the adopted capital goods and the period during which the planner finds profitable to keep in use the adopted technology before replacing it decreases. Equation (24) describes the first order condition with respect to the technological delay and its slope is determined by $\alpha''(B)$.

Assumption 3 *(i)* $\alpha(B) \in [0, \alpha_0]$; $\alpha'(B) < 0$, $\alpha''(B) > 0$.
$(ii) |\alpha'(0)| > \frac{\gamma[1+\alpha(0)(1-e^{-(r-\gamma)D})]}{(1-e^{-(r-\gamma)D})}$

Proposition 6 *A balanced growth path exists if and only if assumption 3 is checked.*

The proof is given in the appendix.

Assumption 3 restricts the values of α and the behavior of $\alpha(B)$ in a reasonable way. Adoption costs are a decreasing and convex function of the technological delay. The older a technology is, the more public information about its use and the lower marginal gains of delaying adoption. We denote by α_0 the additional labor requirement when the economy acquires the leader capital good. Hence, when the economy adopt a technology enough far from the leader one, the adoption requirements tends to zero, *ie.* it exists $B_{\max}$ such that $\alpha(B_{\max}) \to 0$.

It also imposes a necessary condition on the behavior of $\alpha'(0)$ for a stationary equilibrium to exist. As $\alpha''(B) > 0$, equation (24) defines an implicit function $T = f(B)$, with $f'(.) < 0$: an increase in B, reduces the marginal costs of adoption and then T should decrease in order to diminish marginal revenues to adoption. The second part of assumption 3 is required to verify that both decreasing function intercept once.

The existence of an interior solution implies that it is optimum not to invest in the leader technology when implementation costs are heterogeneous across the technological menu. The existence of a positive technological delay is not only suitable for the study of developing countries; it is well known that the largest part of *R&D* in many sectors, information technologies among others, is undertaken in USA, and the European countries adopt technologies already invented with a small delay. In order to deal with some numerical comparative statics exercise, we set the following specification of the adoption function, which satisfies assumption 3:

$$\alpha(B) = \frac{\alpha_o}{\log[e + B]}$$

where α_0 denotes the additional adoption labor requirement when the economy adopts the leader technology. We display below the most interesting sensitivity analysis results, namely those for the parameters D, and γ[5]. We compute the deviations from the initial steady state (in percentage) for the more relevant variables.

$\triangle D$	T	B	TG
1%	0.11	0.66	0.68
5%	0.54	3.2	3.35

$\triangle\gamma$	T	B	TG
1%	-0.26	-0.27	0.23
5%	-2.5	-2.6	2.28

A larger adoption period induces a longer lifetime of capital and a bigger technological delay; consequently the technological gap increases. A rise in D increases the adoption costs and then T rises in order to recoup the additional adoption costs induced by a longer adoption period. An increment in D has two opposing effects of the technological delay; first, higher D tends to augment B to reduce adoption costs; second, a longer lifetime of capital should be compensated with a smaller technological delay (an increase in the productivity of the adopted technology) in order to be profitable to use the adopted technology for a longer period of time. The main difference with the exogenous adoption parameters model is the existence of another channel to adjust the long run variables after the shock: the technological adoption decision, *via* B, has an indirect effect on the optimal lifetime of capital. There exists a trade-off between adopting an older technology and using it for a smaller period of time. As it is shown in the previous table the economy reacts incurring in a larger technological delay in order to reduce the negative effect on T.

We next investigate the long run implications of a technological acceleration. A faster rate of technical progress decreases the lifetime of machines

[5]Setting $r = 0.05$, $\gamma = 0.02$, $D = 3$, $\alpha_0 = 0.25$, we obtain the following stationary values: $T = 13.5$, $B = 2.28$, $TG = 0.047$

and the technological delay, but has a positive effect on the technological gap. Technical progress makes replacement more profitable and reduces the optimal lifetime of machines. Higher γ increases the reservation wage inducing to a rise in the technological delay; however, faster γ also leads to an increment in capital productivity and stimulates the adoption of better technologies to take advantage of the productivity improvements. The numerical results suggests that the wage effect is totally compensated by the productivity effect, and the economy reacts decreasing B and adopting better technologies. Despite the economy reduces the technological delay, the technological gap increases; higher γ rises the productivity differentials across technologies augmenting the equilibrium technological gap.

The model predicts a negative role of adoption costs on the technological delay and on the technological replacement. The positive effect on the lifetime of capital induced by an increment on the adoption period is smoothed by a bigger technological delay. However, it is not totally compensated, since it would imply negative consequences on productivity and hence on the long run level of output. In contrast, technical progress encourages machine replacement and the adoption of better technologies, although the effect on the technological gap is negative.

These numerical findings are quite consistent with the empirical adoption literature. Navaretti et *al.* (1999) models the firm's choice between new and used technologies (that can be interpreted as dominated technologies) assuming that machines last for two periods. They analyze U.S. export data on some types the vehicles, equipment and machinery, and find that the use of dominated technologies is higher the lower the level of development of the importing country and the faster technical change. In Jaumotte (1999) a sample covering 63 developing countries is analyzed, and the main finding is that human capital plays a decisive role in the absorption of technologies. Our model predicts that the technological delay is higher the bigger the adoption cost (as a proxy of the level of development of the country), and hence the lower the frequency of adoptions. In contrast we obtain that faster technical change encourages the adoption of better technologies reducing the technological delay. This result may crucially depends on the homogeneous labor assumption: we lose the specific-investment constraint due to the skills requirements to run a better machine efficiently.

6 Conclusion

In this paper, we incorporate costly adoption in a Ramsey growth vintage capital model with linear utility and Leontieff technology, in order to analyze the effect of adoption costs on the dynamics properties of the model; Moreover, we study the consequences of a costly process of implementing new technologies, already invented, in the replacement process. We find that adoption costs increases the lifetime of machines and delay the technological substitution. This is a very good property of the model since adoption costs have been invoked to explain the technological sclerosis and the higher age of capital in developing countries. The obsolescence effect is a well known feature of creative destruction models: faster growth induces a faster machine replacement, in order to take advantage of the new and more productives capital goods. We prove that when adoption costs depend positively on the technical growth rate, the obsolescence effect can hold up, depending on the size of the adoption costs. Countries with lack of skilled labor or technological barriers (as a proxy of adoption costs), delay the replacement process since the adoption effect compensates the obsolescence one. The problem of costly adoption is more crucial for developing countries which typically lack capital and skill. We extend the model in order to be consistent with the stylized fact that developing countries do not operate with the latest technologies. We found that when adoption costs depend on the adopted technology, it is optimum to incur in a technological delay in order to take advantage of the lower implementation costs. The model predicts a negative role of adoption costs on the technological delay and on the machine substitution, whereas technical progress encourages machine replacement and the use of better technologies. Obviously, it would be highly interesting to check these results in a more complex model, including heterogeneous labor and technology specific skills in the adoption side of the economy.

As theoretically shown by Boucekkine et *al.* (1997), the RVKM with Leontieff technology and lineal utility yields periodic solutions paths for detrended investment and production, beginning at a finite date. This result comes mainly from the fact that the optimal scrapping rule is constant under the previous assumptions. With the adoption function assumed in section 2, we prove that although the optimal scrapping rule remains constant, the introduction of costly adoption distorts the equilibrium dynamics as long as they involve a different timing with respect to the main creation and destruction decisions. As a consequence, fluctuations vanish in the long run and the convergence to the steady state is cyclical, but shows some irregular patterns.

The main implication of the presence of the technological delay in section 5 is that we can not assured that the optimal scrapping rule $T(t)$ (and consequently $J(t)$) is constant for any $t \geq 0$. In order to analyze the dynamics properties, it is required to solve a mixed-delay integro differential equation system with endogenous leads and lags. We got over the analysis of the convergence properties to the steady state and we have restricted our analysis to the steady state equilibrium. Given that an important economic issue is the technological catch-up experienced by developing countries (conditional on their steady state technological gaps), a natural extension of this research is to handle with the technical difficulties and to analyze the role of the optimal scrapping rule and the technological delay in the technological convergence process.

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8 Appendix

1. Proof of Proposition 2

$$\frac{\partial T^*}{\partial D} = \frac{\alpha(r-\gamma)e^{-(r-\gamma)D}}{\gamma\left[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}\right]} > 0$$

Under assumption 1, the optimal lifetime increases with D. To establish the concavity result, much more tedious computations are needed. Let us define the function $Q(.,.)$ in the following way:

$$Q(D,T^*) = \frac{1-e^{-rT^*}}{r} - \frac{e^{-\gamma T^*}-e^{-rT^*}}{r-\gamma} - \frac{\alpha e^{-\gamma T^*}(1-e^{-(r-\gamma)D}}{r-\gamma} - 1 = 0$$

if we denote by $f(.)$ the functional relation between T^* and D, $T^* = f(D)$, the second order derivative should satisfy

$$f''(D) = -\frac{1}{(Q_2)^3}\left[Q_{11}Q_2^2 + Q_{22}Q_1^2 - 2Q_1Q_2Q_{12}\right]$$

with

$$Q_1 = -\alpha e^{-\gamma T^*}e^{-(r-\gamma)D} < 0$$

$$Q_2 = \frac{\gamma e^{-\gamma T^*}\left[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}\right]}{r-\gamma} > 0$$

$$Q_{11} = \alpha(r-\gamma)e^{-\gamma T^*}e^{-(r-\gamma)D} > 0$$

$$Q_{12} = \alpha\gamma e^{-\gamma T^*}e^{-(r-\gamma)D} > 0$$

$$Q_{22} = \frac{-\gamma^2 e^{-\gamma T^*}\left[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}\right]}{r-\gamma} + \gamma e^{-\gamma T^*}e^{-(r-\gamma)D}$$

Now observe that

$$Q_{22}Q_1^2 - 2Q_1Q_2Q_{12} = \alpha^2\gamma e^{-3\gamma T^*}e^{-2(r-\gamma)D}\left[e^{-(r-\gamma)T^*} + \frac{[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}]}{r-\gamma}\right]$$

is bigger that zero. As $Q_{22}Q_1^2 > 0$ and $Q_{22}Q_1^2 - 2Q_1Q_2Q_{12} > 0$, we do get $f'(D) < 0$

2. Comparative Static Results (i) Differentiating equation (13) with respect to the adoption cost parameters we obtain:

$$\frac{\partial T^*}{\partial D} = \frac{\alpha(r-\gamma)e^{-(r-\gamma)D}}{\gamma\left[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}\right]} > 0$$

$$\frac{\partial T^*}{\partial \alpha} = \frac{1-e^{-(r-\gamma)D}}{\gamma\left[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}\right]} > 0$$

Differentiating equation (13) with respect to γ gives:

$$\begin{aligned}\frac{\partial T^*}{\partial \gamma} &= \frac{1-e^{-(r-\gamma)T^*}-T^*(r-\gamma)}{(r-\gamma)\gamma\left[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}\right]} + \\ & \frac{\alpha\left[1-e^{-(r-\gamma)D}-D(r-\gamma)e^{-(r-\gamma)D}-T^*(r-\gamma)(1-e^{-(r-\gamma)D}\right]}{(r-\gamma)\gamma\left[1-e^{-(r-\gamma)T^*}+\alpha(1-e^{-(r-\gamma)D}\right]}\end{aligned}$$

As $1-e^{-x}-x<0$ and $[1$ - $e^{-(r-\gamma)D}$-$D(r-\gamma)e^{-(r-\gamma)D}$ - $T^*(r-\gamma)(1-e^{-(r-\gamma)D})]$ is a decreasing function of T^* and is zero for $T^*=0$, we obtain a negative relation between the technical progress rate and the lifetime of machines.

(ii) Differentiating equation (14):

$$\begin{aligned}\frac{\partial H}{\partial D} &= -\frac{\left(\frac{\partial T^*}{\partial D}+\alpha\right)}{H^2} < 0 \\ \frac{\partial H}{\partial \alpha} &= -\frac{\left(\frac{\partial T^*}{\partial \alpha}+T\right)}{H^2} < 0 \\ \frac{\partial H}{\partial \gamma} &= -\frac{1}{H^2}\frac{\partial T^*}{\partial \gamma} > 0\end{aligned}$$

(iii) We first check that adoption costs affect detrended output in a negative way. Replacing H in equation (15), and differentiating with respect to the adoption parameters:

$$\frac{\partial Y}{\partial D} = \frac{1}{\gamma(T^*+\alpha D)}\left[(\gamma e^{-\gamma T^*}(T^*+\alpha D)-1+e^{-\gamma T^*})\frac{\partial T^*}{\partial D}-\alpha(1-e^{-\gamma T^*})\right]$$

$$\frac{\partial Y}{\partial \alpha} = \frac{1}{\gamma(T^*+\alpha D)^2}\left[(\gamma e^{-\gamma T^*}(T^*+\alpha D)-1+e^{-\gamma T^*})\frac{\partial T^*}{\partial \alpha}-D(1-e^{-\gamma T^*})\right]$$

As $\frac{\partial T^*}{\partial D}$, $\frac{\partial T^*}{\partial \alpha}>0$, and the function $[\gamma e^{-\gamma T^*}(T^*+\alpha D)$ - 1 + $e^{-\gamma T^*}]$ takes the value 0 for $T^*=0$ and decreases for $T^*>0$, then $\frac{\partial Y}{\partial D}$, $\frac{\partial Y}{\partial \alpha}>0$.

To check the negative relation between technical progress growth rate and detrended output, we first define $x = \gamma T^*$, and we rewrite Y as a function of x and γ:

$$Y = \frac{1 - e^{-\gamma T^*}}{\gamma T^* + \alpha \gamma D} = \frac{1 - e^{-x}}{x + \alpha \gamma D} = G(x(\gamma), \gamma)$$

then

$$\frac{\partial Y}{\partial \gamma} = \frac{\partial G}{\partial x}\frac{\partial x}{\partial \gamma} + \frac{\partial G}{\partial \gamma}$$

Differentiating G with respect to γ, and x, we obtain an inverse relation in both cases:

$$\frac{\partial G}{\partial \gamma} = -\frac{\alpha D(1 - e^{-x})}{(x + \alpha \gamma D)^2} < 0$$

$$\frac{\partial G}{\partial x} = \frac{e^{-x} + x e^{-x} - 1 + \alpha \gamma D e^{-x}}{(x + \alpha \gamma D)^2} < 0$$

we need to prove that $\frac{\partial x}{\partial \gamma} > 0$; that is, γT^* is increasing with respect to γ.

$$\frac{\partial (\gamma T^*)}{\partial \gamma} = T^* + \gamma \frac{\partial T^*}{\partial \gamma}$$

developing the previous equation, we find that its sign depends on the sign of the following expression:

$$\left[1 - e^{-(r-\gamma)T^*} - T^*(r-\gamma)e^{-(r-\gamma)T^*}\right] + \alpha \left[1 - e^{-(r-\gamma)D} - D(r-\gamma)e^{-(r-\gamma)D}\right]$$

which is zero for $T^* = 0$, and in increasing in T^*, then $\frac{\partial Y}{\partial \gamma} < 0$

3. Lagrangian associated to the planner's problem (section 5)

$$\begin{aligned}
L(t \quad = \quad & \int_{t=0}^{\infty} [Y(t) - H(t)e^{\gamma(t-B(t))} - \phi(t)Y(t) + \omega(t)]e^{-rt}dt + \\
& \int_{0}^{\infty} H(t)\left[\int_{t}^{t+J(t)} \left(\phi(s)e^{\gamma(t-B(t))} - \omega(s)\right) e^{-r(s-t)}ds\right] e^{-rt}dt - \\
& \int_{0}^{\infty} H(t)\left[\int_{t}^{t+D} \alpha(B(t)\omega(s)e^{-r(s-t)}ds\right] e^{-rt}dt + \\
& \int_{-T(0)}^{0} H(t)\left[\int_{0}^{t+J(t)} \left(\phi(s)e^{\gamma(t-B(t))} - \omega(s)\right) e^{-r(s-t)}ds\right] e^{-rt}dt - \\
& \int_{-T(0)}^{0} H(t)\left[\int_{0}^{t+D} \alpha(B(t)\omega(s)e^{-r(s-t)}ds\right] e^{-rt}dt
\end{aligned}$$

4. Proof of proposition 6 The steady state equilibrium values of (T,B) are obtained from the following equations:

$$F(T,B) = \frac{1-e^{-rT}}{r} - \frac{e^{-\gamma T}-e^{-rT}}{r-\gamma} - \frac{\alpha(B)e^{-\gamma T}[1-e^{-(r-\gamma)D}]}{r-\gamma} - 1 = 0 \quad (25)$$

$$G(T,B) = \frac{1-e^{-rT}}{r} + \frac{\alpha'(B)e^{-\gamma T}[1-e^{-(r-\gamma)D}]}{\gamma} - 1 = 0 \quad (26)$$

We denote by $f(.)$,($g(.)$) the functional relation between T and B given by $F(T,B)$ $(G(T,B))$; it is easily checked that both functions define a decreasing relation between the optimal lifetime of machines and the technological delay:

$$f'(T) = \frac{\alpha'(B)[1-e^{-(r-\gamma)D}]}{\gamma[1-e^{-(r-\gamma)T}+\alpha(B)(1-e^{-(r-\gamma)D})]} < 0$$

$$g'(T) = -\frac{\alpha'(B)[1-e^{-(r-\gamma)D}]}{\gamma(r-\gamma)e^{-(r-\gamma)T}-\alpha'(B))[1-e^{-(r-\gamma)D}]} < 0$$

When $B \to 0$, under assumption 3, G curve is above F curve, and when $B \to B_{\max}$, G curve is below F curve, then both functions intercept once, and the steady state is unique.

5. Short Run Dynamics of Investment

In all the figures we set $r = 0.05$, $\gamma = 0.03$, and we vary D, and α to isolate the effect of each one of them in the dynamics of $H(t)$. In order to show the effect of adoption costs on the dynamics properties, we also plot the zero adoption costs case. In figure 2.1, we assume cyclical initial conditions: $H_0(t) = a + b\cos(\frac{2\pi t}{T})$; In figure 2.2 we set the following increasing initial conditions: $H_0 = ce^{dt}$

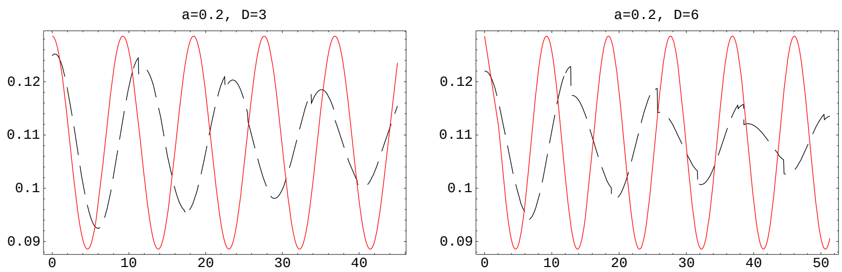

Figure 1: Detrended investment under cyclical initial conditions.

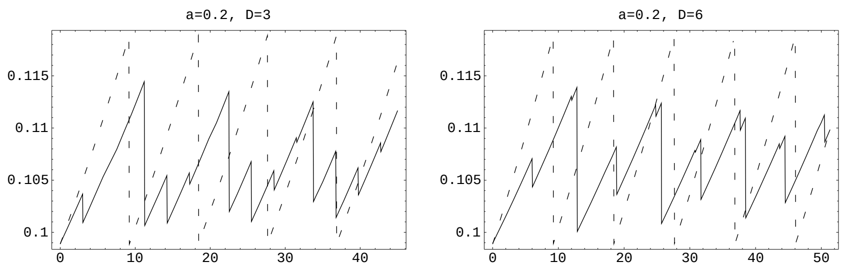

Figure 2: Detrended investment under increasing initial conditions.