Why the Kuznets Curve Will Always Reverse [*]

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Abstract

In this paper, we develop a model of innovation-based growth to address the issue of skill-biased technical change over the long run. We show that innovations fluctuate endogenously from skill-intensive to unskilled-intensive sectors, thereby generating periods of increasing and decreasing wage inequality. This could contribute to explain that technological progress exerts a non monotonic pressure on wage inequality over the long run.

Keywords: Innovation-Driven Growth, Wage Inequality, Kuznets Curve, Cycles.

JEL Classification: J31, O31, O41.

Résumé

Dans cet article, nous developpons un modèle de croissance avec innovations en vue de rendre endogène le sens du biais de progrès technique. Nous montrons que les innovations ne se produisent pas systématiquement dans le secteur qualifié et non qualifié. Au contraire, la nature des innovations change, entraînant des périodes d'accroissements et de reductions des inégalités de salaires, ce qui permet de reproduire l'évolution des inégalités aux Etats-Unis au cours du vingtième siècle.

Mots clefs: Croissance fondée sur l'innovation, Inégalités de salaires, Courbe de Kuznets, Cycles.

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1 Introduction

Many have seen in the recent coincidence of computerization and widening wage inequality a skill-biased revolution, in the form of technology-skill or technology-ability complementarity. This technological bias has shed a contradicting light on the Kuznets curve, according to which, along the process of development, income inequality initially increases but then declines. This suggests that wage inequality was largely cyclical over the twentieth century.

Figure 1 reproduces the evolution of the wage ratio of starting engineers to average low-skilled workers in the beginning of the twentieth century in the U.S., using the data provided by Goldin and Katz (1999). We see that the wage differential between skilled and unskilled labor declined in the 1910s, increased in the 1920s, and decreased again in the 1930s and the 1940s. The evolution of the wage structure in the U.S. in the second half of the century also is consistent with a cyclical picture. The wage differential between skilled and unskilled workers widened in the 1950s and the 1960s, narrowed until the 1970s, and then widened until the 1990s (see Autor, Katz and Krueger (1998), Goldin and Margo (1992), Katz and Murphy (1992)). The cyclical pattern is all the more striking as the extent of wage inequality was sometimes as high as in the 1980s (or even higher in the 1940s according to Goldin and Katz (1998)).

Figure 1: Wage ratio of starting engineers to average low-skilled workers in the U.S. (Source: our own computations, from Goldin and Katz (1999))

The purpose of this paper is to provide a theoretical explanation for the cyclical behavior of the wage structure observed in the US over the 20^{th} century. In the skill-biased technical change literature, several papers address this issue (representative approaches of this literature are for instance Galor and Moav (2000), Caselli (1999), Acemoglu (1998), Aghion and Howitt (1998), and Galor and Tsiddon (1997)). Basically, these approaches consider the long-run interaction between technology and skills, and it is the changing nature of this interaction over time that explains the non monotonic behavior of the wage structure.

From this point of view, Caselli (1999) analyzes revolutionary technological change that can be either skill-biased or de-skilling. A technological revolution is the adoption of a new type of machines, which by essence consists in a radical technical change. It is skill-biased if the skills required by the new machines are more costly to acquire than the skills needed to operate previous machines. These revolutions increase the skill premium because they lead to higher capital-labor ratio in high skill technological sectors (capital flows from low skill technologies to high skill ones) and because these technologies are more productive. Whether inequality persists or not then depends on the number of people who upgrade their skills. If there is incomplete upgrading because individuals find that learning costs are too high, the adoption of new technologies is very slow and wage inequality persists for long time after the arrival of the technological revolution. Otherwise, the adoption of the leading edge technology is universal and wage inequality reverses downwards very rapidly. Conversely, a de-skilling technological revolution implies a more or less persistent reduction of the skill premium.

Closer to the issue of cycles in wage inequality, Galor and Tsiddon (1997) propose a model in which the life cycle of technology governs the evolution of wage inequality. Along this life cycle, the nature of the returns to human capital (including ability and parental human capital) changes, thereby exerting a non monotonic pressure on the wage structure. More precisely, in periods of major technological progress (that is in periods of technological inventions) the returns to ability increase, driving wage inequality upward. Once existing technologies become more accessible (that is in periods of technological innovations) the role of ability in individual earnings declines, and so does wage inequality. The transition from inventions to innovations implies a cyclical evolution of wage inequality: first it increases with inventions, but then it declines when innovations make technological breakthrough more accessible.

According to these articles, it is the interplay between technological progress and the returns to human capital that governs the evolution of inequality. In Caselli's approach, revolutionary technological changes modify the costs of acquiring skills needed to use new machines, thereby altering the wage structure. In Galor and Tsiddon's model, inventions and innovations exert a different impact on the two components of individual earnings (parental human capital and ability), explaining the non-monotonic evolution of the wage structure over time. These explanations are supported by empirical evidence. For instance, Goldin and Katz (1999) observe that in the history of many inventions, like in the automobile production industry, technological change successively increased and decreased the relative demand for skilled workers.

The literature on the interactions between technical change and the returns to human capital provides a comprehensive analysis of the non-monotonic response of wage inequality to technological revolutions. This paper explores a different but complementary link between innovation and wage inequality. We consider a model where growth is driven by successive radical innovations that improve labor productivity either in the skill-intensive or in the unskilled-intensive sector. We focus on the endogenous determination of the sector that benefits from technological change. For simplicity we consider that a radical innovation has an instantaneous diffusion, so we neglect the interplay between technological progress and the returns to human capital emphasized in the previous literature. The present analysis exhibits the role of patent lifetime as a major source of technological fluctuations. By changing the sector in which radical innovations are introduced, researchers can increase the lifetime of patents. Cycles in technological adoption arise not in response to fluctuations in the returns to human capital but in response to incentives in the research sector. Alternation in the innovative sector indeed reduces the negative effect of creative destruction. We then argue that the wage distribution behaves cyclically because the sectors that alternatively benefit from innovations are of different skill intensities.

The argument that cycles arise due to endogenous technological change is also related to the literature on growth and output fluctuations (e.g. Aghion and Saint-Paul (1991), Caballero and Hammour (1994), Aghion and Howitt (1998), Li (2001)). During contraction (expansion) phases, the risk of obsolescence is low (high) and incentives to engage in research is high (low). Hence, the following period is therefore characterized by a high (low) number of firms in the final output sector. In both approaches, cycles arise because of the negative externality (business stealing effect) that obsolescence exerts on current research. We extend our model in section 5 to include these Schumpeterian sources of fluctuations. The irrelevance of dynamics where innovations always benefits to a single sector is robust to this extension.

Our model builds upon the basic framework of Aghion and Howitt (1992) in which two sectors of intermediate goods are differentiated. Hence, the closest model to ours is Acemoglu (1998). Acemoglu wishes to explain why an increase in the relative supply for skilled labor implies a rise in the skill premium. In his framework, an increase in the supply of skills increases the market size for technologies, thereby rising the relative returns of inventing technologies complementary to skilled labor and driving the skill premium upward. Our ambition is different and is more concentrated on a very-long run issue. Hence, innovations are radical in our model, whereas they are incremental in

Acemoglu's framework.

This paper is organized as follows. Section 2 presents the basic structure of the model. Section 3 defines the competitive equilibrium concept used and shows what determines fluctuation of innovations between sectors. Section 4 analyzes the simple case of a two-cycle dynamics and its impact on wage inequality. Section 5 extends the model to allow for mobility of skilled workers between research and intermediate manufacturing. Section 6 concludes.

2 The model

2.1 Description of the environment

The framework considered is a closed economy where growth is driven by productivity gains associated with successive radical innovations. The economy is composed of a final good sector, two intermediate good sectors and a research sector. The final good is produced in a perfectly competitive environment, using both types of intermediate goods as inputs. Each intermediate good sector is composed of a continuum of firms that monopolistically produce imperfectly substitutable goods.

The outcome of the research process is a radical innovation. Each successful innovator obtains a patent on its radical innovation which enables him to sell without any additional cost a continuum of licences to the intermediate good producers. A radical innovation has different consequences on the productivity of intermediate good sectors, and for simplicity, we restrict our attention to radical innovations which are specific to a single intermediate good sector.

Intermediate good sectors also differ in their skill requirement. The intermediate good sectors are indexed by $i = H, L$. We assume that firms in the L sector only rely on unskilled workers, whereas firms in the H sector only rely on skilled workers. We do this for the sake of simplicity, although smaller differentiation could be more realistic. Hence, we consider that any radical innovation specifically increases the productivity of a single type of workers. This hypothesis has some empirical relevance. For instance, the diffusion of electricity was a radical innovation that induced many incremental innovations (new machines) that has more specifically benefited to unskilled workers during the beginning of the twentieth century. On the contrary, electronics was a radical innovation that led to many incremental innovations (new softwares) that nowadays benefits more specifically to skilled workers.

The research sector is characterized by a patent race and innovations are both radical and uncertain, occurring according to a Poisson process. An innovation in sector $i = H$ or L gives the innovator, instantaneously and without any additional cost, a continuum of licenses, one for each intermediate good j of type i. This simplifying assumption neglects the mechanisms that induce fluctuations of wage inequality through the slow adoption of major radical change emphasized in the previous literature. Time is continuous. Subscript t denotes the number of innovations until the current period [1]. Let A_t^i be the productivity of workers employed in sector i after t innovations. Innovations improve the production processes of intermediate goods that replace the old ones, and raise the technology level (i.e. the labor productivity parameter) in the corresponding intermediate good sector by a factor $1/(1-\gamma)$, with $0 < \gamma < 1$. If the $(t+1)^{st}$ innovation occurs in sector i, then $A_{t+1}^i = \frac{1}{1-\gamma} \cdot A_t^i$ and $A_{t+1}^{-i} = A_t^{-i}$.

2.2 Labor market and resources constraints

The economy is populated by a fixed mass of individuals. A worker can be employed in the skilled intermediate good sector, in the unskilled intermediate good sector or in the research sector. Let N_t^i be the mass of workers employed in the intermediate good sector i and R_t^i be the mass of skilled workers engaged in the research for an innovation directed to sector i.

Two issues arise regarding individuals' decisions. On the one hand, the supply of skills can be either exogenous or endogenous. We will consider the latter, namely that the allocation between skilled and unskilled workers is endogenous. On the other hand, the allocation of workers between research and intermediate productive sectors can be exogenous (no mobility for workers between research and intermediate manufacturing), or endogenous (mobility). Both cases will be considered (no mobility in sections 2, 3 and 4 and mobility in section 5). As will become apparent, this latter case does not change the qualitative results.

Individuals are risk-neutral, discount time at the exogenous rate r and can neither save nor borrow. Let w_t^H and w_t^L denote the wages of skilled and unskilled workers (in the intermediate goods sectors). For now, we consider that workers are exogenously either productive or researchers. All productive workers x face at any point in time an occupational choice. They can supply a single unit of labor either in the sector H (at the expenses of a disutility cost) or in sector L (at the expense of a lower wage). Productive workers have different abilities to work in the skill-intensive sector which consist in heterogeneous disutility factors $\eta(x)$ (with $0 < \eta(x) < 1$) associated with working in the skill-intensive sector. Since individuals are risk-neutral, the utility flow of a productive worker x with ability $\eta(x)$ is then $\eta(x) \cdot w_t^H$ if he works in the skill-intensive sector H and w_t^L if he works

[1] What we call a period is the time interval between two successive innovations, which is of random duration.

in the unskilled-intensive sector L [2]. An individual x therefore chooses to work in sector H as long as:

$$w_t^H \cdot \eta(x) \geq w_t^L \qquad \Leftrightarrow \qquad \eta(x) \geq \frac{w_t^L}{w_t^H}$$

It follows from this inequality that there exists a unique, interior, threshold ability $\bar{\eta} = \frac{w_t^L}{w_t^H}$ such that all individuals with ability above $\bar{\eta}$ choose to work in the skill intensive sector and vice versa. The distribution of ability $\eta(j)$ in the population is assumed to be continuous and stationary, with cumulative distribution function denoted by $G(.)$. Besides, normalizing the size of each category of workers to 1, the labor market clearing conditions write [3]:

$$N_t^H + N_t^L = 1 \qquad \text{and} \qquad R_t^H + R_t^L = 1 \tag{1}$$

Thus, the employment level in the skilled and in the unskilled intermediate good sector (sectors H and L) are respectively given by:

$$N_t^L = G\left(\frac{w_t^L}{w_t^H}\right) \quad , \quad N_t^H = 1 - G\left(\frac{w_t^L}{w_t^H}\right) \quad \text{and} \quad \frac{N_t^H}{N_t^L} = \frac{1 - G\left(\frac{w_t^L}{w_t^H}\right)}{G\left(\frac{w_t^L}{w_t^H}\right)} \tag{2}$$

In turn, N_t^H increases, N_t^L decreases and $\frac{N_t^H}{N_t^L}$ increases with the wage premium $\frac{w_t^H}{w_t^L}$ [4].

2.3 Final consumption good sector

The final good is the numeraire in this economy. It is produced competitively under a standard constant return to scale technology using both types of intermediate goods as inputs. The production function is given by:

$$Y_t = F\left(C_t^H, C_t^L\right) \qquad F_i' > 0 \qquad F_{12}'' > 0 \qquad F_{ii}'' < 0 \quad , \quad i = H, L \tag{3}$$

where $F_i'(.,.)$ denotes partial derivative of F with respect to C_t^i. C_t^i is a CES basket of the continuum of intermediate goods i $C_t^i(s)$ and is defined by:

$$C_t^i = \left(\int_0^1 \left[C_t^i(s)\right]^\beta ds\right)^{\frac{1}{\beta}} \qquad 0 < \beta < 1$$

[2] The lower the disutility factor $\eta(x)$, the higher the disutility of working in the productive sector for worker x.

[3] Recall that we first consider that workers are not mobile between research and intermediate production. This assumption is relaxed in section 5.

[4] In this setting, the efficiency units of labor supplied by workers in the skill-intensive and unskilled-intensive sectors are independent of the rate of technological progress. In contrast, in Galor and Moav (2000), technological progress complements ability in the formation of human capital. The higher the rate of technical change, the higher the returns to ability.

with $C_t^i(s)$ the quantity of the intermediate good s in sector i. Let $p_t^i(s)$ denote the price of intermediate good s in sector i. Profit maximization by a representative firm in the final good sector yields the following inverse demand function:

$$p_t^i(s) = F_i'\left(C_t^H, C_t^L\right) \cdot \left(\int_0^1 \left[C_t^i(s)\right]^\beta ds\right)^{\frac{1}{\beta}-1} \cdot \left[C_t^i(s)\right]^{\beta-1} \tag{4}$$

2.4 Intermediate goods sector

In each sector, there is a continuum of mass 1 ($s \in [0,1]$) of intermediate goods. Each of these goods is produced monopolistically, with a technology featuring constant returns to scale as follows:

$$C_t^i(s) = A_t^i \cdot n_t^i(s), \qquad i = H, L \tag{5}$$

where $n^i(s)$ is the number of workers employed by firm s in sector i, and A_t^i is the productivity of any firm s in sector i. We consider that innovations are radical. Hence, the patent owner of the latest technology required to produce the s^{th} intermediate good i enjoys a monopoly position. Each intermediate good producer maximizes her profit stream $\pi_t^i(s) = p_t^i(s) \cdot C_t^i(s) - w_t^i \cdot n_t^i(s)$, given equations (4) and (5). The first order condition of this program yields[5]:

$$\beta \cdot p_t^i(s) = \frac{w_t^i}{A_t^i} \tag{6}$$

In turn, individual equilibrium profits write $\pi_t^i(s) = (1-\beta) \cdot p_t^i(s) \cdot C_t^i(s)$. Hence, orices are symmetric $p_t^i(s) = p_t^i = F_i'\left(C_t^H, C_t^L\right)$, therefore $C_t^i(s) = C_t^i$ and $n_t^i(s) = n_t^i$. Since firms in sector H only employ skilled labor, and firms in sector L only employ unskilled labor, labor market clearing conditions imply that

$$N_t^i \equiv \int_0^1 n_t^i(s) \cdot ds = n_t^i \quad , \quad i = H, L$$

[5] If innovations are non-drastic, the price set by the owner of the last technology is the highest price that drives the owner of the next to last technology out of the market. Such a limit-price strategy leads to

$$p_t^i(s) = \frac{w_t^i(s)}{(1-\gamma) \cdot A_t^i(s)}$$

and nothing in the model is altered provided that $1 - \beta = \gamma$. Hence, the condition for innovations to be drastic is that $\gamma > 1 - \beta$.

Taking into account equations (4), (5) and (6), the relative inverse demand function for skilled labor is given by:

$$\frac{w_t^H}{w_t^L} = \frac{A_t^H}{A_t^L} \cdot \frac{F_H' \left(A_t^H N_t^H, A_t^L N_t^L\right)}{F_L' \left(A_t^H N_t^H, A_t^L N_t^L\right)} = \frac{A_t^H}{A_t^L} \cdot \phi \left(\frac{A_t^H N_t^H}{A_t^L N_t^L}\right) \tag{7}$$

where function ϕ is defined as follows:

$$\phi(x) \equiv \frac{F_H'(x,1)}{F_L'(x,1)}$$

Because F exhibits constant returns to scale, the marginal productivities F_i' are homogenous of degree 0. Hence, the relative price p_t^H/p_t^L only depends on the ratio of the effective units of labor employed in both sectors $A_t^H N_t^H / A_t^L N_t^L$, and so does function $\phi(.)$. Furthermore, function ϕ is decreasing. The inverse demand function is indeed standard and exhibits a familiar substitution effect: when there are more skilled workers relatively to unskilled workers, the relative demand for skilled workers decreases. This mechanism relates to Acemoglu (1998)'s *price effect*. In his model, when there are more skilled workers, the price of the goods they produce is low (so there should be less technologies complementary to skilled labor). Here as well, when skilled workers are relatively more abundant, the price of the goods they produce (captured by function $\phi(.)$) decreases.

Combining (2) and (7), the equilibrium on the labor market is determined by the solution of the system:

$$\frac{N_t^H}{N_t^L} = \frac{1 - G\left(\frac{w_t^L}{w_t^H}\right)}{G\left(\frac{w_t^L}{w_t^H}\right)} \quad \text{and} \quad \frac{w_t^H}{w_t^L} = \frac{A_t^H}{A_t^L} \cdot \phi\left(\frac{A_t^H N_t^H}{A_t^L N_t^L}\right) \tag{8}$$

The first equation describes an increasing relationship between the relative labor supply and the wage premium, whereas the second one emphasizes a negative relationship between the relative labor demand and the wage premium (see figure 2). Hence, the labor market equilibrium is unique and is denoted by: $\left(\frac{w_t^H}{w_t^L}\right)^*$ and $\left(\frac{N_t^H}{N_t^H}\right)^*$.

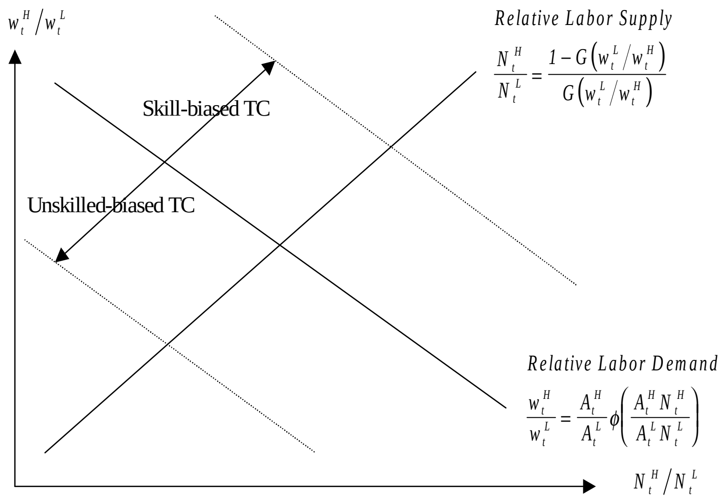

Figure 2: Dynamics of the skill premium with skill-biased and unskilled-biased technical change.

Aggregate intermediate profits in each sector are then given by:

$$\begin{aligned}\pi_t^i &= \Pi_i\left(A_t^H, A_t^L\right) \\ &\equiv (1-\beta)\cdot A_t^i \cdot N^i\left(A_t^H/A_t^L\right)\cdot F_i'\left(\mathcal{N}^H\left(A_t^H/A_t^L\right)\cdot A_t^H, \mathcal{N}^L\left(A_t^H/A_t^L\right)\cdot A_t^L\right)\end{aligned} \tag{9}$$

Functions $\Pi^i(.,.)$ are homogenous of degree 1 and increasing in both arguments. The impact of the direction of the technical bias on the relative skilled wage is reproduced in Figure 2. Skill-biased technological progress consists in a change in A_t^H/A_t^L that shifts the relative demand for skilled workers upward. Using the property of function Π_i, (9) can be re-expressed as follows:

$$\Pi_i\left(A_t^H, A_t^L\right) = (1-\beta)\cdot A_t^i \cdot \mathcal{N}^i\left(A_t^H/A_t^L\right)\cdot F_i'\left(\frac{\mathcal{N}^H\left(A_t^H/A_t^L\right)\cdot A_t^H}{\mathcal{N}^L\left(A_t^H/A_t^L\right)\cdot A_t^L}, 1\right) \tag{10}$$

We restrict the functional forms of $F(.)$ and $G(.)$ such that the following assumption is satisfied:

$$\frac{\partial \Pi_i\left(A_t^H, A_t^L\right)}{\partial A_{-i}} \geq 0 \qquad \text{for } i = H, L \text{ and } -i \neq i \tag{11}$$

This assumption states that if innovation t occurs in sector i, the profits earned by intermediate good producers in the other sector $(-i)$ increase: when A_t^i increases, A_t^{-i}/A_t^i decreases. If $F_{-i}'(.,.)$ is decreasing in A_t^{-i}/A_t^i , then $\Pi_{-i}(.,.)$ increases with A_t^i. With specific functional forms, as those adopted in section 4 and 5, that is with $F = \left[\left(A^H N^H\right)^\sigma + \left(A^L N^L\right)^\sigma\right]^{\frac{1}{\sigma}}$, $\sigma < 1$ and $G(x) = \frac{1}{1+x^{-\upsilon}}$, assumption (11) is numerically satisfied for a wide range of plausible values of parameters σ and υ.

2.5 Research sector

Innovations are radical and raise the technological parameter (labor productivity) in sector i, A_t^i, by a constant factor $\frac{1}{1-\gamma}$ [6]. The research process is portrayed as a patent race with a probability of success following a Poisson process. Research decisions regarding the $(t+1)^{st}$ innovation are made in period t. The length of a period between two successive innovations (t and $t+1$) is stochastic and follows a Poisson process of parameter $\lambda \cdot \left(R_t^H + R_t^L\right)$, where λ is the individual Poisson probability of innovating and R_t^i is the amount of research devoted to sector $i = H, L$. Because time is continuous, the probability that two innovations occur at the same time in both sectors is null.

There is a continuum of researchers normalized to 1 ($j \in [0,1]$), each of whom devoting a fraction of his time to the development of project directed to sector L or H. Let $T_t^i(j)$ denotes the fraction of time devoted by researcher j to sector i. With unit time endowment, we have $T^H(j) + T^L(j) = 1$. Besides, summing over all researchers yields the aggregate amount of research devoted to sector i: $R_t^i \equiv \int_0^1 T_t^i(j) \cdot dj$, $i = H, L$.

The research technology is a one-for-one relationship. The individual rate of successfully innovating in sector i for researcher j is given by:

$$y_t^i(j) = \lambda \cdot T_t^i(j)$$

Such a technology implies constant returns to scale in the research sector. Besides, it relies on the assumption that the productivity of the time devoted to research in sector i is not affected by the time devoted to sector $-i$ [7].

Let V_t^i denote the sum of the prices of licences in sector i for the t^{th} innovation. Researchers allocate their time between both sectors by maximizing their expected profits, taking as given the price of licences, according to the following program:

$$\begin{aligned} &\max_{T_t^H(j), T_t^L(j)} \quad \lambda \cdot T_t^H(j) \cdot V_t^H + \lambda \cdot T_t^L(j) \cdot V_t^L \\ &\text{s.t.} \quad T_t^H(j) + T_t^L(j) = 1 \end{aligned} \tag{12}$$

The solution of this program is straightforward: whenever $V_t^H > V_t^L$, researchers are incited to allocate their entire time endowment to sector H, that is $V_t^H > V_t^L \Leftrightarrow T_t^H(j) = 1$

[6] Recall that if the t^{th} innovation occurs in sector i, then $A_t^i = \frac{1}{1-\gamma} \cdot A_{t-1}^i$ whereas $A_t^{-i} = A_{t-1}^{-i}$, and vice versa.

[7] As in Aghion and Howitt (1992), there are no contemporaneous spillovers in research. However, there are intertemporal spillovers once a researcher successfully innovates via the increase in the technological parameter A_t^i.

and $T_t^L(j) = 0$ and vice versa if $V_t^H < V_t^L$ [8]. Aggregate research decisions then consist in the following rule:

$$V_t^H < V_t^L \Leftrightarrow R_t^H = 0 \quad \text{and} \quad R_t^L = 1 \quad ; \quad V_t^L < V_t^H \Leftrightarrow R_t^L = 0 \quad \text{and} \quad R_t^H = 1 \qquad (13)$$

according to which, in each period research is conducted in a single sector.

When innovating in sector i, the $(t+1)^{st}$ innovator sells his patents to the corresponding intermediate good firms and earns V_t^i. Each patent enables to produce intermediate good s of type i at a lower cost, thereby driving out of the market the old intermediate good producers. Assuming perfect financial markets, V_t^i must equals the sum of the intertemporal expected profits earned monopolistically by each intermediate good producer in sector i until replacement by the next innovation in sector i. Let $W_t^i(s)$ denote the value of firm s in sector i during period t. The incumbent monopolist sells the continuum of licences at the highest price an intermediate good producer is willing to pay. Patent price and firms' values hence are related according to the following appropriation equation:

$$V_t^H = \int_0^1 W_{t+1}^H(s) \cdot ds \qquad \text{and} \qquad V_t^L = \int_0^1 W_{t+1}^L(s) \cdot ds \qquad (14)$$

Unlike Aghion and Howitt (1992), if the t^{th} innovation occurs in sector i, the $(t+1)^{st}$ innovation is not bound to occur in sector i. The patent length may be finite or infinite, depending on whether the $(t+2)^{nd}$, $(t+3)^{rd}$ ... innovations take place in sector i or in sector $-i$. This is a fundamental difference between our approach and the previous literature. Hence, V_t^i is conditional upon the expected dynamics of R_t^H and R_t^L in future periods, and therefore upon the expected length of patents. The incentives to do research are such that the patent price equals the expected present value of an intermediate firm. The value $W_t^i(s)$ is defined by the following asset equation:

$$r \cdot W_t^i(s) = \pi_t^i(s) - \lambda \cdot R_t^i(s) \cdot W_t^i(s) + \lambda \cdot R_t^{-i}(s) \cdot \left[W_{t+1}^i(s) - W_t^i(s)\right] \qquad (15)$$

Because of the symmetry property all indexes s can be dropped. The interpretation of equation (15) is the following. The expected income generated by the s^{th} licence in sector i, associated with innovation t, during a unit time interval is composed of three elements (appearing in the right hand side).

· The first one is the flow of aggregate profits π_t^i attainable by the t^{th} intermediate good monopolist s in sector i, where:

$$\pi_t^i(s) = \Pi_i\left(A_t^H \mathcal{N}_t^H, A_t^L \mathcal{N}_t^L\right) \equiv \Pi_i\left(A_t^H, A_t^L\right)$$

[8] The case $V_t^H = V_t^L$ implies very extreme assumptions on initial conditions regarding the value of productivity and profits in the intermediate goods sectors. These assumptions are not robust to small variations (any infinitely small change in these initial conditions would render the strict equality between V_t^H and V_t^L false). The particular case $V_t^H = V_t^L$ is therefore ruled out.

since $\mathcal{N}_t^H$ and $\mathcal{N}_t^L$ only depend on A_t^H and A_t^L.

· According to a Poisson process of rate $\lambda \cdot R_t^i$, the $(t+1)^{st}$ innovation occurs in sector i, and incumbent monopolists are replaced by new innovators in sector i. The second element in the right hand side of (15) is therefore the expected "capital loss" that will occur when the t^{th} innovator is replaced by a new innovation in his/her sector. This loss corresponds to the standard Aghion and Howitt's "business-stealing" effect, whereby the next innovator destroys the surplus attributable to the previous generation of intermediate good i by making it obsolete.

· However, according to a Poisson process of rate $\lambda \cdot R_t^{-i}$ the $(t+1)^{st}$ innovation occurs in sector $-i$, and the incumbent monopolist is not replaced. Licences on the t^{th} innovation are then still valid when the $(t+1)^{st}$ innovation occurs. The third element in the right hand side of (15) is thus the expected "capital gain" that will occur when the t^{th} innovator is not replaced by the next innovation and therefore when the patent lasts more than one period. By symmetry with the business-stealing effect, we call this gain a "business-continuing" effect, whereby the next innovator does not make the preceding generation of intermediate good i obsolete.

3 Equilibrium dynamics

An equilibrium in this economy is defined as follows.

Definition 1: *An equilibrium is a sequence of* A_t^H, A_t^L, R_t^H, R_t^L, V_t^H, V_t^L, $W_t^H(.)$, $W_t^L(.)$, N_t^H and N_t^L *defined by equations (9) to (14), where*

1. *when* $V_t^L < V_t^H$, *the economy's stock of researchers between innovations* t *and* $t+1$ *is allocated to sector* H *only*: $R_t^H = 1$, $R_t^L = 0$,

2. *when* $V_t^L > V_t^H$, *the economy's stock of researchers between innovations* t *and* $t+1$ *is allocated to sector* L *only*: $R_t^H = 0$, $R_t^L = 1$.

Hence, the equilibrium consists in a fixed point between 3 relations. First, the dynamics of the number of researchers $\left\{R_t^L, R_t^H\right\}_{t\geq 0}$ defines $\left\{W_{t+1}^L, W_{t+1}^H\right\}_{t\geq 0}$ according to the asset equations (15). Second, appropriation equations (14) define $\left\{V_t^L, V_t^H\right\}_{t\geq 0}$ as functions of $\left\{W_{t+1}^L, W_{t+1}^H\right\}_{t\geq 0}$. Lastly, patents prices $\left\{V_t^L, V_t^H\right\}_{t\geq 0}$ determine $\left\{R_t^L, R_t^H\right\}_{t\geq 0}$ at any point in time according to which intermediate goods sector yields the highest profits (H or L) in the researchers' decision rule (13). The corner stone of this definition is that all researchers are atomistic and ignore the impact of their decisions on other researchers' behavior. Thus, they take as given the patent prices $\left\{V_t^L, V_t^H\right\}_{t\geq 0}$ when choosing which technology (H or L) they want to improve. They don't take into account the influence of their decisions on the dynamics of patent prices, which is a standard assumption in competitive equilibria.

This paper questions the relevance of the hypothesis of a permanent adoption of skill-biased technologies. This issue requires to determine the conditions under which researchers permanently improve the productivity of a single sector, for instance the skill-intensive one. In our economy, this is equivalent to establish the conditions under which, in each period, innovations occur in sector H [9]. In this case, the equilibrium condition is given by:

$$\text{for all } t, \qquad R_t^H = 1,\ R_t^L = 0 \quad \text{and} \quad V_t^L < V_t^H \tag{16}$$

The present value of innovations in sector H, V_t^H, is determined by substituting condition (16) in the asset equation (15):

$$V_t^H = \int_0^1 W_{t+1}^H(s) \cdot ds = \frac{1}{r+\lambda}\Pi^H\left(\frac{A_t^H}{1-\gamma}, A_t^L\right)$$

Since innovations occur systematically in the skill-intensive sector, each patent in this sector lasts only one period and we obtain Aghion and Howitt's asset equation as a particular case of our model.

The present discounted value of an innovation in sector L, V_t^L, is determined as follows. Since all innovations occur in sector H, an innovation in sector L is expected to yield a patent that lasts indefinitely. If innovation t occurs in sector L, this is considered by any atomistic innovator as a deviation from the equilibrium behavior which only happens once. Thus, V_t^L is given by $V_t^L = W_{t+1}^{d,L,t}$, where $W_{t+k}^{d,L,t}$, $k \geq 1$ is defined recursively by:

$$r \cdot W_{t+k}^{d,L,t} = \Pi^L\left(\frac{A_t^H}{(1-\gamma)^{k-1}}, \frac{A_t^L}{1-\gamma}\right) + \lambda\left[W_{t+k+1}^{d,L,t} - W_{t+k}^{d,L,t}\right]$$

which implies:

$$V_t^L = W_{t+1}^{d,L,t} = \frac{1}{r+\lambda}\sum_{k=1}^{+\infty}\left(\frac{\lambda}{r+\lambda}\right)^{k-1}\Pi^L\left(\frac{A_t^H}{(1-\gamma)^{k-1}}, \frac{A_t^L}{1-\gamma}\right)$$

[9] The case where innovations permanently appear in sector L is perfectly symmetric and is also analyzed in order to draw general propositions regarding fluctuations in technological adoption.

Proposition 1 *A dynamic such that for any $t \geq t_0$ innovation permanently appears in the same sector is an equilibrium iff:*

$$\forall t \qquad \sum_{k=1}^{+\infty} \left(\frac{\lambda}{r+\lambda}\right)^{k-1} \Pi^L \left(\frac{A_{t_0}^H}{(1-\gamma)^{k-1}}, \frac{A_{t_0}^L}{1-\gamma}\right) < \Pi^H \left(\frac{A_{t_0}^H}{1-\gamma}, A_{t_0}^L\right) \quad (17)$$

$$or \qquad \forall t \qquad \sum_{k=1}^{+\infty} \left(\frac{\lambda}{r+\lambda}\right)^{k-1} \Pi^H \left(\frac{A_{t_0}^H}{1-\gamma}, \frac{A_{t_0}^L}{(1-\gamma)^{k-1}}\right) < \Pi^L \left(A_{t_0}^H, \frac{A_{t_0}^L}{1-\gamma}\right) \quad (18)$$

These inequalities are very unlikely to be satisfied. In particular, as the discount rate r tends to 0, the left hand side of equations (17) and (18) tend to $+\infty$. Indeed, from equation (9) and assumption (11), $\Pi^L\left(A^H, A^L\right)$ increases in A^H. Hence, for any $k \geq 1$, $\Pi^L\left(\frac{A_t^H}{(1-\gamma)^{k-1}}, \frac{A_t^L}{1-\gamma}\right) > \Pi^L\left(A_t^H, \frac{A_t^L}{1-\gamma}\right)$. This implies that, as r tends to 0, the right hand side of equation (17) tends to $+\infty$ (and symmetrically for equation (18)). This is precisely the reason why this kind of dynamics is very implausible. In other words, there is a trade-off between a continuum of patents that lasts a single period and one that lasts infinitely. With no discounting, deviating once leads to a rent that tends to $+\infty$. Hence, permanent innovations in the same sector are associated with highly implausible dynamics.

Proposition 2 *If equations (17) and (18) are false, any equilibrium dynamics is characterized by infinitely alternation of periods where innovations occur in sector H and periods where innovations occur in sector L.*

The proof relies on the contrapositive of proposition 1. (17) or (18) are necessary conditions for the existence of a permanent bias in the same sector. If these conditions are ruled out, innovations will never occur indefinitely in the same sector. □

In other words, at any point in time, there is a date at which the direction of the technological bias will change. Hence, there always exists a moment when alternation occurs. Fluctuation in technology adoption is endogenous and stems from the business-continuing effect whereby the rents generated by a particular innovation last as long as future innovations occur in a different sector. The Kuznets curve is then bound to reverse because of this incentive to innovate in a different sector.

4 Two-Cycle dynamics

Our concept of equilibrium is rather weak, implying probably a high multiplicity of possible dynamics. Hence, we know what the dynamics is not (permanent innovations in a single sector) but this framework does not allow for unique dynamics if we do not restrict the nature of fluctuations or if we do not specify a selection criteria. Nevertheless, a criteria based on the simplicity of the dynamics might be realistic, since the simpler the dynamics is, the easier should it be for agents to learn it and therefore the easier should it be for agents to coordinate their behavior on it. Actually, the two-cycle dynamics, where alternation occurs each time an innovation appears is both intuitive and very simple. We hence study in this section such a dynamics, knowing that it only constitutes a proxy for the discontinuity in radical innovation adoption. Yet, this kind of dynamics is analytically tractable and provides a simple stylized version of the patterns of cycles of innovation and inequality.

4.1 Characteristics of a two-cycle

The kind of cycles analyzed is such that during even intervals research is directed toward sector H, and during odd intervals, research is directed toward sector L. Under such a dynamics and using the constant return to scale property of functions $\Pi^i(.,.)$, the variables evolve in the following way:

t	**0**	**1**	**2**	**3**	...	**2 t**	**2 t + 1**
R_t^H	1	0	1	0	...	1	0
R_t^L	0	1	0	1	...	0	1
A_t^H	A^H	$\frac{1}{1-\gamma}A^H$	$\frac{1}{1-\gamma}A^H$	$\left(\frac{1}{1-\gamma}\right)^2 A^H$	...	$\left(\frac{1}{1-\gamma}\right)^t A^H$	$\left(\frac{1}{1-\gamma}\right)^{t+1} A^H$
A_t^L	A^L	A^L	$\frac{1}{1-\gamma}A^L$	$\frac{1}{1-\gamma}A^L$	...	$\left(\frac{1}{1-\gamma}\right)^t A^L$	$\left(\frac{1}{1-\gamma}\right)^t A^L$
$\mathcal{N}_t^H$	$\mathcal{N}_0^H$	$\mathcal{N}_1^H$	$\mathcal{N}_0^H$	$\mathcal{N}_1^H$	...	$\mathcal{N}_0^H$	$\mathcal{N}_1^H$
$\mathcal{N}_t^L$	$\mathcal{N}_0^L$	$\mathcal{N}_1^L$	$\mathcal{N}_0^L$	$\mathcal{N}_1^L$	...	$\mathcal{N}_0^L$	$\mathcal{N}_1^L$
π_t^H	π_0^H	π_1^H	$\frac{1}{1-\gamma}\pi_0^H$	$\frac{1}{1-\gamma}\pi_1^H$	...	$\left(\frac{1}{1-\gamma}\right)^t \pi_0^H$	$\left(\frac{1}{1-\gamma}\right)^t \pi_1^H$
π_t^L	π_0^L	π_1^L	$\frac{1}{1-\gamma}\pi_0^L$	$\frac{1}{1-\gamma}\pi_1^L$	...	$\left(\frac{1}{1-\gamma}\right)^t \pi_0^L$	$\left(\frac{1}{1-\gamma}\right)^t \pi_1^L$

Table I: Two-cycle dynamics

where according to figure 2, $\mathcal{N}_t^i$ depends on w_t^H/w_t^L, $\mathcal{N}_t^i$ depends on w_{t+1}^H/w_{t+1}^L, $\pi_0^i = \Pi_i\left(A^H, A^L\right)$, $\pi_1^i = \Pi_i\left(\frac{A^H}{1-\gamma}, A^L\right)$ for $i = H, L$ and functions $\Pi_i(.,.)$ are defined in (10).

Proposition 3 *A two-cycle dynamics is an equilibrium iff:*

$$\pi_1^H + \frac{\lambda}{r+\lambda}\frac{\pi_0^H}{1-\gamma} > \pi_1^L \qquad \textit{and} \qquad \pi_0^H < \pi_0^L + \frac{\lambda}{r+\lambda}\pi_1^L \tag{19}$$

Proof: see appendix 7.1. □

4.2 Application with functional specifications

In order to characterize the relationship between fluctuation in technological adoption and the dynamics of wage inequality, specific functional forms have to be adopted. We consider that the final good is produced according to the following CES technology:

$$Y_t = \left[\left(C_t^H\right)^{\sigma} + \left(C_t^L\right)^{\sigma} \right]^{\frac{1}{\sigma}} \tag{20}$$

where $\sigma \in]-\infty, 1[$, $\sigma \neq 0$. Hence, $1/(1-\sigma) > 0$ is the elasticity of substitution between skill-intensive and unskilled-intensive intermediate goods [10]. Besides, we consider that the distribution of ability within individuals is characterized by the following cumulative distribution function:

$$G(x) = \frac{1}{1 + x^{-\upsilon}} \tag{21}$$

With the production function (20), the prices of intermediate goods are given by [11]:

$$p_t^i = F_i' = \left(C_t^i\right)^{\sigma-1} \left[\left(C_t^H\right)^{\sigma} + \left(C_t^L\right)^{\sigma} \right]^{\frac{1-\sigma}{\sigma}} \tag{22}$$

In equilibrium, the relative inverse demand function writes: $p \equiv \frac{p^H}{p^L} = \left(\frac{C^L}{C^H}\right)^{1-\sigma}$. Profits in the intermediate good sectors are given by:

$$\pi_t^i = (1-\beta)\ \left(C_t^i\right)^{\sigma} \left[\left(C_t^H\right)^{\sigma} + \left(C_t^L\right)^{\sigma} \right]^{\frac{1-\sigma}{\sigma}} \tag{23}$$

To determine whether a two-cycle dynamics is supported by our equilibrium concept, that is whether condition (19) is realistic, we can consider a particular case of (20) where the elasticity of substitution between both intermediate goods tends to 1 (i.e. $\sigma \to 0$), and A^H tends to A^L. The difference between π_t^H and π_t^L then tends to 0, implying that condition (19) is satisfied. Hence, by continuity, a two-cycle is an equilibrium, at least when the differentiation between intermediate goods is not too wide (that is when the gap between A^H and A^L is not too wide and σ is sufficiently close to 0).

4.3 Growth rates of output and wages

Depending on the value of σ, along a two-cycle dynamics, the skill premium evolves as follows (see appendix 7.2).

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[10] This particular CES technology is used only for the sake of simplicity. Relying on a general CES technology $Y_t = \left[\alpha^H \left(C_t^H\right)^{\sigma} + \alpha^L \left(C_t^L\right)^{\sigma}\right]^{\frac{1}{\sigma}}$ would not alter our qualitative results, though making equations sometimes too complicated without adding useful insights.

[11] Recall that because of the symmetry property, we have dropped index s.

w^H/w^L	$2t \to 2t+1$	$2t+1 \to 2t+2$
$0 < \sigma < 1$	$\nearrow$	$\searrow$
$\sigma < 0$	$\searrow$	$\nearrow$

Table II: Evolution of the skill premium along a two-cycle

It is obvious from table 2 that wage inequality is cyclical, but its fundamental causes vary according to whether both categories of workers are relatively substitutable or not [12]. When $0 < \sigma < 1$, skill-intensive and unskilled-intensive intermediate goods are more substitutable than in the Cobb-Douglas case. In such a case, firms are incited to reorganize production and demand more workers who experience productivity gains, which increases their wage premium. A skill-biased technological change then corresponds to an increase in A^H/A^L which occurs between odd and even intervals.

When $\sigma < 0$, skill-intensive and unskilled-intensive intermediate goods are less substitutable than in the Cobb-Douglas case. In this case, a rise in labor productivity incites firms to save worker who experience productivity gains because of the weak substitutability between both inputs, which reduces the wage premium for this type of labor (skilled workers in odd intervals, unskilled workers in even intervals). A skill-biased technological change then corresponds to a decrease in A^H/A^L which occurs between even and odd intervals.

5 Mobility between research and manufacturing

In this section we relax the assumption of absence of mobility for skilled workers between research and skill-intensive intermediate manufacturing. Workers have now to choose between three alternatives: working in the unskilled productive sector, in the skilled productive sector or in the research sector. The labor resources constraint writes:

$$N_t^H + N_t^L + R_t = 2 \tag{24}$$

We assume that the disutility to work in the research sector equals the disutility to work in the skilled productive sector. The utility flow of worker x is then w_t^L if he/she works in the unskilled productive sector, $\eta(x) \cdot w_t^H$ if he/she works in the skilled productive sector and $\eta(x) \cdot w_t^R$ if he/she works in the research sector. An individual x therefore chooses

[12] Many estimates of the elasticity of substitution between skilled and unskilled workers $\left(\frac{1}{1-\sigma}\right)$ correspond to the first case. In particular, the usual estimate is close to 1.7, which corresponds to a value of 0.4 for σ (Krusell et al. (2000)). However, taking into account the role of physical capital, most studies show a very high complementarity between skilled labor and physical capital, and an elasticity of substitution between capital and unskilled labor smaller than 1 (see Krusell et al. (2000)). This suggests that we can not rule out the case where $\sigma < 0$.

to work in the unskilled productive sector as long as: $\eta(x) \cdot \max\left(w_t^H, w_t^R\right) \leq w_t^L$. The unskilled labor supply therefore equals:

$$N_t^L = 2 \cdot G\left(\frac{w_t^L}{\max\left(w_t^H, w_t^R\right)}\right)$$

The wage rate in the research sector, w_t^R, maximizes the expected income of researchers defined by program (12). Hence, the research decisions and equation (13) still hold. The equilibrium on the labor market now is such that skilled workers must be indifferent between research and skill-intensive intermediate manufacturing. In turn, the wages of skilled workers must equal the earnings of researchers in each period. The arbitrage equation (between research and intermediate manufacturing) therefore writes:

$$w_t^H\left(A^H, A^L, N^H, N^L\right) = \lambda \cdot \max\left[V_t^H, V_t^L\right] \tag{25}$$

and the unskilled labor supply is $N_t^L = 2 \cdot G\left(\frac{w_t^L}{w_t^H}\right)$. Finally, the relative demand equation (7) still holds.

The allocation of the labor force between the research sector, the skilled productive sector and the unskilled productive sector depends on the returns of research, that is on future profits in intermediate good sectors. Formally, equation (25) sets wages in the research sector w_t^R and in the skilled intensive sector w_t^H. One therefore obtains the employment levels in the two intermediate good sectors N_t^H and N_t^L as well as the wage rate in the unskilled productive sector w_t^L as the solutions of the following system:

$$\begin{aligned} N_t^L &= 2 \cdot G\left(\frac{w_t^L}{w_t^H}\right) \\ w_t^L &= \beta \cdot A_t^L \cdot F_L'\left(A_t^H N_t^H, A_t^L N_t^L\right) \\ w_t^H &= \beta \cdot A_t^H \cdot F_H'\left(A_t^H N_t^H, A_t^L N_t^L\right) \end{aligned}$$

Finally, equation (24) gives the number of researchers R_t.

Our concept of equilibrium now is affected by the fact that the allocation of workers in period t between research and the two productive sectors $\left\{N_t^H, N_t^L, R_t\right\}$ depends on the expected returns of research, $\max\left(V_t^H, V_t^L\right)$, that is on the expected values of new firms in periods $t+1$, $\left\{W_{t+1}^H(s), W_{t+1}^L(s)\right\}$. This causality is consistent with the literature on growth driven by creative destruction (e.g. Aghion and Saint-Paul (1991), Caballero and Hammour (1994), Aghion and Howitt (1998), Li (2001)). However, our concept of equilibrium remains weak and many different dynamics might belong to the set of possible equilibria. We again restrict our attention to the two-cycle dynamics where alternation occurs following any innovation (see Table 1).

The asset equations are now given by (see appendix 7.3):

$$W_{2t}^H = \frac{\pi_0^H}{(r+\lambda R_0)(1-\gamma)^t} \qquad W_{2t+1}^H = \frac{\pi_1^H + \frac{\lambda R_1}{(r+\lambda R_0)(1-\gamma)}\pi_0^H}{(r+\lambda R_1)(1-\gamma)^t}$$

$$W_{2t}^L = \frac{\pi_0^L + \frac{\lambda R_0}{r+\lambda R_1}\pi_1^L}{(r+\lambda R_0)(1-\gamma)^t} \qquad W_{2t+1}^L = \frac{\pi_1^L}{(r+\lambda R_1)(1-\gamma)^t}$$

The equilibrium of the economy is a fixed point between the arbitrage equations, the asset equations and the labor market clearing conditions. It is not analytically tractable. In order to determine under which conditions a two-cycle iimplies a plausible dynamics, only numerical simulations can be conducted. In this exercise, the parameters are set the following way.

$$\sigma = 0.41 \quad r = 0.02 \quad \beta = 0.6 \quad \upsilon = 1 \quad \gamma = 0.5 \quad \lambda = 0.1 \quad A^H = 1.1 \quad A^L = 1$$

The elasticity of substitution between both inputs in the final good sector is the usual one, a value of 0.41 for σ corresponding to an estimate around 1.7 (see Krusell et al. (2000)). The interest rate is fixed at 2%. The markup rate in the intermediate goods sectors is set to 0.6. The elasticity of relative supply with respect to relative wages, υ, is normalized to 1. The average length of a unit time interval $(1/\lambda)$ when there is no mobility (when $R_{2t+1}^H = 0$ and $R_{2t}^L = 0$) is equal to 10 years, which implies that $\lambda = 0.1$. γ is set such that the average annual growth rate $\lambda\gamma/2$ is equal to 2.5%. Lastly, we impose $A_{2t}^H = 1.1\ A_{2t}^L$.

Numerical simulations then lead to the following results [13]:

N_0^H	N_0^L	N_1^H	N_1^L	R_0	R_1	V_0^H	V_0^L	V_1^L	V_1^H
0.39	0.78	0.558	0.754	0.829	0.687	21.4	10.64	31	10.83

Besides, the average length of even intervals is 12 years and the average length of odd intervals is 14.5 years.

First, it is checked that, in both cases $V_0^H > V_0^L$ and $V_1^H < V_1^L$. Since. $V_t^H < V_t^L$ during odd intervals, no innovation occurs in sector H during these intervals; and vice versa during even intervals. By the way, odd periods last longer than even ones and imply higher wage inequality. Intuitively, since there is less researchers during odd periods (skilled labor is allocated between both sectors), there is less intermediate producers as well (compared to even periods), which exerts an upward pressure on the relative skilled wage [14].

[13] The Mathematica 3.0 program used for this exercise is downloadable at http://eurequa.univ-paris1.fr/fr/membres /lehmann/offrelastique.nb

[14] In contrast to the literature on output and growth flucutations, our model draws upon radical innovations adoption. Cycles are therefore longer and output does not grow faster in contraction than in expansion (as in Li (2001)).

The two-cycle dynamics highlighted in this numerical example seems consistent with empirical evidence on U.S. data. In particular, our example reproduces the stylized fact that periods of skill-biased inventions increase wage inequality and last longer than periods of inventions directed in favor of unskilled workers.

6 Conclusion

We have built a model of endogenous growth where fluctuation in the innovative sector is bound to occur because researchers are incited to increase the lifetime of patents in the presence of creative destruction. A permanent skill-bias is therefore not plausible in our framework. Under a two-cycle dynamics, patents last two periods and the wage differential between skilled and unskilled workers successively increases and decreases, reproducing a stylized feature of the evolution of wage inequality over the twentieth century in the U.S.

We have derived implications which are positive and not normative. Indeed, we do not argue that since alternation in technology adoption is bound to occur, then the recent skill-bias does not raise puzzling issues. In particular, skill-biased technological change seems to increase wage inequality in the U.S. and unemployment in Europe. The institutional environment therefore matters, implying different reactions of the relative skilled wages to a common change in the relative skilled labor demands. One should therefore incorporate imperfect wage settings and their political determinants to draw normative conclusions on welfare issues regarding the relationship between innovation and wage inequality. This constitutes a direction for our future research.

7 Appendix

7.1 Two-cycle

Using Table 1 and the properties of functions $\Pi^i(.;.)$, equation (15) becomes:

$$\begin{aligned} r\,W_{2t}^H &= \left(\frac{1}{1-\gamma}\right)^t \pi_0^H - \lambda\,W_{2t}^H \\ r\,W_{2t+1}^H &= \left(\frac{1}{1-\gamma}\right)^t \pi_1^H + \lambda\,\left[W_{2t+2}^H - W_{2t+1}^H\right] \\ r\,W_{2t}^L &= \left(\frac{1}{1-\gamma}\right)^t \pi_0^L + \lambda\,\left[W_{2t+1}^L - W_{2t}^L\right] \\ r\,W_{2t+1}^L &= \left(\frac{1}{1-\gamma}\right)^t \pi_1^L - \lambda\,W_{2t+1}^L \end{aligned}$$

Hence, we get:

$$W_{2t}^H = \frac{\pi_0^H}{(r+\lambda)(1-\gamma)^t} \qquad W_{2t+1}^H = \frac{\pi_1^H + \frac{\lambda}{(r+\lambda)(1-\gamma)}\pi_0^H}{(r+\lambda)(1-\gamma)^t} \tag{26}$$

$$W_{2t}^L = \frac{\pi_0^L + \frac{\lambda}{r+\lambda}\pi_1^L}{(r+\lambda)(1-\gamma)^t} \qquad W_{2t+1}^L = \frac{\pi_1^L}{(r+\lambda)(1-\gamma)^t} \tag{27}$$

This kind of cyclical dynamics is an equilibrium if and only if, for any t, $V_{2t}^H > V_{2t}^L$ and $V_{2t+1}^H < V_{2t+1}^L$. This implies:

$$\text{for all } t, \qquad W_{2t+1}^H > W_{2t+1}^L \qquad \text{and} \qquad W_{2t}^H < W_{2t}^L$$

Together with equations (26) and (27), this leads to:

$$\pi_1^H + \frac{\lambda}{r+\lambda}\frac{\pi_0^H}{1-\gamma} > \pi_1^L \qquad \text{and} \qquad \pi_0^H < \pi_0^L + \frac{\lambda}{r+\lambda}\pi_1^L$$

7.2 Growth rates

The average growth rate is approximately:

$$g_{t\to t+1} \simeq \lambda \ln \frac{Y_{t+1}}{Y_t}$$

With $G(x) = \frac{1}{1+x^{-\upsilon}}$ and $Y = \left[\left(A^H N^H\right)^\sigma + \left(A^L N^L\right)^\sigma\right]^{\frac{1}{\sigma}}$ $\sigma < 1$, we obtain:

$$\left(g^{w^H}-g^{w^L}\right)_{2t\to 2t+1} = \lambda\cdot\gamma\cdot\sigma-\lambda\cdot\gamma\cdot\varepsilon\cdot(1-\sigma)\cdot\left(\left(\frac{A^H}{A^L}\right)^{-\varepsilon}+\left(\frac{A^H}{A^L}\right)^{\varepsilon}\right)$$

$$\left(g^{w^H}-g^{w^L}\right)_{2t+1\to 2t+2} = -\lambda\cdot\gamma\cdot\sigma+\lambda\cdot\gamma\cdot\varepsilon\cdot(1-\sigma)\cdot\left(\left(\frac{A^H}{A^L}\right)^{-\varepsilon}+\left(\frac{A^H}{A^L}\right)^{\varepsilon}\right)$$

$$g^{Y}_{2t\to 2t+1}-g^{Y}_{2t+1\to 2t+2} = -2\cdot\lambda\cdot\gamma\cdot\varepsilon\cdot\left(\frac{A^H}{A^L}\right)^{\varepsilon}+\lambda\cdot\gamma^2\cdot(1+\varepsilon)\cdot\Gamma$$

$$g^{w^H}_{2t\to 2t+1}-g^{w^H}_{2t+1\to 2t+2} = \lambda\cdot\gamma\cdot\sigma-2\cdot\lambda\cdot\gamma\cdot\varepsilon\cdot(1-\sigma)\cdot\left[\left(\frac{A^H}{A^L}\right)^{-\varepsilon}+\left(\frac{A^H}{A^L}\right)^{\varepsilon}\right] +(1-\sigma)\cdot\lambda\cdot\gamma^2\cdot(1+\varepsilon)\cdot\Gamma$$

$$g^{w^L}_{2t\to 2t+1}-g^{w^L}_{2t+1\to 2t+2} = -\lambda\cdot\gamma\cdot\sigma+(1-\sigma)\cdot\lambda\cdot\gamma^2\cdot(1+\varepsilon)\cdot\Gamma$$

where $\Gamma\equiv\frac{\left(A^H\right)^{\sigma\cdot(1+\varepsilon)}}{\left(A^H\right)^{\sigma\cdot(1+\varepsilon)}+\left(A^L\right)^{\sigma\cdot(1+\varepsilon)}}$ and $\varepsilon\equiv\sigma\frac{\upsilon}{1+\upsilon(1-\sigma)}$.

7.3 Equilibrium with mobility between research and manufacturing

The asset equations are given by:

$$r\cdot W^H_{2t} = \left(\frac{1}{1-\gamma}\right)^t\cdot\pi^H_0-\lambda\cdot R_0\cdot W^H_{2t} \tag{28}$$

$$r\cdot W^H_{2t+1} = \left(\frac{1}{1-\gamma}\right)^t\cdot\pi^H_1+\lambda\cdot R_1\cdot\left[W^H_{2t+2}-W^H_{2t+1}\right] \tag{29}$$

$$r\cdot W^L_{2t} = \left(\frac{1}{1-\gamma}\right)^t\cdot\pi^L_0+\lambda\cdot R_0\cdot\left[W^L_{2t+1}-W^L_{2t}\right] \tag{30}$$

$$r\cdot W^L_{2t+1} = \left(\frac{1}{1-\gamma}\right)^t\cdot\pi^L_1-\lambda\cdot R_1\cdot W^L_{2t+1} \tag{31}$$

Using (28) we have:

$$r\cdot W^H_{2t} = \left(\frac{1}{1-\gamma}\right)^t\cdot\pi^H_0-\lambda\cdot R_0\cdot W^H_{2t}$$

$$W^H_{2t} = \frac{\left(\frac{1}{1-\gamma}\right)^t\cdot\pi^H_0}{r+\lambda\cdot R_0}$$

Replacing into (29), we obtain:

$$
\begin{aligned}
r \cdot W^H_{2t+1} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \pi^H_1 + \lambda \cdot R_1 \cdot \left[W^H_{2t+2} - W^H_{2t+1}\right] \\
r \cdot W^H_{2t+1} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \pi^H_1 + \lambda \cdot R_1 \cdot \left[\frac{\left(\frac{1}{1-\gamma}\right)^{t+1} \cdot \pi^H_0}{r + \lambda \cdot R_0} - W^H_{2t+1}\right] \\
(r + \lambda R_1)\, W^H_{2t+1} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \pi^H_1 + \lambda \cdot R_1 \cdot \frac{\left(\frac{1}{1-\gamma}\right)^{t+1} \cdot \pi^H_0}{r + \lambda \cdot R_0} \\
W^H_{2t+1} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \left(\frac{1}{r+\lambda R_1}\right) \cdot \left[\pi^H_1 + \frac{\lambda R_1}{r+\lambda R_0} \cdot \left(\frac{\pi^H_0}{1-\gamma}\right)\right]
\end{aligned}
$$

Using (31), we have:

$$
\begin{aligned}
r \cdot W^L_{2t+1} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \pi^L_1 - \lambda \cdot R_1 \cdot W^L_{2t+1} \\
W^L_{2t+1} &= \frac{\left(\frac{1}{1-\gamma}\right)^t \cdot \pi^L_1}{r + \lambda R_1}
\end{aligned}
$$

Replacing into (30):

$$
\begin{aligned}
r \cdot W^L_{2t} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \pi^L_0 + \lambda \cdot R_0 \cdot \left[W^L_{2t+1} - W^L_{2t}\right] \\
(r + \lambda R_0) \cdot W^L_{2t} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \pi^L_0 + \lambda \cdot R_0 \cdot \frac{\left(\frac{1}{1-\gamma}\right)^t \cdot \pi^L_1}{r + \lambda R_1} \\
W^L_{2t} &= \left(\frac{1}{1-\gamma}\right)^t \cdot \left(\frac{1}{r+\lambda R_0}\right) \cdot \left[\pi^L_0 + \frac{\lambda R_0}{r+\lambda R_1} \cdot \pi^L_1\right]
\end{aligned}
$$

Therefore, we get:

$$
\begin{aligned}
W^H_{2t} &= \frac{\pi^H_0}{(r+\lambda R_0)(1-\gamma)^t} \\
W^H_{2t+1} &= \frac{\pi^H_1 + \frac{\lambda R_1}{(r+\lambda R_0)(1-\gamma)} \pi^H_0}{(r+\lambda R_1)(1-\gamma)^t} \\
W^L_{2t} &= \frac{\pi^L_0 + \frac{\lambda R_0}{r+\lambda R_1} \pi^L_1}{(r+\lambda R_0)(1-\gamma)^t} \\
W^L_{2t+1} &= \frac{\pi^L_1}{(r+\lambda R_1)(1-\gamma)^t}
\end{aligned}
$$

8 References

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