# Embodied technical progress and unemployment

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November 12, 2001

#### Abstract

In this paper we build up a canonical vintage capital model with embodied and disembodied technical progress and generalized Nash bargaining in the labor market. First, we handle both types of technical progress as exogenous, but we endogenize them after. In these setups, we comprehensively study the relations between technical progress, unemployment, and job creation and destruction in the short and long run.

Keywords: unemployment, job creation and destruction, embodied technical progress.

Journal of Economic Literature: E24, J60, O33.

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<sup>&</sup>lt;sup>y</sup>This article was partly written when I was visiting IRES, Universit<sup>®</sup> Catholique de Louvain in Spring 2001.

# 1 Introduction

The pioneering growth model of Solow (1956) assumed disembodied technical progress, which makes new and old capital goods more productive in the same way and in the same proportion. In a later work, Solow (1960) developed a growth model with embodied technical progress, which makes only new capital goods more  $e \pm cient$ . The embodied nature of a substantial fraction of technical progress was considered unimportant by Denison (1964) and irrelevant in the long run by Phelps (1962) three decades ago. However, recent theoretical and empirical contributions to growth and business °uctuations have shown its importance in explaining several stylized facts of the US economy, such as the productivity slowdown, the decline in the relative price of investment goods and the persistent rise in the equipment to output ratio.<sup>1</sup> It is now widely admitted that embodiment is crucial in the understanding of these phenomena and several other related issues. In this paper, we wish to show that the nature of technical progress is relevant to understanding the labor market and realize a comprehensive analyses of its implications on unemployment, and job creation and destruction.

The quantitative importance of embodied technical progress has been recently documented by Greenwood, Hercowitz and Krusell (1997), they have found that around 60% of US productivity growth can be attributed to embodied technical progress. Moreover, the empirical evidence supports the view of a change in the composition of technical progress post 1974. The decline of the relative price of equipment, which can be seen as a proxy of the importance of the embodied technical change, has experienced a signi<sup>-</sup>cant acceleration after 1974 in the US economy, passing from 3:3% per year to 4%, as documented by Greenwood and Yorukoglu (1997). Sectorial empirical evidence also supports that post 1974 technical progress is more embodied in capital goods (see for example Baily, Barstelsman and Haltiwanger, (1994) and Gordon (1999)). As Kortum (1997) noticed, \two indus-

<sup>&</sup>lt;sup>1</sup>The contributions of Boucekkine, del Rio and Licandro (1999b), Cooley, Greenwood and Yorukoglu (1997), Greenwood, Hercowitz and Krusell (1997, 1998), Greenwood and Jovanovic (1998), Greenwood and Yorukoglu (1997), Hornstein and Krusell (1996), and Krusell (1998) are some appealing examples of this recent trend in the macroeconomic literature. More theoretical contributions in this <sup>-</sup>eld using explicit vintage capital settings can be found in Benhabib and Rustichini (1991), Boucekkine, Germain and Licandro (1997) and Boucekkine, del Rio and Licandro (1999a).

tries display much more rapid productivity growth after 1974 than before, industrial machinery (which include computing equipment) and electrical equipment. All the rest either had roughly constant productivity growth or slower productivity growth after 1974". This change in the composition of technological progress may well be the major characteristic of the post rst oil shock period, ingeniously referred to as the Third Industrial Revolution by Greenwood and Yorukoglu (1997). Recently, Boucekkine, del Rio and Licandro (1999b) have built up an endogenous growth model in which learning by doing is the engine of both embodied and disembodied technical progress. In sharp contrast to the Phelps (1962)'s exogenous growth model, they show that a change in the composition of technical progress a<sup>®</sup>ects the long run growth rate. They also show this reassignment e<sup>®</sup>ect provides an alternative explanation to the productivity slowdown. The e<sup>®</sup>ects of technological reassignment on unemployment, and job creation and destruction are comprehensively explored in this paper. We show that composition of the technical progress is important to long run employment and employment reallocation. However, technological reassignment has opposite e<sup>®</sup>ects in exogenous and endogenous growth settings. Indeed, under assumptions of exponential exogenous technical progress, an increase in the fraction of embodied technical progress involves higher unemployment, higher employment reallocation and faster speed of convergence. But, if both states of embodied and disembodied technical progress are isoelastic functions of per worker cumulative e±cient investment, e<sup>®</sup>ects of technological reassignment on previous variables are completely opposite.

An objective of the present paper is to build up a simple model integrating all types of technical progress to study its e<sup>®</sup>ects on long run unemployment and job creation and destruction. In the nineties several works have analyzed the relation between technical progress and unemployment. Pissarides (1990, chapter 2) showed that faster disembodied technical progress reduces the long run unemployment rate through the so-called capitalization e<sup>®</sup>ect. On the other hand, Aghion and Howitt (1994) showed that faster embodied technical change generally leads to a rises in long run unemployment through a creative destruction e<sup>®</sup>ect.<sup>2</sup> Recently, Mortensen and Pissarides (1998) built

<sup>&</sup>lt;sup>2</sup>Postel-Vinay (1997) analyzes the short and long run e<sup>®</sup>ects of embodied technical progress on employment in a matching model. King and Welling (1995) analyzes the relation between disembodied technical progress and unemployment when search is costly

up a canonical matching model with embodied technical progress in which both types of results can be obtained. They showed that the capitalization e<sup>®</sup>ect rests on the assumption that <sup>-</sup>rms are able to update their technology continuously and at no expense, which precludes technological obsolescence, whereas creative destruction arises from the extreme opposite assumption of total irreversibility in the <sup>-</sup>rms' technological choices. Our <sup>-</sup>ndings con<sup>-</sup>rm that of Mortensen and Pissarides. However, unlike Pissarides (1990), we obtain that higher disembodied technical progress generally involves higher unemployment.

We also analyze the relations between technical progress and job creation and destruction in the short run. Most of the empirical literature on job creation and destruction has focused on their cyclical properties. Davis and Haltinwanger (1990, 1992) and Davis, Haltinwanger and Schuh (1996) found on US data that job creation is procyclical and that job destruction is countercyclical and more volatile than job creation. However, the cyclical properties of job reallocation depend on the national economy data. Indeed, job destruction is more volatile than job creation on US data (Davis and Haltiwanger (1992, 1994), and Davis, Haltiwanger and Schuh (1996)) and on UK data (Konings (1993)), whereas volatility appears to be the same on Spanish data (Dolado and G@mez (1995)) or on German data (Boeri and Cramer (1991)). This non-synchronization (or decoupling) between job creation and destruction and higher volatility of job destruction in US economy data have been theoretically analyzed by Caballero and Hammour (1996) in a creative destruction framework.<sup>3</sup> The authors show how decoupling and higher volatility of job destruction can result from the existence of ine±cient decentralized equilibria and increasing marginal creation costs. Boucekkine, del Rio and Licandro (1999) combining endogenous and exogenous °uctuations sources in the Caballero and Hammour (1996)'s creative destruction model show that endogenous replacement echoes generally dominate the short run dynamics and that the combination of the two °uctuation sources favors the appearance of asymmetries in job creation and job destruction patterns. Here, we study the response of job creation and destruction to permanent and transitory technological shocks. We obtain that destruction is more volatile than

and shocks are positively correlated.

<sup>&</sup>lt;sup>3</sup>On this issue see also Mortensen and Pissarides (1994).

creation, and job creation and destruction are not synchronized.<sup>4</sup>

The rest of paper is organized as follows. In Section 2 we describe the model. It is a canonical vintage capital model with embodied and disembodied technical progress and generalized Nash bargaining in the labor market. As in Caballero and Hammour (1996, 1997, 1998), we assume there is an appropriability problem which introduces a kind of wage rigidity and makes the decentralized equilibrium ine±cient.<sup>5</sup> Di®erently to Caballero and Hammour's papers and like Solow (1960), we assume a Cobb-Douglas production function, which allows substitution of factors and aggregation. In Section 3 an Section 4 we respectively relate our long run and short run <sup>-</sup>ndings in an exogenous growth setting. Section 5 analyzes an endogenous growth setting. Section 6 concludes.

## 2 The Economy

At time z a plant is built (we call it vintage z). Each vintage combines capital and workers. The vintage production function is Cobb-Douglas:

$$y_{z;t} = x_t q_{z;t} I_{z;t}^{\text{\tiny (B)}} i_z e^{i \ \pm (t_i \ z)} f_{1_i \text{\tiny (B)}}^{\text{\tiny (C)}};$$

where  $y_{z;t}$  is production in vintage z at time t,  $i_z$  is investment at time z, which undergoes a constant depreciation rate given by  $\pm$ ,  $I_{z;t}$  is employment in vintage z at time t and 0 < @ < 1. Both  $x_t$  and  $q_{z;t}$  express the state of technical progress at time t in vintage z, and it is assumed that

$$\mathbf{x}_{t} = e^{\circ t}; \tag{1}$$

$$q_{z:t} = e^{z+t(t_i z)}$$
: (2)

These technological hypothesis can be interpreted in the following way: Capital used by the plant built at time z embodies a technical progress given by  $e^{z}$ , > 0, and it is updated costlessly at rate 2 > 0. Additionally, we assume that there is an exponential disembodied technical progress given by

<sup>&</sup>lt;sup>4</sup>Di<sup>®</sup>erently to Caballero and Hammour (1996), our results do not depend on an assumption of increasing marginal creation costs.

<sup>&</sup>lt;sup>5</sup>Caballero and Hammour (1998) study the macroeconomic implications of rent appropriation.

 $e^{\circ t}$ ,  $\circ > 0$ , which makes new and old plants more productive in the same way and in the same proportion. These hypothesis of exponential exogenous technical progress will be modi<sup>-</sup>ed in section 5 to introduce endogenous growth in the model. Because the vintage production function is Cobb-Douglas, the vintage life time is in<sup>-</sup>nite and at time t all former vintages are producing. So, aggregate output and employment are respectively given by

$$y_{t} = \int_{i}^{z} x_{t} q_{z;t} I_{z;t}^{*} i_{z} e^{i \pm (t_{i} z)} dz; \qquad (3)$$

and

$$I_{t} = \sum_{i=1}^{Z} I_{z;t} dz;$$
 (4)

There is a continuum of in nitely lived individuals in the interval [0; 1], each endowed with one unit of labor. We assume that all individual have the same linear utility function in consumption. Then, their intertemporal utility at time 0 is given by  $\mathbf{Z}_{1}$ 

$$e^{i rt} c_t dt;$$

where r is the subjective discount rate equal to the interest rate. Let  $g = \frac{1}{\textcircled{0}}(\degree + \_)$  be the stationary growth rate of consumption, investment and output under the technological hypothesis (1) and (2). The following assumption, which is needed to guarantee  $\_$ nite utility, is realized:

Assumption 1

$$r > \frac{\circ + r}{\Re}$$

The good market equilibrium condition is

$$\mathbf{y}_{t} = \mathbf{c}_{t} + \mathbf{i}_{t}$$
(5)

And unemployment is given by

$$u_t = 1 i I_t:$$
 (6)

We assume that a share of investment is speci<sup>-</sup>c and whose quasi rents cannot be protected by contract. So, the appropriable surplus of vintage t is

$$s_{t} = \int_{t}^{z} x_{z} q_{t;z} I_{t;z}^{\circledast} i_{t} e^{i \pm (z_{i} t)} f_{t;z}^{\circledast} i_{t} e^{i \pm (z_{i} t)} f_{t;z}^{(1)} e^{i r(z_{i} t)} dz_{i} (1 i A) i_{t;z} (7)$$

where the parameter A = (0; 1] captures the share of investment  $i_t$  which is speci<sup>-</sup>c.<sup>6</sup> ! <sub>z</sub> is the worker's shadow wage at time z. To form a plant, we think of the <sup>-</sup>rm and workers as entering into a self-enforcing agreement. The two parties recognize that building a plant creates an appropriable surplus and agree to maximize it. So, for all z the following <sup>-</sup>rst order condition is satis<sup>-</sup>ed:

$$^{\mathbb{B}} \mathsf{x}_{\mathsf{t}} \mathsf{q}_{z;\mathsf{t}} \mathsf{I}_{z;\mathsf{t}}^{\mathbb{B}_{i} \ \mathbf{i}} \mathbf{i}_{\mathsf{z}} \mathsf{e}^{i \ \pm (\mathsf{t}_{i} \ z)} \mathbf{c}_{\mathsf{1}_{i} \ \mathbb{B}} = \mathsf{I}_{\mathsf{t}}. \tag{8}$$

This condition states that the marginal productivity of labor in each vintage equals the worker's shadow wage. It determines labor allocation across vintages that maximizes the surplus generated by all vintages. In equilibrium all vintages z < t destroy jobs and only the latest vintage creates new jobs. Therefore,  $I_{t;t}$  equal job creation and job destruction is  $d_t = i \int_{i=1}^{k} I_{z;t} dz$ . We assume that all hired workers by a plant come from the unemployment pool and all -red workers by a plant go to unemployment.

In the labor market,  $\neg$ rm and workers bargain over the appropriable surplus.<sup>7</sup> A generalized Nash bargaining solution, with a share  $\neg 2$  (0; 1) of the surplus going to workers and (1 i  $\neg$ ) going to the  $\neg$ rm, yields the following standard equilibrium conditions:

$$!_{t}I_{t;t} = \frac{I_{t;t}}{u_{t}} - s_{t},$$
(9)

and

$$\mathcal{U}_{t} = (1_{i}^{-})s_{ti}^{-} Ai_{ti};$$
 (10)

where  $\frac{1}{4}$  is the present-discounted value of pro<sup>-</sup>t from vintage t. First equation states that the equilibrium shadow wage is equal to the expected utility °ow received by an unemployed worker. Second equation states that the rm's share of the appropriable surplus minus the unprotected investment must be equal to the present-discounted value of pro<sup>-</sup>t from a vintage.

Firm chooses investment in vintage t in order to maximize its discounted pro<sup>-</sup>t from vintage t,  $\frac{1}{4}$ t, subject to (7) and (10). The <sup>-</sup>rst order condition

<sup>&</sup>lt;sup>6</sup>We assume  $\acute{A}$  higher than zero, which guarantees that unemployment is strictly positive. If  $\acute{A} = 0$ , descentralized and central planning allocations are equal, and unemployment is zero.

<sup>&</sup>lt;sup>7</sup>We assume that workers in each vintage forms a coalition that bargains as a single agent.

to this maximization problem is :

$$Z_{1} (1_{i} \otimes X_{z}q_{t;z}I_{t;z} i_{t} \otimes e^{i \pm (z_{i} t)(1_{i} \otimes e^{i r(z_{i} t)})} e^{i r(z_{i} t)} dz = 1 + \frac{-}{1_{i}} \hat{A}:$$
(11)

Equation (11) states that the discounted marginal return to investment equals its cost, which includes the appropriability costs.

In this economy capital is heterogeneous because is composed by a continuum of vintages of capital goods with di<sup>®</sup>erent embodied productivities. However, as in the Solow's vintage capital model, aggregation is possible, which allows to simplify the analyses.<sup>8</sup> So, if (8) is solved for  $I_{z,t}$  and then substituted in (4) we get

$$I_{t} = \frac{\mu_{t}}{\mathbb{R}_{X_{t}}} \prod_{i=1}^{q} \frac{q_{i}}{\mathbb{R}_{i}} \frac{1}{1} q_{z;t}^{\frac{1}{1}} i_{z} e^{i \pm (t_{i} - z)} dz:$$
(12)

Let be de ned the e ± cient capital at time t as the sum of survival investments at time t weighted by their respective productivities,

$$k_{t} = \int_{i=1}^{t} q_{z;t}^{\frac{1}{1_{i} \otimes}} i_{z} e^{i \pm (t_{i} z)} dz.$$
(13)

Under (1) and (2), its evolution is given by

$$\hat{\mathbf{k}}_{t} = q_{t;t}^{\frac{1}{1,0}} \mathbf{i}_{t \ i} \pm \mathbf{i}_{t} \frac{1}{1 \ \mathbf{k}^{\mathbb{R}}} \mathbf{q}_{z;t}^{\frac{1}{1,0}} \mathbf{k}_{t}:$$
(14)

From the de<sup>-</sup>nition of  $e\pm$ cient capital, given by equation (13), and (12) we obtain

$${}^{\otimes}\boldsymbol{k}_{t}^{1_{i}}{}^{\otimes} = \boldsymbol{k}_{t}; \tag{15}$$

where  $\mathbf{e}_t$  is the worker's detrended shadow wage,  $\mathbf{e}_t = \mathbf{I}_t (\mathbf{x}_t q_{t;t})^{\frac{i-1}{\otimes}}$ , and  $\mathbf{k}_t$  is the detrended e±cient capital per employed worker,  $\mathbf{k}_t = \frac{\mathbf{k}_t}{\mathbf{I}_t} \mathbf{x}_t^{\frac{i-1}{\otimes}} \mathbf{q}_{t;t}^{\frac{i-1}{(1_i \otimes)\otimes}}$ . Detrended aggregate production per employed worker, which denote by  $\mathbf{g}_t$ , can

<sup>&</sup>lt;sup>8</sup>Caballero and Hammour (1996, 1997, 1998) assume a Leontie<sup>®</sup> production function, and its model is hard to handle because the state of the economy at each moment includes the distribution of jobs across vintages, which has a dimension equal to the number of surviving vintages. However, our assumption of a Cobb-Douglas production function allows aggregation, which makes the model analitically simple and manageable.

be rewritten as a function of  $\Re_t$ . If (8) is solved for  $I_{z,t}$  and then substituted in (3), using (12) and the de<sup>-</sup>nition of e±cient capital, we get

$$\mathbf{\mathfrak{F}}_{t} = \mathbf{k}_{t}^{\mathbf{1}_{i}}^{\mathbf{\$}}.$$
 (16)

1

The <sup>-</sup>rst order condition (11) can be rewritten as:

0

$$(1_{i} \ ^{\text{e}}) \ \mathbf{R}_{t}^{i} \ ^{\text{e}} = \mathbf{B}_{t}^{i} \ \mathbf{R}_{t}^{i} \ ^{\text{e}} = \mathbf{B}_{t}^{i} \ \mathbf{R}_{t}^{i} \ ^{\text{e}} = \mathbf{R}_{t}^{i} \ \mathbf{R}_{t}^{i} \ \mathbf{R}_{t}^{i} \ ^{\text{e}} = \mathbf{R}_{t}^{i} \ \mathbf{R}_{t}^{i} \ ^{\text{e}} \ \mathbf{R}_{t}^{i} \ ^{\text{e}} \ \mathbf{R}_{t}^{i} \ ^{\text{e}} \ ^{\text{$$

10

Equation (17) states that marginal productivity of  $e\pm$ cient capital equal its user cost and it is obtained in the following way: equation (8) is solved for  $I_{z,t}$  and then substituted in (11), after (15) is substituted in the resulting equation. Finally, di®erentiating and using (1) and (2), after a little of algebra (17) is obtained. Note that the user cost of  $e\pm$ cient capital depends, additionally to the interest and depreciation rate, on the obsolescence and appropriability costs. So, higher embodied technical progress rate increases the user cost of capital, which a®ects job creation in two opposite ways:<sup>9</sup> (i) higher user cost of capital reduces investment, and job creation also decreases because lower marginal productivity of labor. But, since labor and capital are substitutive, (ii) higher user cost of capital provokes substitution of capital for labor and job creation rises. First e®ect, which call obsolescence e®ect, increases unemployment and second e®ect, which call substitution effect, reduces it.<sup>10</sup>

The following assumption together with Assumption 1 guarantees that the user cost of capital is strictly positive:

Assumption 2

$$\frac{\overset{\circ}{}+\underline{}}{\overset{\circ}{}+\underline{}}+\frac{\underline{}}{1}\frac{\dot{}}{\overset{\circ}{}}\overset{\circ}{}+\pm>0.$$

<sup>&</sup>lt;sup>9</sup>Under (1) and (2),  $\frac{\dot{q}_{t;z}}{q_{t;z}} = 1$ . Note that faster technological updating in °uences user cost of capital with opposed sign to an increase in the embodied technical progress rate.

<sup>&</sup>lt;sup>10</sup>The obsolescence e<sup>®</sup>ect is in our setup the counterpart of the so-called indirect creative destruction e<sup>®</sup>ect by Aghion and Howitt (1994).

Assumption 1 and Assumption 2 also guarantee that job creation and destruction are positive, as will be seen below. From (17) is clear that  $\mathbf{R}_t$  is constant in equilibrium. However, this is not true for employment (and unemployment), which presents transitional dynamics. From the equilibrium conditions, a simple linear di<sup>®</sup>erential equation governing behavior of employment can be obtained:

Proposition 1 Dynamics of employment are given by the following rst order di®erential equation

$$I_{t}^{c} = I_{i}^{c} (I_{t}^{c} + \mu) I_{t};$$
 (18)

and job creation and destruction are respectively given by

$$h_t \quad I_{t;t} = "(1_i \ I_t);$$
 (19)

$$d_t = \mu I_t; \tag{20}$$

where

$$= r + \frac{1}{1_{i}} + \frac{1}{1_$$

$$\mu = \frac{\circ + \frac{1}{2}}{\mathbb{R}} + \frac{\frac{1}{2}}{1}\frac{1}{\mathbb{R}} + \pm.$$
 (22)

 $\mu$  is the job separation rate, which from Assumption 2 is positive, and " is the exit rate from unemployment, which from Assumption 1 and Assumption 2 is also positive.

**Proof:** Since  $\frac{\hat{\mathbf{R}}_t}{\mathbf{R}_t} = 0$  for all t, from the evolution law of capital under the technological assumptions (1) and (2), given by equation (14), and de<sup>-</sup>nition of  $\hat{\mathbf{R}}_t$  follow that

$$\dot{\mathbf{I}}_{t} = \frac{\mathbf{\Phi}_{t}}{\mathbf{R}_{t}} \mathbf{i} \quad \mu$$
(23)

where  $\mathbf{f}_t$  is detrended investment per employed worker,  $\mathbf{f}_t = \frac{i_t}{l_t} (x_t q_{t;t})^{\frac{i-1}{\otimes}}$ . At equilibrium  $\frac{1}{4} = 0$ , since there is constant returns to scale, and equilibrium conditions (9) and (10) yield  $\frac{u_t}{l_t} = \frac{1}{1_i} - \hat{A} \frac{\mathbf{f}_t}{\mathbf{b}_t}$ . From (6), (16), (17), previous equation and given that under (2)  $\frac{\mathbf{f}_{t;z}}{q_{t;z}} = \frac{1}{1_i} - \hat{A} \frac{\mathbf{f}_t}{\mathbf{b}_t}$ , the ratio  $\frac{\mathbf{f}_t}{\mathbf{f}_t}$  can be rewritten as a function of

the parameters and employment,  $\frac{\varphi_t}{R_t} = "\frac{1i \ l_t}{l_t}$ . Substituting previous equation in (23) follows (18). From (8), (15) and (23) the evolution of employment is given by  $l_t = l_{t;t} i \ \mu l_t$ . Evolution of employment equal job creation minus job destruction. Therefore, given that job creation is equal to  $l_{t;t}$ , from (18) and previous equation follows (19) and (20). From Assumption 2,  $\mu$  is positive. From Assumption 1 and Assumption 2, " is also positive.2

From (20) follows that higher embodied and disembodied technical progress rates rise the job separation rate, and job destruction increases, which rises unemployment. This e<sup>®</sup>ect is called by Aghion and Howitt (1994) direct creative destruction e<sup>®</sup>ect. We must stress that in our model this e<sup>®</sup>ect is tied to both types of technical progress. Pissarides (1990) has shown that higher disembodied technical progress rate reduces unemployment because the so-called capitalization e®ect: He found that higher rate of disembodied technical progress rises the present-discounted value of jobs, which increases job creation and consequently reduces unemployment. He assumed an exogenous and constant job separation rate, consequently the creative destruction e<sup>®</sup>ect is not present in his model. In our model, the job separation rate is endogenous and disembodied technical progress in °uences unemployment through the direct creative destruction e<sup>®</sup>ect. But, it does not stimulate job creation because the user cost of capital does not depend on the rate of disembodied technical progress. Note that the user cost of capital can be rewritten as  $R = (r + \mu_i g) 1 + \frac{1}{1i} \dot{A}$ . So, higher ° increases g, which reduces the user cost of capital. But, this reduction is just o<sup>®</sup>set by increasing job separation rate. Therefore, job creation is not in °uenced by disembodied technical progress. In our model, as in Mortensen and Pissarides (1998), higher ability of plants to update their technology reduces unemployment, as will be seen in the next section. It in our unemployment through both job creation and destruction, just in opposite direction to an increase in the embodied technical progress rate.

## 3 The Long Run

Under technological hypothesis (1) and (2) consumption, investment and output in the long run grow at the same constant rate  $g = \frac{\circ + \cdot}{\circ}$ , which we will call it growth rate, and the growth rate of e±cient capital is also constant

and equal to  $g_k = g + \frac{1}{1_i \otimes k}$ . Employment, unemployment and job creation and destruction remain constant. Using equation (18) existence, uniqueness and stability of the steady state can be easily established.

Proposition 2 Under Assumption 1 and Assumption 2 there is an unique and stable steady state, in which consumption and investment are strictly positive, and

$$0 < u = \frac{\mu}{\mu} < 1;$$
 (24)

where  $\mu$  is the job separation rate, given by (22), and " is the exit rate from unemployment, given by (21).

Proof: Assumption 1 guarantees <code>-nite</code> utility. Existence, uniqueness and stability follow from (18) since " > 0 and  $\mu$  > 0 under Assumption 2 and Assumption 1. Equation (24) follows from (18) and (6) if  $\mathbf{i}_t = 0$ . And if  $\mathbf{i}_t = 0$  from (23) follows that  $\mathbf{e}_t = \mathbf{k}_t \mu$ . Solving (17) for  $\mathbf{k}_t$  and substituting in previous equation we obtain that  $\mathbf{e}_t = \mathbf{k}_t \mu$ . Solving (17) for  $\mathbf{k}_t$  and substituting in previous equation we obtain that  $\mathbf{e}_t = \mathbf{k}_t \mu$ , which under Assumption 1 and Assumption 2 is strictly positive. From (5), (16), (17) and previous equation,  $\mathbf{e}_t = \mathbf{i}_{1} \frac{\mathbf{k}_t \mathbf{k}}{\mathbf{k}_t} \mathbf{i}_t \frac{\mathbf{k}_t}{\mathbf{k}_t} \mathbf{k}_t \mathbf{k}$ . Using definitions of R and  $\mu$ ,  $\mathbf{e}_t > \mathbf{i}_{1} \frac{\mathbf{k}_t \mathbf{k}}{\mathbf{k}_t} \mathbf{k}_t \mathbf{k}_t \mathbf{k}_t$ , which from Assumption 1 is higher than zero.2

Although technical progress has opposite e<sup>®</sup>ects on unemployment, relations between long run unemployment and di<sup>®</sup>erent types of technical progress are no ambiguous.

Proposition 3 Under Assumption 1 and Assumption 2, (i) long run unemployment is an increasing function of the disembodied and embodied technical progress rates, and (ii) faster technological updating decreases long run unemployment.

**Proof:** Proposition 3 follows di<sup>®</sup>erentiating (24) with respect to  $^{\circ}$ ; and  $^{\prime}$ , using (21) and (22).2

Unlike Pissarides (1990), disembodied technical progress increases unemployment in our model because the direct creative destruction e<sup>®</sup>ect described in previous section. However, like in the Aghion and Howitt (1994)'s model, embodied technical progress increases unemployment and, like in the Mortensen and Pissarides (1998)'s model, faster technological updating decreases unemployment. But, in our model there is an e<sup>®</sup>ect of technical progress on unemployment, the substitution e<sup>®</sup>ect, that is not present in both Aghion and Howitt (1994)'s model and Mortensen and Pissarides (1998)'s model because they assume that capital and labor are complementary. Note from (24) that u depends negatively on ", which means that the substitution e<sup>®</sup>ect always dominates on the obsolescence e<sup>®</sup>ect. Therefore, Proposition 3 establishes that the creative destruction e<sup>®</sup>ect is higher than the sum of two previous e<sup>®</sup>ects. We have assumed that all employment reallocation needs to be made throughout unemployment. This supposition could be no very realist, since employment reallocation could be made from job to job.

Relations between long run employment reallocation and the technical progress rates are established by the following proposition :

Proposition 4 Long run employment reallocation is given by

$$a + d = \frac{2''\mu}{'' + \mu}$$
: (25)

Under Assumption 1 and Assumption 2, (i) it is an increasing function of the disembodied and embodied technical progress rates, and (ii) faster technological updating reduces long run employment reallocation.

**Proof:** Equation (25) is obtained, after a little of algebra, from the sum of (19) and (20) using (6) and (24). (i) and (ii) follow by di<sup>®</sup>erentiating (25) with respect to  $^{\circ}$ , and  $^{\circ}$ , using (21) and (22).2

### 3.1 Endogenous Interest Rate

We can relax our assumption of linear intertemporal preferences so as to allow for a rate of interest that varies with the growth rate. Suppose that the utility function is replaced by the isoelastic generalization:

$$\int_{0}^{2} e^{i \frac{y_{t}}{2}t} \frac{c_{t}^{1i}}{1 \frac{y_{t}}{34}} dt, \qquad \frac{y_{t}}{34} > 0.$$

Then is easily seen that the rate of interest in steady state equilibrium must be:

$$r = \frac{3}{4} \frac{\circ}{\otimes} + \frac{1}{2} + \frac{1}{2}.$$
 (26)

This generalization adds a new e<sup>®</sup>ect of technical progress on job creation and unemployment: higher embodied or disembodied technical progress raises the rate of interest and consequently the user cost of capital, which reinforces both obsolescence and substitution e<sup>®</sup>ects. As we explain above, the substitution e<sup>®</sup>ect always dominates obsolescence e<sup>®</sup>ect. Therefore, the assumption of endogenous interest rate introduces a new way in which embodied and disembodied technical progress reduce unemployment, which could change the sign of relation between embodied and disembodied technical progress and unemployment. The following proposition establishes parameter conditions that allow a decreasing relation between unemployment and embodied and disembodied technical progress if the interest rate is given by (26).

Proposition 5 Under Assumption 1 and Assumption 2 and the interest rate given by (26), (i) unemployment is a decreasing function of the disembodied technical progress rate if only if

$$\frac{3}{4} > 1 + \frac{\frac{1}{2}}{\frac{1}{10} + \pm};$$
 (27)

and (ii) unemployment is a decreasing function of the embodied technical progress rate if only if

$$\frac{1}{34} > 1 + \frac{\frac{1}{1}}{\frac{1}{1}} \frac{\mu}{R} + \frac{\mu}{1} + \frac{\mu}{1} + \frac{\pi}{R} \frac{\Pi}{R}$$
 (28)

Proof: Di<sup>®</sup>erentiating  $\frac{\mu}{\mu}$  with respect to ° and ; using (26), (21), (22), we obtain:  $\underline{a}_{\mu} = \frac{\mu}{1} \mu \mu$   $\underline{b}_{\mu} = \frac{\eta}{1} \mathbf{a}_{\mu} \mathbf{a}_{\mu}$ 

$$\frac{\overset{@}{\mu}}{\overset{@}{\mu}} = \frac{1}{\overset{@}{\mu}} \overset{\%}{4} 1 + \frac{1}{\overset{@}{-}\dot{A}} \frac{\overset{@}{-}}{(1 \overset{@}{i})} \overset{\#}{i} \frac{}{\mu}$$

which is strictly positive if only if (27) is hold, and

$$\frac{e_{\mu}}{e_{s}} = \frac{1}{e_{\mu}} \frac{\mu}{1_{i}} \frac{e_{\mu}}{e_{s}} + \frac{\pi}{4} \frac{\mu}{1_{i}} + \frac{\pi}{4} \frac{1_{i}}{1_{i}} - \frac{\pi}{4} \frac{e_{\mu}}{1_{i}} \frac{e_{\mu}}{e_{s}} \frac{\mu}{1_{i}} + \frac{\pi}{4} \frac{e_{\mu}}{1_{i}} \frac{e_{\mu}}{e_{s}} \frac{\pi}{4} \frac{\mu}{1_{i}} + \frac{\pi}{4} \frac{e_{\mu}}{1_{i}} \frac{e_{\mu}}{e_{s}} \frac{1_{i}}{e_{\mu}} \frac{\mu}{1_{i}} \frac{e_{\mu}}{e_{s}} \frac{\pi}{4} \frac{\mu}{1_{i}} \frac{e_{\mu}}{e_{s}} \frac{1_{i}}{e_{\mu}} \frac{\mu}{1_{i}} \frac{e_{\mu}}{e_{s}} \frac{1_{i}}{e_{\mu}} \frac{\mu}{1_{i}} \frac{e_{\mu}}{e_{s}} \frac{1_{i}}{e_{\mu}} \frac{\mu}{1_{i}} \frac{1_{i}}{e_{\mu}} \frac{e_{\mu}}{1_{i}} \frac{1_{i}}{e_{\mu}} \frac{1_{i}}{e_{\mu}}$$

which is strictly positive if only if (28) is hold. From (24),  $\frac{@u}{@^{\circ}} < 0$  if only if  $\frac{@\frac{u}{@^{\circ}}}{@^{\circ}} > 0$  and  $\frac{@u}{@^{\circ}} < 0$  if only if  $\frac{@\frac{u}{@^{\circ}}}{@^{\circ}} > 0$ . Then, Proposition 5 follows.2

Therefore, under some parameter conditions, there could be a decreasing relation between unemployment and embodied and disembodied technical progress if the . However, note that employment reallocation is an increasing function of the disembodied and embodied technical progress rates although the interest rate is endogenous and (27) and (28) are ful<sup>-</sup>Iled.

## 3.2 Technological Reassignment

The empirical evidence suggests that US economy has experienced a change in the composition of technical progress post 1974. Boucekkine, del Rio and Licandro (1999b) have shown that this fact can a<sup>®</sup>ect growth and explain the productivity slowdown. Could this fact also a<sup>®</sup>ect unemployment? The following proposition and its corollary provide an a±rmative response to this question.

**Proposition 6** Under Assumption 1, Assumption 2 and a constant growth rate, long run unemployment and employment reallocation are increasing functions of the embodied technical progress rate.

**Proof:** Using (21) and (22), Proposition 6 follows by di<sup>®</sup>erentiating (24) and (25) with respect to  $_{a}$ , holding  $g = \frac{-a}{R} ... 2$ 

**Corollary** Under Assumption 1, Assumption 2 and a constant growth rate, long run employment and employment reallocation are decreasing functions of the disembodied technical progress rate.

# 4 The Short Run

In this section we study the in<sup>°</sup>uence of technological change on the speed of convergence and the response of job creation and destruction to permanent and transitory technological shocks.

## 4.1 The Speed of Convergence

Proposition 7 The speed of convergence for  $I_t$  to its steady state is given by

$$^{\circ} = ('' + \mu);$$
 (29)

(i) it is an increasing function of the embodied and disembodied technical progress rates, and (ii) faster technological updating involves lower speed of convergence for  $I_t$  to its steady state.

Proof: The solution of (18) is:

$$I_t = I + (I_0 i I) e^{i ("+\mu)t}$$

where  $0 < I_0 < 1$  is an initial condition for  $I_t$  and (" +  $\mu$ ) is the speed of convergence for  $I_t$  to its steady state. Using (21) and (22), (i) and (ii) follow by di®erentiating " +  $\mu$  with respect to \_, ° and ´.2

The e<sup>®</sup>ect of faster technological updating on the speed of convergence is opposed to the e<sup>®</sup>ect of higher embodied and disembodied technical progress rates, as follows from previous proposition. The following proposition and its corollary establish the e<sup>®</sup>ects of technological reassignment on the speed of convergence:

Proposition 8 Given a constant growth rate, the speed of convergence for  ${\sf I}_t$  to its steady state is an increasing function of the embodied technical progress rate.

**Proof:** Proposition 8 follows by di<sup>®</sup>erentiating " +  $\mu$  with respect to \_, using (21) and (22), and holding  $g = \frac{a + \mu}{B}$  constant.2

Corollary Given a constant growth rate, the speed of convergence for  $I_t$  to its steady state is a decreasing function of the disembodied technical progress rate.

A consequence of Proposition 8 is that stochastic employment °uctuations will be sharper and less persistent if the fraction of embodied technical progress is larger.

## 4.2 Technological Shocks

Now, we analyze the e<sup>®</sup>ects of permanent and transitory technological shocks on job creation and destruction. Dynamics of these variables are given by (18), (19) and (20). We will see that technological shocks can explain higher volatility of job destruction and non-synchronization between job creation and destruction as observed in data. Permanent embodied shocks In Figure 1 we draw equation (19), which gives job creation as a function of employment, before an embodied shock (line H) and after an embodied shock (line H'), and equation (20), which gives job destruction as a function of employment, before a embodied shock (line D) and after a embodied shock (line D'). Economy is initially in the steady state  $s_0 = (I_0; h_0 = d_0)$ , if a permanent embodied shock occurs it shifts to the new steady state  $s_2 = (I_2; h_2 = d_2)$ , characterized by lower employment and higher employment reallocation, as Proposition 3 and 4 establish. After an embodied shock, job creation and destruction at once jump to  $h_1$  and  $d_1$  and adjust monotonically to their new stationary levels along lines H' and D'. Therefore, job destruction is more volatile than job creation in response to permanent embodied shocks, and both job creation and destruction patterns are decoupled.

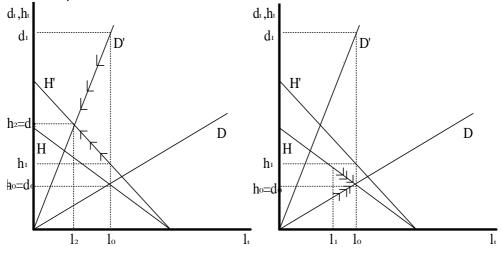


Figure 1: Permanent Embodied Shock.

Figure 2: Transitory Embodied Shock.

Transitory embodied shocks In Figure 2 we draw equation (19), which gives job creation as a function of employment, before an embodied shock (line H) and after an embodied shock (line H'), and equation (20), which gives job destruction as a function of employment, before an embodied shock (line D) and after an embodied shock (line D'). Economy is initially in the steady state  $s_0 = (I_0; h_0 = d_0)$ , if a transitory embodied shock occurs job creation and destruction at once jump to  $h_1$  and  $d_1$ . Employment shifts  $I_1 < I_0$  and job creation and destruction adjust monotonically to their stationary levels

along lines H and D. Therefore, job destruction is more volatile than job creation in response to transitory embodied shocks, and both job creation and destruction patterns are decoupled.

Permanent disembodied shocks In Figure 3 draw equation (19), which gives job creation as a function of employment and remains constant before and after a permanent disembodied shocks (line H), and equation (20), which gives job destruction as a function of employment, before a disembodied shock (line D) and after a disembodied shock (line D'). Economy is initially in the steady state  $s_0 = (I_0; h_0 = d_0)$ , if a permanent disembodied shock occurs it shifts to the new steady state  $s_2 = (I_2; h_2 = d_2)$ , characterized by lower employment and higher employment reallocation, as Proposition 3 and 4 establish. After a disembodied shock, job destruction at once jumps to  $d_1$  and job creation remains in  $h_0$  and both variables adjust monotonically to their new stationary levels along lines H and D'. Therefore, job destruction is more volatile than job creation in response to permanent embodied shocks, and both job creation and destruction patterns are decoupled.

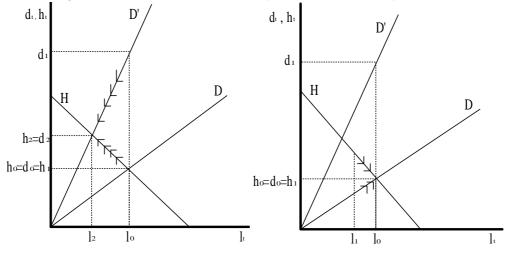


Figure 3: Permanent Disembodied Shock.

Figure 4: Transitory Disembodied Shocks.

Transitory disembodied shocks In Figure 4 we draw equation (19), which gives job creation as a function of employment and remains constant before and after a disembodied shock (line H), and equation (20), which gives job destruction as a function of employment, before a disembodied shock

(line D) and after a disembodied shock (line D'). Economy is initially in the steady state  $s_0 = (I_0; h_0 = d_0)$ , if a transitory disembodied shock occurs, job destruction at once jumps to  $h_1$  and job creation remains constant in  $d_0$ . Employment shifts  $I_1 < I_0$  and job creation and destruction adjust monotonically to their stationary levels along lines H and D. Therefore, job destruction is more volatile than job creation in response to transitory disembodied shocks, and both job creation and destruction patterns are decoupled.

Other technological shocks Reassignment shocks (increases in the fraction of embodied technical progress) and negative shocks in ability of plants to update its technology induce dynamics of job creation and destruction like that described in Figure 1 and Figure 2.

## 5 Endogenous Growth

In this section we substitute technological hypothesis (1) and (2) by the following hypothesis:  $\Pi_{\alpha}$ 

$$\mathbf{x}_{t} = \mathbf{x} \quad \frac{\mathbf{k}_{t}}{\mathbf{l}_{t}} = \mathbf{x} \; \mathbf{k}_{t}^{\circ}, \tag{30}$$

$$q_{z;t} = q_{z;z} = q \frac{\boldsymbol{\mu}_{\boldsymbol{k}_z}}{\boldsymbol{l}_z} = q \boldsymbol{k}_{\hat{z}}.$$
 (31)

With z, q, ° and four strictly positive real numbers. Additionally we assume that: i) social returns to capital are constant, namely  $^{\circ} + \frac{1}{1_{i}} = ^{\circ}$ , and ii) the e<sup> $\circ$ </sup> ects of capital accumulation on technical progress are not internalized by  $^{-}$ rms. As usual, condition i) is needed for a balanced growth paths to exist, and condition ii) is consistent with the existence of a competitive equilibrium. With these assumptions, the system describing the equilibrium of the considered economy is:<sup>11</sup>

$$\overset{\mathfrak{t}}{\mathbf{k}_{t}} = \mathfrak{q}^{\frac{1}{1_{i}} \otimes} \mathfrak{k}_{t}^{\frac{1}{1_{i}} \otimes} \mathbf{i}_{t} \mathbf{i}_{t} \pm \mathbf{k}_{t}, \qquad (32)$$

<sup>&</sup>lt;sup>11</sup>Identical technological assumptions are made by Boucekkine, del Rio and Licandro (1999). The engine of growth is learning by doing, where both states of technology are isoelastic functions of per worker cumulative e±cient investment. The technological specircation  $x_t = x \ \mathbf{k}_t^\circ$  and  $\mathbf{q}_{z;z} = \mathbf{q}$  is consistent with Romer (1986) whereas specircation  $x_t = x \ \mathbf{a}_t \ \mathbf{q}_{z;z} = \mathbf{q} \ \mathbf{k}_z$  is close to Greenwood and Jovanovic (1998), a slightly modired version of Arrow (1962).

$$!_{t} = \frac{-\dot{I}_{t}}{1_{i} - \dot{A}} \frac{\dot{I}_{t}}{1_{i} - I_{t}}$$
(33)

$$^{\otimes}\mathbf{z}\mathbf{k}_{t}^{\mathbf{1}_{i}}\mathbf{\overline{1}_{i}}^{\otimes} = \mathbf{!}_{t}$$

$$(34)$$

$$\mathbf{Z} \mathbf{R}_{t}^{\mathbf{1}_{i}} \stackrel{\overline{\mathbf{1}_{i}} \otimes}{\longrightarrow} \mathbf{I}_{t} = \mathbf{C}_{t} + \mathbf{i}_{t}$$
(35)

$$(1_{i} \ ^{\text{\tiny (B)}}) zq^{\frac{1}{1_{i}} \otimes} = \bigotimes^{\text{\tiny (B)}} r + \frac{1_{i}}{1_{i}} \otimes^{\text{\tiny (B)}} \frac{k_{t}}{k_{t}} + \pm X^{\text{\tiny (D)}} 1 + \frac{1_{i}}{1_{i}} - A^{\text{\tiny (I)}}; \qquad (36)$$

Equation (32) follows from (14) under (30), (31). Equation (33) follows from (6), (9) and (10) given that at equilibrium  $\frac{1}{4t} = 0$ . Equation (34) follows from (15) under (30), (31) and i). Equation (35) follows from (5) and (16) under (30), (31) and i). Finally, equation (36) follows from (17) under (30), (31) and i). The steady state growth rate can be very easily computed from (32) since  $g = \frac{1}{1} \frac{1}{1} \otimes g_{k}$ :

$$g = \frac{\mu_{1}}{\frac{1}{1}} \frac{\mathbb{R}}{\mathbb{R}} \frac{\Pi}{\mathbb{R}} \frac{\tilde{A}}{1 + \frac{1}{1}} \frac{(1 + \mathbb{R}) Z q^{\frac{1}{1}}}{1 + \frac{1}{1}} \frac{1}{\mathbb{R}} \frac{1}{\mathbb{R}}$$

To guarantee that the growth rate is positive at the balanced growth path and that utility at the decentralized equilibrium is bounded, we impose the following assumption:

#### Assumption 3

$$\frac{(1_{i} \ ^{\text{\tiny (B)}}) zq^{\frac{1}{1_{i} \ ^{\text{\tiny (B)}}}}}{1 + \frac{1}{1_{i} \ ^{\text{\tiny (A)}}} A} > r + \pm > \frac{(1_{i} \ ^{\text{\tiny (B)}}) zq^{\frac{1}{1_{i} \ ^{\text{\tiny (B)}}}}{1 + \frac{1}{1_{i} \ ^{\text{\tiny (A)}}} H^{-\frac{1}{1_{i} \ ^{\text{\tiny (B)}}}} \frac{\mu}{1_{i} \ ^{\text{\tiny (B)}} i \ ^{1} + \pm \frac{1}{1_{i} \ ^{\text{\tiny (B)}}}$$

The rst part of this condition implies g > 0. The second part ensures that r > g and equilibrium utility is bounded.

Evolution of employment is equal job creation minus job destruction. The following proposition establishes that both job creation and destruction are linear functions of employment, and consequently evolution of employment is governed by a simple linear di<sup>®</sup>erential equation.

Proposition 9 Dynamics of employment are given by the following rst order di®erential equation

$$\int_{t}^{t} = 3 i (3 + \#) I_{t};$$
 (38)

and job creation and destruction are respectively given by

$$h_t \cap I_{t;t} = {}^3 (1_i \ I_t);$$
 (39)

$$\mathsf{d}_{\mathsf{t}} = \#\mathsf{I}_{\mathsf{t}}; \tag{40}$$

where

$${}^{3} = zq \frac{1}{1_{i} \circ (1_{i} - ) \circ (1_{$$

$$# = \frac{1_{i} @}{1_{i} @} \frac{A}{1 + \frac{1}{1_{i}} @} zq^{\frac{1}{1_{i}} @} i r_{i} \pm \pm .$$
(42)

# is the job separation rate, which from Assumption 3 is positive, and  $^{3}$  is the exit rate from unemployment, which is also positive.

Proof: From (8) and (34) under (30) and (31) follow that  $I_{t,t} = q^{\frac{1}{1_i \otimes}} i_t R_t^{\frac{1}{1_i \otimes}} i^{\frac{1}{1}}$ . From (33) and (34) follow that  $i_t R_t^{\frac{1}{1_i \otimes}} i^{\frac{1}{1}} = z^{\frac{(1_i -)}{A}} (1_i I_t)$ . Substituting last equation in former equation, we get (39), which is job creation because only newest vintage hires workers. From de nition of  $R_t$  and (32),  $\frac{I_t}{I_t} = q^{\frac{1}{1_i \otimes}} \frac{I_t}{I_t} R_t^{\frac{1}{1_i \otimes}} i^{\frac{1}{1}}$   $i \pm i$  $\frac{k_t}{R_t}$ . Solving (36) for  $\frac{k_t}{R_t}$  and substituting in previous equation we get that  $\frac{I_t}{I_t} = q^{\frac{1}{1_i \otimes}} \frac{I_t}{I_t} R_t^{\frac{1}{1_i \otimes}} i^{\frac{1}{1_i}} R_t^{\frac{1}{1_i \otimes}} I^{\frac{1}{1_i \otimes}} I^{$ 

Indeed, although Assumption 1 guarantees the positivity of consumption's growth rate, it does not ensure at rst glance the positivity of consumption as computed from the resource constraint (35). The following proposition provides a condition for this property to hold.

Proposition 10 Under Assumption 3 a unique and stable steady state with positive consumption to output share exists if only if the following condition is ful<sup>-</sup>lled

#### Assumption 4

$$Zq^{\frac{1}{1_i \otimes \infty}} > \#.$$

Stationary unemployment and employment reallocation are respectively given by

$$0 < u = \frac{\#}{3 + \#} < 1; \tag{43}$$

$$a = \frac{2\#}{{}^3 + \#} \tag{44}$$

where <sup>3</sup> is the exit rate from unemployment, given by (41), and # is the job separation rate, given by (42).

Proof: Assumption 3 guarantees - nite utility. Existence, uniqueness and stability follow from (38) since  ${}^3 > 0$  and # > 0 under Assumption 3. Equation (43) follows from (38) and (6) if  $I_t = 0$ . From the sum of (39) and (40), using (43) and (6), follows (44). From (35) follows that  $\hat{A}_t = z_i \frac{i_t}{I_t} R_t^{\overline{1_i \otimes} i}$ . From (33) and (34) follows that  $i_t R_t^{\overline{1_i \otimes} i} = z \frac{(1_i^{-1})^{\otimes}}{Z_A}$  (1 i  $I_t$ ): Then, from two last equations follows that  $\hat{A}_t = z_i \frac{I_i}{I_t} R_t^{\overline{1_i \otimes} i} = \frac{(1_i^{-1})^{\otimes}}{Z_A}$  (1 i  $I_t$ ): Then, from two last equations follows that  $\hat{A}_t = z_i \frac{(1_i^{-1})^{\otimes}}{I_t} \frac{(1_i^{-1})^{\otimes}}{I_t}$ . From (43) and (6) follows that  $\frac{(1_i^{-1})}{I_t} = \frac{\#}{I_t}$ . Therefore, using (41), the stationary value of  $\hat{A}_t$  is given by  $\hat{A} = z_i \frac{1_i}{Z_q \frac{\pi}{1_i \otimes}}$ , which is positive if only if Assumption 4 is ful-lled.2

#### 5.1 Technological Reassignment

In this subsection we analyze the e<sup>®</sup>ects of a change in the composition of technical progress on unemployment, employment reallocation and the speed of convergence for employment to its steady state. <sup>12</sup> Proposition

<sup>&</sup>lt;sup>12</sup>Under identical technological assumptions, Boucekkine, del Rio and Licandro (1999) analyze as a change on the composition of technological progress a®ects the long run growth rate.

11, Proposition 12 and their corollaries establish our <code>-ndings</code>. The response of job creation and destruction to reassignment shocks is analyzed below. Thereafter, technological reassignment is identi<sup>-</sup>ed with changes in <code>\_</code>. Note that we have assumed ° +  $\frac{1}{1_i}$  = <sup>®</sup>. A change in <code>\_</code> must imply an opposite change in ° if <sup>®</sup> remains constant. Therefore, changes in <code>\_</code>, given <sup>®</sup> constant, involve changes in the composition of thecnical progress.

Proposition 11 Under Assumption 3 and Assumption 4, higher fraction of embodied technical progress involves lower long run unemployment and lower long run employment reallocation.

**Proof:** Proposition 11 follows by di<sup>®</sup>erentiating (43) and (44) with respect to  $_{,}$  using (41) and (42). 2

Corollary Under Assumption 3 and Assumption 4 higher fraction of disembodied technical progress involves higher long run unemployment and higher long run employment reallocation.

Proposition 12 The speed of convergence for  $I_{t}$  to its steady state is given by

$$P = (3 + \#);$$
 (45)

and it is a decreasing function of the fraction of embodied technical progress.

Proof: The solution of (38) is:

$$I_t = I + (I_0 i I) e^{i (3 + \#)t}$$

where  $0 < I_0 < 1$  is an initial condition for  $I_t$  and  ${}^3 + \#$  is the speed of convergence for  $I_t$  to its steady state. Proposition 12 follows by di<sup>®</sup>erentiating  ${}^3 + \#$  with respect to  ${}_{a,t}$  using (41) and (42).2

Corollary The speed of convergence for  $I_t$  to its steady state is an increasing function of the fraction of disembodied technical progress.

Technological reassignment has opposite e<sup>®</sup>ects in endogenous and exogenous growth settings. In an exogenous growth setting, as that analyzed in previous sections, higher fraction of embodied technical progress involves higher unemployment and employment reallocation, and faster speed of convergence. However, in an endogenous growth setting, an increase in the fraction of embodied technical progress has opposite e<sup>®</sup>ects on employment reallocation, employment and the speed of convergence. The key economic mechanism behind this result is related to the obsolescence cost speci<sup>-</sup>c to embodied technological progress: Since a drop in the fraction of embodied technical change has a direct e<sup>®</sup>ect on the obsolescence costs, it implies a change in the user cost of capital, which typically determines the growth rates in endogenous growth settings. Therefore, an increase in the fraction of embodied technical progress reduces the growth rate of e±cient capital and consequently the job separation rate (note that  $\# = g_{\mathbb{R}}$ ), which decreases unemployment.

The response of job creation and destruction to reassignment shocks in an endogenous setting is interesting and di<sup>®</sup>erent to that in an exogenous growth setting. It is also veri<sup>-</sup>ed that job destruction is more volatile than job creation and both job creation and destruction patterns are decoupled, but in an endogenous setting only the job separation rate reacts to a reassignment shock, while the exit rate from unemployment remains constant.

Permanent reassignment shocks In Figure 5 we draw equation (39), which gives job creation as a function of employment and remains constant before and after a reassignment shock (line H), and equation (40), which gives job destruction as a function of employment, before a reassignment shock (line D) and after a reassignment shock (line D'). Economy is initially in the steady state  $s_0 = (I_0; h_0 = d_0)$ , if a permanent reassignment shock occurs it shifts to the new steady state  $s_2 = (I_2; h_2 = d_2)$ , characterized by lower employment reallocation and higher employment as Proposition 11 establishes. After a shock, job destruction at once jumps to  $d_1$  and job creation remains in  $h_0$ . Afterward, both variables adjust monotonically to their new stationary levels along lines H and D'. Therefore, job destruction is more volatile than job creation in response to permanent embodied shocks, and both job creation and destruction patterns are decoupled.

Transitory reassignment shocks In Figure 6 we draw equation (39), which gives job creation as a function of employment and remains constant before and after a reassignment shock (line H), and equation (40), which gives job destruction as a function of employment, before a reassignment shock (line D) and after a reassignment shock (line D'). Economy is initially

in the steady state  $s_0 = (I_0; h_0 = d_0)$ , if a transitory reassignment shock occurs, job destruction at once jumps to  $d_1$  and job creation remains constant in  $h_0$ . Employment shifts  $I_1 < I_0$  and after job creation and destruction adjust monotonically to their stationary levels along lines H and D. Therefore, job destruction is more volatile than job creation in response to transitory disembodied shocks, and both job creation and destruction patterns are decoupled.

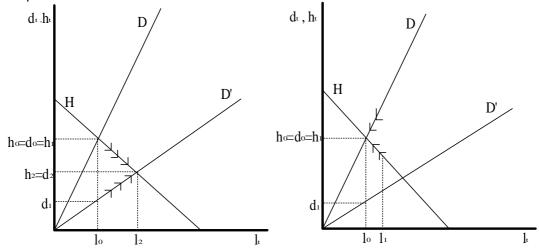


Figure 5: Permanent Reassignment Shocks Figure 6: Transitory

#### Figure 6: Transitory Reassignment Shocks

## 6 Conclusions

In this paper we built up a simple growth model which allows comprehensively to explore the relations between technical progress, unemployment and employment reallocation in the long and short run. Our model includes embodied and disembodied technical progress. Two alternative technological assumptions are realized and its implications analyzed.

In rst place, we develope a vintage exogenous growth model, in which is assumed exponential embodied and disembodied technical progress. Additionally, technology of each vintage is allowed to update at constant rate. In this setup, our rndings can be synthesized as follows: (i) higher embodied and disembodied technical progress rates generally involves higher long run unemployment and employment reallocation, and faster convergence for employment to its steady state. (ii) Faster technological updating involves lower long run unemployment and employment reallocation, and slower convergence for employment to its steady state. (iii) Higher fraction of embodied (resp. disembodied) technical progress involves higher (resp. lower) long run unemployment and employment reallocation, and faster (resp. slower) convergence for employment to its steady state. (iv) Both permanent and transitory technological shocks provoke a response of job destruction more volatile than job creation and the response of both variables is decoupled.

In second place, we develope an endogenous growth model, in which the engine of growth is learning by doing, where both states of technology are isoelastic functions of per worker cumulative e±cient investment. In this setup, we focus on the e<sup>®</sup>ects of technological reassignment (changes in the composition of technical progress) on unemployment and job creation and destruction. We show that an increase in the fraction of embodied technical progress has opposite e<sup>®</sup>ects on employment reallocation, employment and the speed of convergence that in an exogenous growth model. The key economic mechanism behind this result is related to the obsolescence cost speci<sup>-</sup>c to embodied technological progress: Since a drop in the fraction of embodied technical change has a direct e<sup>®</sup>ect on the obsolescence costs, it implies a change in the user cost of capital, which typically determines the growth rates in endogenous growth settings. Therefore, an increase in the fraction of embodied technical progress reduces the growth rate of e±cient capital and consequently the job separation rate, which lows unemployment. The response of job creation and destruction to reassignment shocks in an endogenous setting is di<sup>®</sup>erent to that in an exogenous growth setting. It is also veri<sup>-</sup>ed that job destruction is more volatile than job creation and both job creation and destruction patterns are decoupled, but in an endogenous setting only the job separation rate reacts to a reassignment shock, while the exit rate from unemployment remains constant.

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