

# Bargaining with Endogenous Deadlines<sup>a</sup>

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## Abstract

We develop a two-person negotiation model with complete information which makes endogenous both the deadline and the level of surplus destruction after the deadline. We show that the equilibrium outcome is always unique but might be inefficient. Moreover, as the bargaining period becomes short or as the players become very patient, the unique outcome is always inefficient.

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## 1 Introduction

Negotiations often take place under the pressure of a deadline. The deadline may be exogenously imposed, or one of the parties to the negotiation may have chosen the deadline and made a credible commitment to it. Recent work has focused on bargaining models with exogenous deadlines after which there is no surplus to be divided. See Fershtman and Seidmann [3], Ma and Manove [7]. A key feature of our paper is that we provide a first attempt to endogenize the deadline in negotiation models. We consider a more general definition of a deadline. It is a point in time after which the surplus to be shared is permanently reduced.

Moreover, we allow the players to choose the level of surplus destruction after the deadline. So, in case an agreement is not reached before the deadline, the value of the underlying relationship will be permanently reduced and the level of surplus destruction will depend on the actions of the players. Then, the surplus available to the players once an agreement is reached may actually be lower than it is at the beginning.

One example of bargaining in the face of such endogenous deadlines are wage negotiations between unions and employers. On the one hand, both parties may have the opportunity to settle a deadline after which starts either a strike or a lockout. The deadline may be subject to labour laws : it is common that a strike or a lockout requires few days' notice in order to be legal. On the other hand, a conflict may reduce permanently the probability of the relationship itself by affecting, for example, the future demand for the firm's products. Indeed, customers may decide to buy from now on to some competitor. So, in order to avoid the loss of customers, the firm may decide moving (partially or entirely) the production to another plant or using replacement workers. But not only the firm has actions at its disposal that directly influence the probability of the future relationship. For example, the union may jeopardized production equipment by the lack of scheduled maintenance or skilled operators.<sup>1</sup>

In this paper, we develop a two-stage negotiation model with complete information between a firm and a union. In the first stage, the deadline in force during the wage bargaining is chosen. That is, the firm and the union choose, respectively, a lockout date (and the intensity of the lockout) and a strike date (and the intensity of the strike). Nevertheless, we allow both parties to choose no deadline. In the second stage, both parties

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<sup>1</sup>Cutcher et al. [1] have examined for the U.S. pressure tactics used by unions and employers to influence the process in collective bargaining and its outcomes. In the past, the threats of a strike and the imminent contract expiration deadline have been central features motivating the parties to reach agreements. But in recent years the observations suggest that management threats regarding replacement workers and plant closings or moving are now also a key part of the collective bargaining landscape.

are bargaining over the division of a surplus which is time dependent. Indeed, before the deadline we will have a peaceful bargaining where in each period until a new agreement is reached both parties continue to produce and the value added is shared following the old wage contract. After the deadline we will have an open-contract bargaining (a strike or a lockout has occurred), where in each period until a new agreement is reached both parties get nothing and the value added in later periods (once an agreement is reached) will be affected by the intensity of the contract occurred. The wage bargaining proceeds following Rubinstein's [9] alternating offer bargaining procedure with the firm making the first offer.

We show that the equilibrium outcome of our negotiation model is always unique but might be inefficient. The condition to get inefficiency is satisfied whenever the old wage is relatively small, each player has at his disposal both actions that reduce substantially the value added in the future and actions that have only a minor impact on the future value added, and the players are not impatient. Which is the intuition behind the result? Both players would like that the other player is the last mover at the deadline and preferably facing the threat of a contract of a strong intensity. A strong contract simply means that after the deadline the value added will be reduced substantially. Then, in order to avoid having to accept a very low wage after facing the threat of a severe lockout, where the firm would grab most of the surplus, it becomes optimal for the union to go on strike immediately and to destroy part of the future value added, but not too much. So, at equilibrium we observe both players competing to be the one who will make an offer just before the deadline, but also trying to avoid to have to move at the deadline facing the threat of a very strong contract. This can lead to one party launching the contract immediately and to the conclusion of a Pareto-dominated agreement. Finally, as the bargaining period becomes short or as the players become very patient, the unique outcome is always efficient.

Our two-stage negotiation model is related to papers that derive bargaining inefficiency under complete information<sup>2</sup>, e.g. van Damme, Selten and Winter [11], Fernandez and Gómez [2], Heller and Holden [4]. One important difference is that inefficiency and delay can arise in these other models because there exist multiple efficient equilibria, while the equilibrium is always unique but sometimes inefficient in our model.

Another strand of related literature includes papers on bargaining models with exogenous deadlines. In fact, any finite horizon bargaining model can be interpreted as such. Fershtman and Seidmann [3] study a complete information bargaining model with a random proposer and an exogenous deadline beyond which there is no surplus to divide. They assume that a player cannot accept a lower share of the surplus than she has pre-

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<sup>2</sup> Another source for agreements reached with delay is incomplete information (see e.g. Watson [13]).

viously rejected during the bargaining session. This endogenous commitment assumption together with the deadline imply that, for patient players, there is a unique equilibrium where agreements are delayed until the deadline. This result depends on the interaction between the existence of a deadline and endogenous commitment. Absent the endogenous commitment, the other assumption cannot explain delay. [Maa and Mancove \[7\]](#) construct a bargaining model with complete information, whose unique equilibrium is such that early in the game offers are postponed and late in the game agreements are reached or the deadline is missed with positive probability. To obtain such equilibrium two assumptions are introduced to the infinite-horizon alternating-offer bargaining model. The first one is strategic delay. An alternating-offer model incorporates strategic delay if a player is permitted to postpone the implementation of her move without losing her turn. The second assumption is imperfect player control over the timing of offers during the bargaining session. Offers and counter-offers are exchanged with exogenous random delay.

In these papers just mentioned, the deadline is exogenously determined. Here, we show that once the deadline is endogenous, no other assumptions such as e.g. endogenous commitment or strategic delay is needed to get a unique and efficient equilibrium. Moreover, our result may also justify the existence of Pareto-inferior phenomena other than strikes or lockouts, such as tariff wars, debt moratoria, break-up of cease-fires or wars in general.

The next section presents the basic negotiation model and some preliminary results. In Section 3 we characterize the equilibrium of the deadline stage game and we show that the equilibrium outcome is always unique but might be inefficient. Section 4 concludes.

## 2 The Negotiation Model

We develop a two-stage negotiation model. The timing of this negotiation model is depicted in Figure 1. In the first stage, before the wage bargaining starts at time 0, the

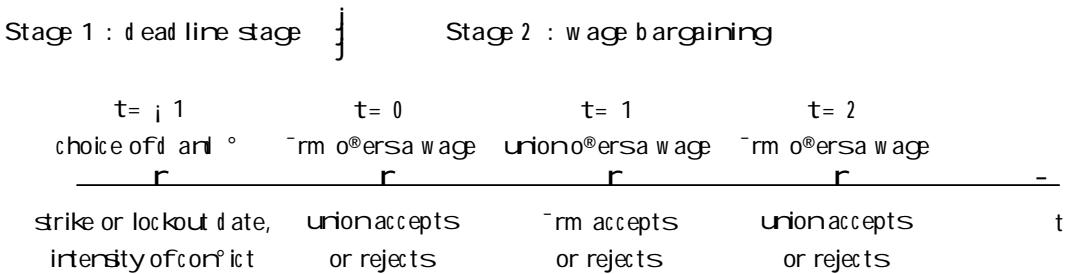


Figure 1: The timing of the negotiation model

Firm and the union choose simultaneously a lockout date  $d_f \in [0; 1; 2; \dots]$ ,  $d_l \in [0; 1; 2; \dots]$  and its intensity  $\alpha_f \in [0; 1; 2; \dots]$  and a strike date  $d_u \in [0; 1; 2; \dots]$ ,  $d_l \in [0; 1; 2; \dots]$  and its intensity  $\alpha_u \in [0; 1; 2; \dots]$ , respectively. We allow both parties to choose no deadline, e.g.  $d_f = 1$  simply means that the firm decides to not choose a lockout date. Afterwards, the players are committed to the deadline and its intensity they have chosen. A deadline rule that seems reasonable and fits perfectly with wage negotiations is the following one. Let  $(d^*)$  be the deadline and its intensity in force during the wage bargaining stage. It is given by

$$(d^*) = \begin{cases} d_f & \text{if } d_f < d_u; \\ d_u & \text{if } d_u < d_f; \\ \min(d_f, d_u) & \text{if } d_u = d_f; \end{cases} \quad (1)$$

That is, the deadline in force and its intensity  $(d^*)$  are determined by the minimum of the deadline choices of the players, and in case of ties, ties are broken in favour of the more intense conflict. This simple deadline rule implies that the wage bargaining will take place facing either the threat of a lockout or the threat of a strike.<sup>3</sup> Finally,  $(1; \emptyset)$  denotes the case where both parties decide to choose nor a strike date nor a lockout date.<sup>4</sup>

In the second stage of the negotiation model, the deadline  $d$  and its intensity  $\alpha$  settled are common knowledge and both parties begin to negotiate. There is an infinite number of periods, and in each period of normal production the firm has a value added of one unit of a good which the firm and the union can divide between them. The union's share is  $W_2[0; 1]$ , the firm's share is  $1 - W_2$ . Two bargaining phases are distinguished. Before the deadline we will have a peaceful bargaining where in each period until a new agreement is reached both parties continue to produce and the value added is shared following the old contract. Initially the wage level in the old contract is  $W_0 > 0$ . After the deadline we will have an open-conflict bargaining (a strike or a lockout has occurred), where in each period until a new agreement is reached both parties get zero and the value added in later periods (once a new agreement is reached) will be affected by the intensity of the conflict occurred,  $\alpha$  (where  $\alpha$  will be equal to  $0$  or  $\alpha < 0 < 1$ ).

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<sup>3</sup>For simplicity and for the sake of presentation we have chosen a specification that excludes the possibility of having simultaneously a lockout and a strike. However, the results we will obtain are, under some weak condition qualitatively robust to another specification where conflicts have an additive destructive impact on the future value added to be shared.

<sup>4</sup>We can also interpret the commitment assumption to start a conflict at the deadline in terms of negotiators' reputation. Imagine that no agreement has been reached and that the union does not go on strike at the chosen deadline. Since the strike date is public knowledge (due to labour laws), the union would lose most of his reputation for the ongoing negotiations as well as for future ones. In other words the results we obtain are robust to the case where the commitment is revocable but the cost of revoking is large enough.

For simplicity both parties are assumed to have linear utility functions, so their payoffs can be represented by the discounted sum of future shares. For the union this is

$$U = \sum_{t=0}^{\infty} \beta^t \mathbb{E} U_t \quad (2)$$

where  $u_t = W_0$  if  $t < d$  and no agreement has yet to be reached,  $u_t = 0$  if  $t \geq d$  and no agreement has been reached, and  $u_t = W_s$  for  $t \geq s$  if an agreement is reached at period  $s$  on  $W_s$ . The common discount factor is  $\pm 1/1.1$ . For the firm we have correspondingly

$$V = \sum_{t=0}^{\infty} \beta^t \Delta V_t \quad (3)$$

where  $v_t = 1$  if  $t < d$  and an agreement has yet to be reached,  $v_t = 0$  if  $t \geq d$  and no agreement has been reached,  $u_t = 1$  if  $t = s$  if an agreement is reached at period  $s < d$  and  $u_t = 0$  if  $t = s$  if an agreement is reached at period  $s \geq d$

The bargaining proceeds following Rubinstein's [9] alternating offer bargaining procedure. The players are assumed to make offers alternately, one offer per period, and without loss of generality the firm is assumed to make an offer in the beginning of period 1. The union can then accept or reject this offer. If the union accepts, the bargaining ends. If the firm's offer is rejected the union makes a new offer in the next period, which the firm accepts or rejects. If the firm accepts, the bargaining ends. If the union's offer is rejected the firm makes a new offer in the next period, and so on until an agreement is reached. Both parties are assumed to have perfect information in the bargaining stage.

We denote  $B(d^\circ)$  the bargaining stage, in other words the alternating offer bargaining game where it is common knowledge that the deadline is  $d$  and its intensity is  $\delta^\circ$ . As in Rubinstein, one can show that the alternating offer bargaining game  $B(d^\circ)$  possesses a unique subgame perfect equilibrium (SPE) and that an agreement is reached without delay at period 1. When the deadline in force is an odd period the SPE wage  $W^\circ(d\text{odd}^\circ)$  and payoffs are

$$W^{\alpha}(d\text{odd}; \cdot) = (1 + \frac{d}{z}) W_0 + \frac{\frac{d}{z} C}{1 + \frac{d}{z}}, \quad (4)$$

$$U^{\pm}(d\text{ odd}; \theta) = \frac{1}{1} \frac{i}{\pm d} \frac{\pm d}{\sin \theta} + \frac{\pm d}{1 - \frac{\pm d}{\sin^2 \theta}}, \quad (5)$$

$$V^{\pm}(d\cos\theta) = \frac{1}{1+i^{\pm}} + \frac{1}{1+i^{\pm}} \Psi_0 i \frac{i^{\pm} C}{1+i^{\pm 2}}. \quad (6)$$

When the deadline in force is an even period the SPE wage $\pi^*(d_{even}, \cdot)$  and payoffs are

$$W^*(d_{even}, \cdot) = (1 - \frac{\pm^d}{1 + \pm}) W_0 + \frac{\pm^d \Phi(1 + \pm i^\circ)}{1 + \pm}, \quad (7)$$

$$U^*(d_{even}, \cdot) = \frac{1 - \frac{\pm^d}{1 + \pm}}{1 - \frac{\pm^d}{1 + \pm} i} W_0 + \frac{\pm^d \Phi(1 + \pm i^\circ)}{1 - \frac{\pm^d}{1 + \pm} i}, \quad (8)$$

$$V^*(d_{even}, \cdot) = \frac{1}{1 - \frac{\pm^d}{1 + \pm} i} \frac{1 - \frac{\pm^d}{1 + \pm}}{1 - \frac{\pm^d}{1 + \pm} i} W_0 + \frac{\pm^d \Phi(1 + \pm i^\circ)}{1 - \frac{\pm^d}{1 + \pm} i}. \quad (9)$$

When the deadline in force is settled at period 0, we enter immediately in an open conflict which affects forever the value added to be shared, and the SPE wage  $W^*(0; \cdot)$  and payoffs are

$$W^*(0; \cdot) = \frac{\pm \Phi}{1 + \pm}, U^*(0; \cdot) = \frac{\pm \Phi}{1 - \frac{\pm^2}{1 + \pm}}, V^*(0; \cdot) = \frac{\cdot}{1 - \frac{\pm^2}{1 + \pm}}. \quad (10)$$

When no deadline is settled, the SPE wage  $W^*(1; \Phi)$  and payoffs are (see Heller and Holden [4]):

$$W^*(1; \Phi) = W_0, U^*(1; \Phi) = \frac{W_0}{1 - \frac{\pm}{1 + \pm}}, V^*(\mu; \Phi) = \frac{1 - \frac{\pm}{1 + \pm} W_0}{1 - \frac{\pm}{1 + \pm}}. \quad (11)$$

Comparing the expressions here above, we obtain the next lemma which gives us some ideas about the preferences of the players over the intensity of the conflict at equilibrium given a deadline  $d$

**Lemma 1** For any deadline  $d$  odd,  $W^*(d, \cdot) < W^*(d^\circ, \cdot)$ ,  $U^*(d, \cdot) < U^*(d^\circ, \cdot)$ , and  $V^*(d, \cdot) > V^*(d^\circ, \cdot)$ . For any deadline  $d$  even,  $W^*(d, \cdot) > W^*(d^\circ, \cdot)$ ,  $U^*(d, \cdot) > U^*(d^\circ, \cdot)$ , and  $V^*(d, \cdot) < V^*(d^\circ, \cdot)$ .

Lemma 1 tells us that, the union prefers the negotiation facing the threat of a conflict of a weak (strong) intensity rather than facing the threat of a conflict of a strong (weak) intensity whenever the deadline is odd (even). A conflict of a strong intensity is simply the case where  $\circ = \underline{\circ}$ , and a conflict of a weak intensity is the case  $\circ = \overline{\circ}$ . On the contrary, the firm prefers the negotiation facing the threat of a conflict of a strong (weak) intensity rather than facing the threat of a conflict of a weak (strong) intensity whenever the deadline is odd (even).

### 3 Endogenous Deadlines

We turn back to stage one of the negotiation model, where the firm and the union choose simultaneously a lookout date  $d \in \{2f_1; 1; 2; \dots; \bar{d}\}$  and its intensity  $\circ \in \{2f_{\underline{\circ}}; \bar{\circ}\}$  and a

strikedated  $d_1$  and its intensity  $\alpha_1$ , respectively.<sup>5</sup> So a strategy for the firm is denoted by  $(d_f; \alpha_f)$  and a strategy for the union is denoted  $(d_u; \alpha_u)$  in the deadline stage game. Remember that the deadline in force during the wage negotiation is the earliest date among the lockout date and the strike date. In case of a tie we simply assume that the conflict with the strongest intensity will start at the chosen deadline. This assumption to break ties is quite reasonable in case of wage negotiations. Nevertheless, the results we obtain are qualitatively robust to a more general assumption where ties are broken with high probability in favour of the strongest conflict.

Since we allow the firm to choose  $(d_f = 0; \alpha_f)$  and the union to choose  $(d_u = 0; \alpha_u)$ , possible inefficiency is not excluded a priori. In Table 1 we represent (very partially) the matrix of the deadline stage game, where the firm is the row player and the union is the column player.

		$0; \alpha_u$	$0; \alpha_u^*$	$1; \alpha_u^*$	$2; \alpha_u^*$	$1; \alpha_u^{\dagger}$
$0; \alpha_f$	$0; \alpha_f$	$0; \alpha_f$	$0; \alpha_f$	$0; \alpha_f$	$0; \alpha_f$	$0; \alpha_f$
$0; \alpha_f^*$	$0; \alpha_f^*$	$0; \alpha_f^*$	$0; \alpha_f^*$	$0; \alpha_f^*$	$0; \alpha_f^*$	$0; \alpha_f^*$
$1; \alpha_f$	$0; \alpha_f$	$0; \alpha_f^*$	$1; \alpha_f$	$1; \alpha_f$	$1; \alpha_f$	$1; \alpha_f$
$2; \alpha_f$	$0; \alpha_f$	$0; \alpha_f^*$	$1; \alpha_f$	$1; \alpha_f$	$2; \alpha_f$	$1; \alpha_f$
$1; \alpha_f^{\dagger}$	$0; \alpha_f$	$0; \alpha_f^*$	$1; \alpha_f$	$1; \alpha_f$	$2; \alpha_f$	$0; \alpha_f$

Table 1: The strategic choice of a deadline

In the matrix we only give the per-period SPE payoff for the firm; the per-period SPE payoff for the union is simply the SPE wage. For solving the deadline stage game, a natural concept would be the Nash equilibrium (NE). But, this concept fails to exclude strategy profiles that seem implausible such as  $f(0; \alpha_f); g(0; \alpha_u)$  and, moreover, there is a multiplicity of NE. As a consequence, we propose to use the trembling-hand perfect equilibrium (THPE) concept as a device to select plausible outcomes among the Nash ones.

In order to characterize the THPE of the deadline stage game, we will use two well-known results (see van Damme [10]). First, since the deadline stage game is a finite two-player game, the strategy profile  $((d_f; \alpha_f); (d_u; \alpha_u))$  is a THPE if and only if it is a NE and neither  $(d_f; \alpha_f)$  nor  $(d_u; \alpha_u)$  is weakly dominated with respect to  $f(1; 2; \dots; d_1) g$ . We say that a player's action is weakly dominated if the player has another action at least

<sup>5</sup>The set of deadlines is assumed to be finite (in order to apply well-known results on finite games) and to be identical for both parties. The upper bound  $\bar{d} = 2N$  is large enough to make the analysis interesting. Obviously, a distinct upper bound for each party would not modify our analysis. Moreover, the results we obtain are not modified if we allow more than two levels of intensity of the conflict.

as good no matter what the other player does and better for at least some action of the other player. Second, every finite game has at least one T H P E.<sup>6</sup>

From Lemma 1 and the weak dominance concept, it is obvious that  $(d; \text{odd}; ^*)$  is weakly dominated by  $(d; \text{odd}; ^\circ)$ . Indeed, (i) against any  $(d_i; ^\circ)$  such that  $d_i > d$  the strategy  $(d; \text{odd}; ^\circ)$  is doing strictly better than  $(d; \text{odd}; ^*)$ , (ii) against any  $(d_i; ^\circ)$  such that  $d_i < d$  the strategy  $(d; \text{odd}; ^\circ)$  is doing as well as  $(d; \text{odd}; ^*)$ , (iii) against any  $(d_i; ^\circ)$  such that  $d_i = d$  the strategy  $(d; \text{odd}; ^\circ)$  is doing strictly better than  $(d; \text{odd}; ^*)$  if  $u = ^*$  and is doing equal if  $u = ^\circ$ . Similarly, one can show that  $(d; \text{even}; ^\circ)$  is weakly dominated by  $(d; \text{even}; ^*)$ ,  $(d; \text{odd}; ^\circ)$  is weakly dominated by  $(d; \text{odd}; ^*)$ ,  $(d_i \in \{0, 1\}; \text{even}; ^\circ)$  is weakly dominated by  $(d_i \in \{0, 1\}; \text{even}; ^*)$ , and  $(d_i = 1; ^\circ)$  is weakly dominated by  $(d_i = 1; ^*)$ .

Lemma 2 The strategies  $(d; \text{odd}; ^*)$ ,  $(d; \text{even}; ^\circ)$ ,  $(d; \text{odd}; ^\circ)$ ,  $(d_i \in \{0, 1\}; \text{even}; ^\circ)$  and  $(d_i = 1; ^\circ)$  are all weakly dominated.

Throughout the paper we will focus on the case where the players are sufficiently patient ( $\pm$  large), which can be reinterpreted as if the interval between offers and counteroffers is short<sup>7</sup> or precisely, we focus on the case where  $\pm > ^* > ^\circ$ .

Assumption 1  $\pm > ^*$ .

Lemma 3 The strategies  $(d; \text{even}; ^*)$ ,  $(d; \text{odd}; ^*)$  are all weakly dominated

Indeed, if  $\pm > ^*$  then  $(d \in \{0, 1\}; \text{even}; ^*)$  is weakly dominated by  $(d+1; ^\circ)$ ,  $(d=1; ^*)$  is weakly dominated by  $(d=1; ^\circ)$ , and  $(d; \text{odd}; ^*)$  is weakly dominated by  $(d+1; ^\circ)$ . Throughout the paper the proofs related to weak dominance results are similar to the one of Lemma 2 and, henceforth, these proofs are omitted. What will be the equilibrium outcomes?

Lemma 4 If  $W_0 < \frac{1+\pm}{1+\pm}^*$  then the strategies  $(d_i = 4 \text{ and even}; ^\circ)$  and  $(d_i = 1; \emptyset)$  are weakly dominated

Lemma 5 If  $W_0 > \frac{1}{1+\pm}^*$  then the strategies  $(d_i = 3 \text{ and odd}; ^\circ)$  and  $(d_i = 1; \emptyset)$  are weakly dominated

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<sup>6</sup>The same results can be obtained using rationalizability concepts (defined in Herings and Vannetelbosch [6],[5], and Vannetelbosch [12]) instead of equilibrium ones for both the bargaining game and the dead line game.

<sup>7</sup>The discount factor can also be expressed by the formula  $\pm = \exp(-r \cdot \ell)$ , where  $r > 0$  is discount rate and  $\ell$  is the length of a single bargaining period.

Could it be that the no deadline situation is an equilibrium outcome? The answer is negative. Indeed, from Lemma 2 to Lemma 5 we already know that, the firm will choose  $(1; \circ)$  for sure if  $W_0 > \frac{\circ}{1+\pm}$  and the union will choose between  $(0; \circ)$  and  $(2; \circ)$  if  $W_0 < \frac{1+\pm\circ}{1+\pm}$ . In order to fully characterize the T H P E outcome we distinguish three cases.

$$\text{Case 1 : } \frac{1+\pm\circ}{1+\pm} > \frac{\circ}{1+\pm}, W_0.$$

It follows from Lemma 2 to Lemma 5 that, at equilibrium, the union could choose either the strategies  $(0; \circ)$  or  $(2; \circ)$  and the firm could choose either the strategies  $(d_f, 1 \text{ odd}; \circ)$  or  $(1; \emptyset)$ . If the union chooses  $(2; \circ)$  then the unique best response for the firm is  $(1; \circ)$ . If the union chooses  $(0; \circ)$  then best responses for the firm are  $(d_f, 1 \text{ odd}; \circ)$  and  $(1; \emptyset)$ . If the firm chooses  $(d_f, 3 \text{ odd}; \circ)$  or  $(1; \emptyset)$  then the unique best response for the union is  $(2; \circ)$ . If the firm chooses  $(1; \circ)$  then the unique best response for the union is  $(0; \circ)$  if  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\circ; i \circ)$  and is  $(2; \circ)$  otherwise. Indeed,  $W^u(0; \circ) = \frac{\pm}{1+\pm} \circ > (1_i \pm)W_0 + \frac{\pm}{1+\pm} \circ = W^u(1; \circ)$  reverts to  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\circ; i \circ)$ . As a consequence, the unique T H P E is  $f(1; \circ); (0; \circ)g$  if  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\circ; i \circ)$  and is  $f(1; \circ); (2; \circ)g$  otherwise.

$$\text{Case 2 : } \frac{1+\pm\circ}{1+\pm} > W_0 > \frac{\circ}{1+\pm}.$$

It follows from Lemma 2 to Lemma 5 that, at equilibrium, the union could choose either the strategies  $(0; \circ)$  or  $(2; \circ)$  and the firm is going to choose for sure the strategy  $(1; \circ)$ . Hence, given that the firm chooses  $(1; \circ)$ , the union will choose  $(0; \circ)$  if  $W^u(0; \circ) = \frac{\pm}{1+\pm} \circ > (1_i \pm)W_0 + \frac{\pm}{1+\pm} \circ = W^u(1; \circ)$ . That is, if  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\circ; i \circ)$ . Otherwise, the union will choose  $(2; \circ)$ .

$$\text{Case 3 : } W_0 > \frac{1+\pm\circ}{1+\pm} > \frac{\circ}{1+\pm}.$$

It follows from Lemma 2 to Lemma 5 that, at equilibrium, the union could choose either the strategies  $(0; \circ)$  or  $(d_u, 2 \text{ even}; \circ)$  or  $(1; \emptyset)$  and the firm is going to choose for sure the strategy  $(1; \circ)$ . Hence, given that the firm chooses  $(1; \circ)$ , the union will choose  $(0; \circ)$  if  $W^u(0; \circ) = \frac{\pm}{1+\pm} \circ > (1_i \pm)W_0 + \frac{\pm}{1+\pm} \circ = W^u(1; \circ)$ . That is, if  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\circ; i \circ)$ . Otherwise, the union will choose  $(d_u, 2 \text{ even}; \circ)$  or  $(1; \emptyset)$ .

Having characterized the equilibrium strategies we can show that the deadline stage game has a unique T H P E. Indeed, if  $W_0 > \frac{\pm}{(1+\pm)(1_i \pm)} (\circ; i \circ)$  then the unique T H P E outcome is  $d = 1$  and  $\circ = \circ$ . So, at equilibrium, the firm and the union will start the wage bargaining under the threat of a severe lockout at period 1. The firm will make a wage offer  $W^u(d=1; \circ) = (1_i \pm)W_0 + \frac{\pm}{1+\pm} \circ$  at period 0, and the union will accept this offer immediately.

**Proposition 1** If  $W_0 > \frac{\pm}{(1+\pm)(1_i \pm)} (\bar{c} i \circ)$  then the negotiation model has a unique and efficient equilibrium outcome. The deadline (a lockout threat of a strong intensity) is settled at period 1 and an agreement on  $W^*(d=1; \circ)$  is reached immediately at period 0.

However, if  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\bar{c} i \circ)$  there is a unique THPE strategy profile where the firm chooses  $(d_f = 1; \circ)$  and the union chooses  $(d_u = 0; \circ)$ . So if  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\bar{c} i \circ)$  the unique THPE outcome is  $d = 0$  and  $\circ = \circ$ , and it is an inefficient outcome since a strike of a weak intensity occurs immediately at the start of the wage bargaining

That is, if the equilibrium wage in case of an immediate strike of a weak intensity is greater than the equilibrium wage in case of the union moving at the deadline and fading the threat of a lockout of a strong intensity, then the union will choose at equilibrium to implement immediately a strike of a weak intensity. Which is the intuition behind this result? Since the odd wage contract is small enough, the difference between  $\circ$  and  $\circ$  is large enough and the players are enough patient, it becomes optimal for the union to go immediately on strike and to destroy part of the available surplus. Indeed, such a conflict of a weak intensity allows the union to avoid having to accept a very low wage offer fading the threat of a severe lockout, where the firm would grab most of the surplus. So, at equilibrium, the union goes into conflict immediately and a Pareto-dominated agreement follows. In fact, this equilibrium outcome is Pareto-dominated by the equilibrium outcome of B ( $d_u = 1; \circ$ ). At equilibrium, the firm will make a wage offer  $W^*(d=0; \circ) = \frac{\pm}{1+\pm}$  at period 0, and the union will accept this offer immediately.

**Proposition 2** If  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\bar{c} i \circ)$  then the negotiation model has a unique and inefficient equilibrium outcome. A conflict of a weak intensity starts immediately at time 0 followed by an immediate agreement on  $W^*(d=0; \circ)$ . The per-period efficiency loss is equal to  $1 - \circ$ .

So the firm at equilibrium chooses  $(d_f = 1; \circ)$  because either it is a dominant strategy or if he does not then the union would choose another deadline and it would be the firm that would face the threat of a severe strike where the union would grab most of the surplus.

The condition  $W_0 < \frac{\pm}{(1+\pm)(1_i \pm)} (\bar{c} i \circ)$  is satisfied whenever the odd wage  $W_0$  is relatively small, each player has at his disposal both actions that reduce substantially the value added in the future and actions that have only a minor impact on the future value added<sup>8</sup> (in

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<sup>8</sup>The actions as well as their impact on the future value added may depend on factors such as the competition on both the product market and the labour market (see Cutcher et al. [1]). Indeed, in case there is a strong competition on the product market, a labour conflict may induce a big loss in market power and future revenues to be generated. However, in case there is a strong competition and mobility on

other words, the difference between  $\hat{\tau}$  and  $\underline{\tau}$  is large enough) and the players are not impatient ( $\pm$  is large enough).

Before concluding we will also consider the limit case of fully patient players as  $\pm$  goes to one.

**Corollary 1** As  $\pm$  goes to one or as the interval between offers and counteroffers vanishes, the negotiation model has a unique and inefficient equilibrium. A conflict of a weak intensity starts at time zero followed by an immediate agreement on  $W^*(d=0; \hat{\tau}) = \frac{1}{2}\hat{\tau}$ .

Notice that we have taken the limit  $\pm \rightarrow 1$  assuming that the players can still reduce permanently the future value added<sup>9</sup>. This assumption may be questionable once we reinterpret the limit as the interval between offers and counteroffers is vanishing. An alternative assumption is to suppose that the level of surplus destruction would be declining with  $\epsilon$  decreasing. Nevertheless, we still obtain the inefficiency result if we assume that the intensities of a conflict are bounded below one. That is, if  $\underline{\tau}(\epsilon) < \hat{\tau}(\epsilon) < \pm$  for all  $\epsilon$  and  $\lim_{\epsilon \downarrow 0} \underline{\tau}(\epsilon) < \lim_{\epsilon \downarrow 0} \hat{\tau}(\epsilon) < 1$ .

## 4 Conclusion

We have developed a two-person negotiation model with complete information which is a first attempt to make endogenous both the deadline and the level of surplus destruction after the deadline. We have shown that the equilibrium outcome is always unique but might be inefficient. Moreover, as the bargaining period becomes short or as the players become very patient, the unique outcome is always inefficient. So, our model may also justify the existence of Pareto-inferior phenomena other than labor conflicts, such as tariff wars, debt moratoria, break-up of cease-fires or wars in general. One very interesting extension would be to consider a more general bargaining procedure as in Perry and Reny [8], which allows the players to choose when and whether or not to make an offer.

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 the labour market, the impact of a conflict could be reduced since the firm may find more easily temporary replacement workers

<sup>9</sup>In case the conflicts have an additive destructive impact on the future value added, a sufficient condition such that, as  $\pm \rightarrow 1$  the THPE outcome is unique and inefficient, is that the intensity of the strongest conflict is large, i.e.  $\underline{\tau}$  is small enough.

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