Markov-Switching Common Dynamic Factor Model with Mixed-Frequency Data

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Abstract

In this paper we consider a coincident economic indicator model with regime-switching dynamics and with the time series observed at different frequencies, for instance, at monthly and quarterly frequencies. Until now the only solution was to drop the lower frequency series and to estimate the model based only on the higher frequency series. This approach leads to the significant information losses. We propose an approach allowing to overcome this problem and to estimate a nonlinear dynamic common factor with the missing observations taking advantage of all the information available.

Keywords: common dynamic factor, Markov switching, mixed frequency data, Kalman filter, composite economic indicator.

JEL Classification: C5, E3.

1 Introduction

The estimation of a coincident economic indicator (CEI) at the regional and national level plays a very important role in measuring and predicting the state of affairs in a given region or country. This indicator can then be used for the political and analytical purposes. Therefore this coincident economic indicator should be readily available, reliable, and representative of the cyclical movements in the main sectors of the economy.

The CEI should satisfy the two defining conditions of the business cycle put forward by Burns and Mitchell and stressed by Diebold and Rudebusch (1996) in their survey of the modern turning points modeling, namely: comovement of the individual macroeconomic series within the cycle and asymmetric business cycle dynamics, when the behavior of the economy during expansions is different from that in the recessions.

Until recently there existed a separation between the model capturing the common dynamics of different macroeconomic variables at the business cycle frequencies, on the one hand, and the nonlinear approach treating different phases of the business cycle asymmetrically, on the other hand. The first approach is, among others, prominently represented by Stock and Watson (1988, 1989 and 1992) who introduced a coincident economic indicator model aiming at extracting a latent, or unobserved, common dynamic model. The second approach was greatly advanced by Hamilton's (1989) breakthrough paper.

However, the need for a model synthesizing these two approaches expressed by Diebold and Rudebusch (1996) called to existence a Markov-switching common dynamic factor model. This approach became feasible thanks to the technique introduced in Kim (1994), which permits to estimate Markov-switching models put in a state-space form. Almost immediately quite a number of applications of this technique to estimating nonlinear common dynamic models surged. Chauvet (1998) and Kim and Nelson (1999) applied it to modeling the common dynamic factor with Markov-switching dynamics, Kim and Yoo (1995) extended this model to the time-varying transition probabilities case.

This model allows to estimate simultaneously both the common factor(s), underlying common dynamics of several macroeconomic time series, and the probabilities of the recessions corresponding to this factor. In other words, this approach incorporates nonlinear dynamics in the common factor extraction by combining the unobserved component model of Stock and Watson with the Markov regime-switching methodology of Hamilton.

However, the practical application of these approach is impeded by the lack of the relevant data measured at high (say, monthly) frequencies. A lot of valuable information is lost because many important time series are only available at the quarterly or annual frequencies. For instance, the CEI estimated with the monthly data does not take into account the information contained in the GDP series which is available only at quarterly or lower frequencies. This problem is especially severe at the regional level, since the regional statistical databases are much more poor than the national ones.

Fortunately, the problem of discrepancy in the frequency of observations seems to be solved. The solution was recently proposed by Mariano and Murasawa (2000). They consider a model where different frequencies, say monthly and quarterly, for different variables entering the model are allowed. This is especially useful if we want our coincident indicator to be a proxy for some aggregate observable variable, e.g. GDP. As a rule the GDP data are released at much lower frequency than individual series characterizing specific sectors of the economy. The Murasawa and Mariano's model enables us to take advantage of the valuable information contained in the lower-frequency time series.

Our idea is to apply this approach to the Markov-switching common dynamic factor model so that to be able to estimate CEI which considers both the comovement of the macroeconomic variables and the asymmetry of the different business cycle phases without losing the important information which is otherwise wasted because of the discrepancies in the observation spacing.

The rest of the paper is structured as follows. In the next section we discuss the technical details of construction and estimation of the Markov-switching common factor models. In the section three we consider application of this methodology to the real data. Section four concludes the paper. All the graphs and tables are put into the Appendix following the list of references.

2 The model and its estimation

The model of the common factor with nonlinear (Markov-switching) dynamics as the one estimated by Kim and Nelson can be expressed as follows:

$$\Delta y_t = \delta + \gamma(L)\Delta c_t + u_t \tag{1}$$

where Δy_t is the $n \times 1$ vector of the first differences of the observed time series in logs; Δc_t is first difference of the unobserved common factor having a regime-switching dynamics; u_t is the $n \times 1$ vector of the specific, or idiosyncratic, components characterizing the individual dynamics of each of the observed series, and $\gamma(L)$ is the lag polynomial in the factor loadings.

The common dynamic factor is modeled as:

$$\phi(L)\Delta c_t = \mu(s_t) + \varepsilon_t \tag{2}$$

where $\phi(L)$ is the $AR(p)$ lag polynomial; $\mu(s_t)$ is the common factor intercept depending on the state variable s_t following a first-order Markov-chain process, and $\varepsilon_t \sim NID(0; \sigma^2(s_t))$ − thus the variance of the common factor shock may also be state-dependent. In a more general specification the coefficients of the autoregressive polynomial $\phi(L)$ may depend on the state too.

The vector of the idiosyncratic components can be represented as follows:

$$\psi(L)u_t = \eta_t \tag{3}$$

where $\eta_t \sim NID(0; \Sigma)$ and both the lag polynomial $\psi(L)$ and variance-covariance matrix Σ have a diagonal structure. Each idiosyncratic component is modelled as $AR(q_i)$ where $i = 1; ...; n$. In principle, the autoregressive order may be different across the specific components and may be equal to zero.

Now assume that different series[1] are observable at different frequencies. Suppose that n_1 time series (y_{1t}) are observed at the lower frequency f, while the rest of the series $n_2 = n - n_1$ (y_{2t}) are measured at a higher frequency, which we may normalize to 1. Denote by y^*_{1t} the unobserved values of the first n_1 measured at the higher frequency. Then the observed series can be expressed in terms of the unobserved as follows:

$$y_{1t} = \frac{1}{f}\sum_{i=0}^{f-1} L^i y^*_{1t} \tag{4}$$

Hence after taking the first difference of the observable lower frequency series, the growth rates of these series would be as:

[1] Here we consider only the case of the flow variables.

$$(1 - L^f) y_{1t} = \frac{1}{f} \left(\sum_{i=0}^{f-1} L^i\right)^2 (1 - L) y_{1t}^* \tag{5}$$

where $(\sum_{i=0}^{f-1} L^i)^2 = \sum_{i=0}^{2f-1} (f + 1 - |i - f|) L^i$ or simpler

$$\left(\sum_{i=0}^{f-1} L^i\right)^2 = 1 + L + 2L^2 + 3L^3 + \dots + 3L^{2f-4} + 2L^{2f-3} + L^{2f-2} + L^{2f-1} \tag{6}$$

Therefore the vector of the growth rates of the observed series may be decomposed as:

$$\begin{pmatrix} (1 - L^f) y_{1t} \\ (1 - L) y_{2t} \end{pmatrix} = \delta + \gamma(L) \begin{pmatrix} \frac{1}{f} (\sum_{i=0}^{f-1} L^i)^2 \\ 1 \end{pmatrix} (1 - L) c_t + \begin{pmatrix} \frac{1}{f} (\sum_{i=0}^{f-1} L^i)^2 \\ 1 \end{pmatrix} u_t \tag{7}$$

In order to be estimated, this model can be expressed in the state-space form.

The measurement equation:

$$\Delta y_t = A s_t + w_t \tag{8}$$

Transition equation:

$$s_t = \mu_t + C s_{t-1} + v_t \tag{9}$$

where $\Delta y_t = \left[(1 - L^f) y_{1t} \quad (1 - L) y_{2t} \right]'$
is the $n \times 1$ vector of observed variables in differences;
$s_t = \left[\Delta c_t^* \quad u_t \right]'$ is the $m \times 1$ state vector containing the common dynamic factor vector $\Delta c_t^* = \left[\Delta c_{t-1} \quad \Delta c_{t-2} \quad \dots \quad \Delta c_{t-r} \right]'$, with $r = \max\{p, 2f - 1\}$;
and the specific components vector
$u_t = \left[u_{1t} \quad \dots \quad u_{1t-l} \quad \dots \quad u_{nt} \quad \dots \quad u_{nt-q_n} \right]'$; with $l = \max\{q_1, 2f - 1\}$;
$\mu_t = \left[\mu(s_t) \quad 0 \quad \dots \quad 0 \right]'$ is the vector of intercepts; and finally
$v_t = \left[\varepsilon_t \quad 0 \quad \dots \quad \eta_{1t} \quad \dots \quad \eta_{nt} \quad \dots \quad 0 \right]'$ is the vector of disturbances:
The dimension of the state vector, m, is determined as:
$m = r + n_1 \times l + \sum_{i=n_1+1}^{n} q_i$
The system matrices have the following structure:
The measurement $n \times m$ matrix:

$$A = \begin{pmatrix} \gamma_1 \; i(f)\delta & & & 0 \\ \gamma_2 & o_{r-1} & i(f)\delta & \\ & & \ddots & \\ \gamma_n & & & i_{q_n} \end{pmatrix}$$

where δ is the $1 \times (2f-1)$ vector of coefficients of the $\left(\sum_{i=0}^{f-1} L^i\right)^2$; o_k is the $k \times 1$ vector of zeros, and i_k is the first row of the $k \times k$ identity matrix, and $i(f)$ is the indicator function:

$$i(f) = \begin{cases} 0, & \text{if } t = 1h, \ldots, (f-1)h \\ 1, & \text{otherwise} \end{cases}$$

where $h = 1, 2, 3, \ldots$

The $m \times m$ transition matrix:

$$C = \begin{pmatrix} \Phi & o_r & & & & 0 \\ I_{r-1} & & & & & \\ & & \phi_1 & o_l & & \\ & & I_{l-1} & & & \\ & & & \ddots & & \\ & & & & \phi_n & o_{q_n} \\ 0 & & & & I_{q_n-1} & \end{pmatrix}$$

where Φ and ϕ_i $(i = 1, \ldots, n)$ are the row vectors of the autoregressive coefficients; I_k is the $k \times k$ identity matrix.

The $n \times n$ variance-covariance matrix of the disturbances to the measurement equation:

$$R = \begin{pmatrix} I(f) & 0 \\ 0 & 0 \end{pmatrix}$$

where $I(f)$ is the diagonal $n_1 \times n_1$ matrix with the indicator functions, $i(f)$, on the main diagonal.

The $m \times m$ variance-covariance matrix of the disturbances to the transition equation:

$$Q = \begin{pmatrix} \sigma^2(s_t) & & & 0 \\ & \ddots & & \\ & & \sigma_1^2 & \\ & & & \ddots \\ 0 & & & \sigma_n^2 \end{pmatrix}$$

We introduce three identifying assumptions in this specification of model. First, the variance-covariance matrix Q is diagonal. Secondly, we may set either $\gamma_1 = 1$ or $\sigma^2(s_t = 1) = 1$. We chose the first option.

The unobserved values of the lower-frequency time series are treated as missing. As Mariano and Murasawa (2000) have shown, they can be replaced by any random variable as soon as it is not correlated with the parameters of the model we are going to estimate. In particular, these missing observations may be substituted by zeros. Thus, the first n_1 observed variables will be constructed as follows:

$$y_{1t} = \begin{cases} 0, & \text{if } t = 1h, \ldots, (f-1)h \\ y_{1t}, & \text{otherwise} \end{cases}$$

In principle, we can do this kind of substitution not only for the observations between the observed values of the lower-frequency time series, but also in case of the series which are shorter than the others. In the general case we may define the indicator function as:

$$i(f) = \begin{cases} 1, & \text{if } t \in \Xi \\ 0, & \text{otherwise} \end{cases}$$

where Ξ is the set of dates for which the shortest time series is observable. For instance, when the t_1 initial observations are missing, the set Ξ will be defined as:

$$\Xi = \{t \mid t > t_1\}$$

In the case when the same variable is also the one which is measured at the lower frequency, the definition of Ξ will be as:

$$\Xi = \{t \mid t > t_1 \text{ and } t \neq 1h, \ldots, (f-1)h\}, \text{ where } h \in \mathbb{Z}$$

We estimate the model using maximum likelihood method. For the derivation of the approximate likelihood function for the common dynamic factor models with Markov switching we refer our reader to Kim and Nelson (1999).

3 Application

3.1 Simulated example

First, in order to check our model, we have simulated a simple common dynamic factor model with a Markov-switching dynamics. The parameters used to simulate the artificial time series are presented in the second column of the Table 1 of Appendix. There were five time series with 540 observations in each generated. Then the first series was chosen to be the low-frequency series. Therefore there were "quarterly" observations calculated as the means over each "quarter". Thus, for this time series we may observe only the data aggregated over each three observations, while the remaining four time series are observed at the "monthly" frequency.

We estimated a nonlinear dynamic common factor model with different observation frequencies, whose structure replicates the DGP of the simulated series. The estimates of the parameters together with the corresponding standard errors, and p-values, are contained in the columns 3 through 5 of the Table 1. The estimated parameters, save for the variance of the idiosyncratic component corresponding to the quarterly observed series, are very close to the true parameters.

Figure 1 comparing the true and estimated common component as well as the true regime with the smoothed conditional probabilities of the regime 2 (recession), also shows striking similarity between the true and estimated series. The conditional regime probabilities sometimes miss the recession when its duration is very short.

Thus, our model, when it corresponds to the DGP of the series, estimates the unknown parameters of the process sufficiently well.

3.2 Real example

Having tested the performance of our model on the artificial data, we applied it to the actual data. The data used are the same as in Mariano and Murasawa (2000) study. These are the quarterly US real GDP series from the first quarter of 1959 till the last quarter of 1998 and four monthly US macroeconomic time series stretching from January 1959 to December 1998, namely: employees on nonagricultural payrolls; personal income less transfer payments; index of industrial production; and manufacturing and trade series[2].

To select the lag order, we applied Akaike information criterion (AIC) and Schwarz Bayesian information criterion (SBIC) computed as follows:

Akaike information criterion:

$$AIC = 2\log[L(\mu)] - 2[n_1 p + n_2 q] \qquad (10)$$

where $L(\mu)$ is the likelihood function value at maximum; n_1 number of the low-frequency series (in this case we have only one such time series - quarterly GDP); n_2 is the number of the high-frequency series; p and q are the orders of the AR polynomials of the low- and high-frequency series, respectively.

Schwarz Bayesian information criterion:

$$SBIC = 2\log[L(\mu)] - [n_1 p + n_2 q]\log(T) \qquad (11)$$

where T is the number of observations.

The values of the log-likelihoods for the various autoregressive order combinations $(p;q)$ as well as the two information criteria are presented in Table 2 of the Appendix. The AIC chooses (3,3) while SBIC selects (1,2) combination as the optimal one. We are going to use the latter combination as a more parsimonious. This is the same combination which was suggested by the SBIC in the linear case (see Mariano and Murasawa (2000)).

We represent the estimates of the parameters of the linear common factor model (taken from Mariano and Murasawa (2000)) and our own estimates of the common factor model with Markov switching in the Table 3 of the Appendix. The estimated parameters for the linear and nonlinear models are very similar, with the exception of the autoregressive parameter of the common dynamic factor which is slightly smaller when the Markov switching is introduced.

Based on the parameter estimates of the nonlinear common factor model with different observation frequencies, we calculated the estimate of the common factor in the same way as it is done in Kim and Nelson (1999), that is,

[2] The data were demeaned and normalized to have unit variance. They were kindly provided to us by Y. Murasawa.

$$c_t = c_{t-1} + \Delta c_t + \delta \qquad (12)$$

where δ is the mean of the common factor computed as in Stock and Watson (1988).

Figure 2 shows the evolution of the common factor and the conditional recession probabilities obtained from the estimation of our model plotted against the NBER recession dates, where the latter are represented by the shading. The smoothed recession probabilities exactly correspond to the NBER recession chronology, the only difference being the recession detected by our model in the very beginning of the sample and missed by the NBER.

4 Summary

In this paper we introduce a Markov-switching common dynamic factor model with missing observations. Until now only the data of the same frequency and with the same length were used to estimate the latent common factor models with Markov-switching dynamics. Building on the extension of the linear common factor model to the case of the data with different observation frequencies proposed by Mariano and Murasawa (2000), we offer a solution to the problem of missing observations in the nonlinear case.

This would allow to prevent the losses of valuable information concerning the evolution of the common dynamic factor which may be contained in the lower-frequency time series and, in general, in the time series with any type of missing values.

References

[1] Chauvet M. (1998) "An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching" International Economic Review 39, 969-96.

[2] Chauvet M., Potter S. (2000) "Coincident and Leading Indicators of the Stock Market" Journal of Empirical Finance 7, 87-111.

[3] Diebold F.X., Rudebusch G.D. (1996) "Measuring Business Cycles: A Modern Perspective" The Review of Economics and Statistics 78, 67-77.

[4] Hamilton J.D. (1994) Time Series Analysis. New Jersey: Princeton University Press.

[5] Kim C.-J. (1994) "Dynamic Linear Models with Markov-Switching" Journal of Econometrics 60, 1-22.

[6] Kim C.-J., Nelson C.R. (1999) State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications. Cambridge: MIT Press.

[7] Kim M.-J., Yoo J.-S. (1995) "New Index of Coincident Indicators: A Multivariate Markov Switching Factor Model Approach" Journal of Monetary Economics 36, 607-30.

[8] Mariano R.S., Murasawa Y. (2000) "A New Coincident Index of Business Cycle Based on Monthly and Quarterly Series" Institute of Economic Research (Kyoto University) discussion paper 518.

[9] Stock J.H., Watson M.W. (1988) "A Probability Model of the Coincident Economic Indicators", NBER working paper 2772.

5 Appendix

Table 1. Simulated example: true and estimated parameters

Parameter	True	Estimated	St. error	p-value
p_{11}	0.95	0.93	0.02	0.0
p_{22}	0.84	0.87	0.03	0.0
¹$_1$	0.4	0.43	0.03	0.0
¹$_2$	-0.6	-0.68	0.05	0.0
°$_2$	0.5	0.44	0.05	0.0
°$_3$	0.8	0.81	0.02	0.0
°$_4$	2.0	2.01	0.06	0.0
°$_5$	1.7	1.73	0.05	0.0
Á	0.6	0.56	0.03	0.0
Ã$_1$	-0.5	-0.61	0.16	0.0
Ã$_2$	0.6	0.59	0.04	0.0
Ã$_3$	-0.1	-0.06	0.05	0.12
Ã$_4$	-0.2	-0.17	0.06	0.0
Ã$_5$	-0.8	-0.84	0.02	0.0
¾$^2_1$	0.25	0.99	0.16	0.0
¾$^2_2$	0.36	0.38	0.02	0.0
¾$^2_3$	0.16	0.16	0.01	0.0
¾$^2_4$	0.49	0.53	0.05	0.0
¾$^2_5$	0.81	0.81	0.06	0.0
¾^2_c	0.16	0.15	0.02	0.0

Table 2. Lag selection analysis

(p,q)	LogLik	AIC	SBIC
(0,0)	-1643.52	-3287.04	-3287.04
(0,1)	-1605.68	-3221.36	-3242.82
(0,2)	-1565.85	-3151.7	-3194.62
(0,3)	-1555.95	-3141.9	-3204.48
(1,0)	-1626.37	-3254.74	-3259.03
(1,1)	-1589.89	-3191.78	-3217.53
(1,2)	-1550.31	-3122.62	-3169.83
(1,3)	-1539.81	-3111.62	-3178.37
(2,0)	-1625.38	-3254.76	-3263.34
(2,1)	-1589.3	-3192.60	-3222.64
(2,2)	-1549.68	-3123.36	-3174.86
(2,3)	-1539.81	-3113.62	-3184.54
(3,0)	-1625.29	-3256.58	-3269.10
(3,1)	-1589.26	-3194.52	-3227.89
(3,2)	-1545.73	-3117.46	-3171.69
(3,3)	-1534.41	-3104.82	-3179.91

Table 3. Results of estimation of linear and Markov-switching models

Parameter	Linear¤	Nonlinear		
	Coe¢cient	Coe¢cient	St. error	p-value
p_{11}	-	0.97	0.01	0.0
p_{22}	-	0.83	0.13	0.0
¹$_{1}$	-	0.06	0.02	0.0
¹$_{2}$	-	-0.39	0.11	0.0
°$_{2}$	0.48	0.49	0.04	0.0
°$_{3}$	0.83	0.83	0.06	0.0
°$_{4}$	2.10	2.10	0.13	0.0
°$_{5}$	1.71	1.72	0.11	0.0
Á	0.56	0.35	0.07	0.0
Ã$_{11}$	-0.02	-0.04	0.10	0.36
Ã$_{12}$	-0.78	-0.79	0.11	0.0
Ã$_{21}$	0.11	0.10	0.05	0.01
Ã$_{22}$	0.45	0.45	0.05	0.0
Ã$_{31}$	-0.04	-0.05	0.05	0.17
Ã$_{32}$	0.02	0.02	0.08	0.43
Ã$_{41}$	-0.03	-0.02	0.07	0.41
Ã$_{42}$	-0.06	-0.06	0.07	0.21
Ã$_{51}$	-0.44	-0.44	0.05	0.0
Ã$_{52}$	-0.22	-0.22	0.05	0.0
¾$_{1}^{2}$	0.19	0.19	0.04	0.0
¾$_{2}^{2}$	0.02	0.02	0.00	0.0
¾$_{3}^{2}$	0.10	0.10	0.01	0.0
¾$_{4}^{2}$	0.26	0.27	0.03	0.0
¾$_{5}^{2}$	0.60	0.60	0.04	0.0
¾$_{c}^{2}$	0.08	0.06	0.01	0.0

* The estimates of the parameters for the linear model are taken from Mariano and Murasawa (2000).

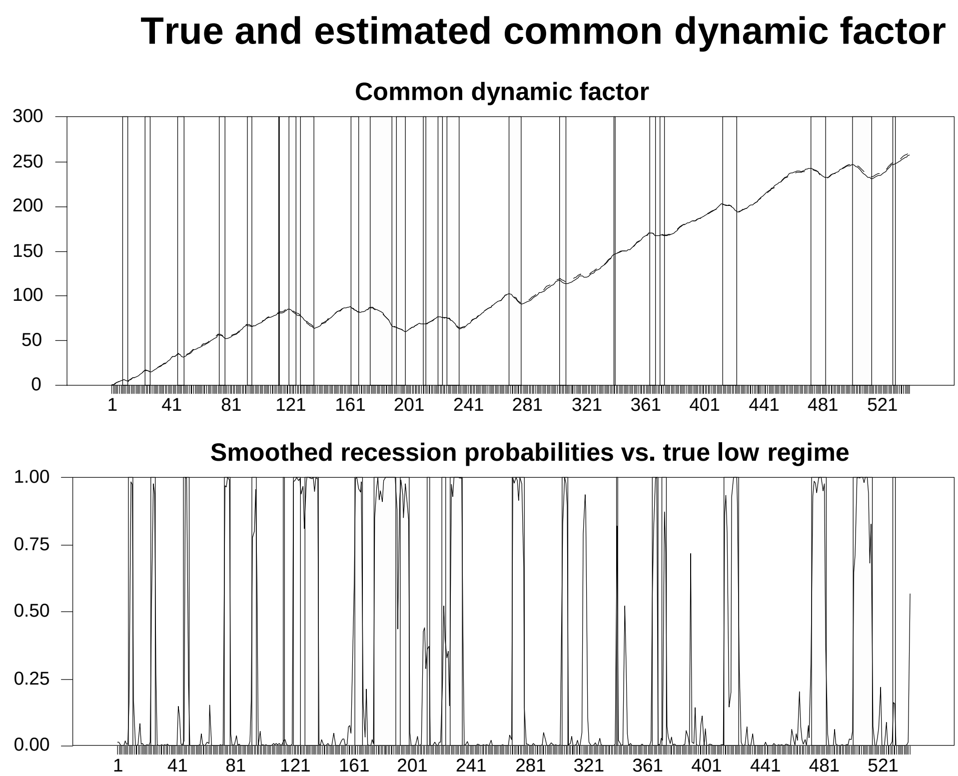
True and estimated common dynamic factor
Common dynamic factor
300
250
200
150
100
50
0
1 41 81 121 161 201 241 281 321 361 401 441 481 521
Smoothed recession probabilities vs. true low regime
1.00
0.75
0.50
0.25
0.00
1 41 81 121 161 201 241 281 321 361 401 441 481 521

Figure 1:

US coincident indicator, 1959:1-1998:12

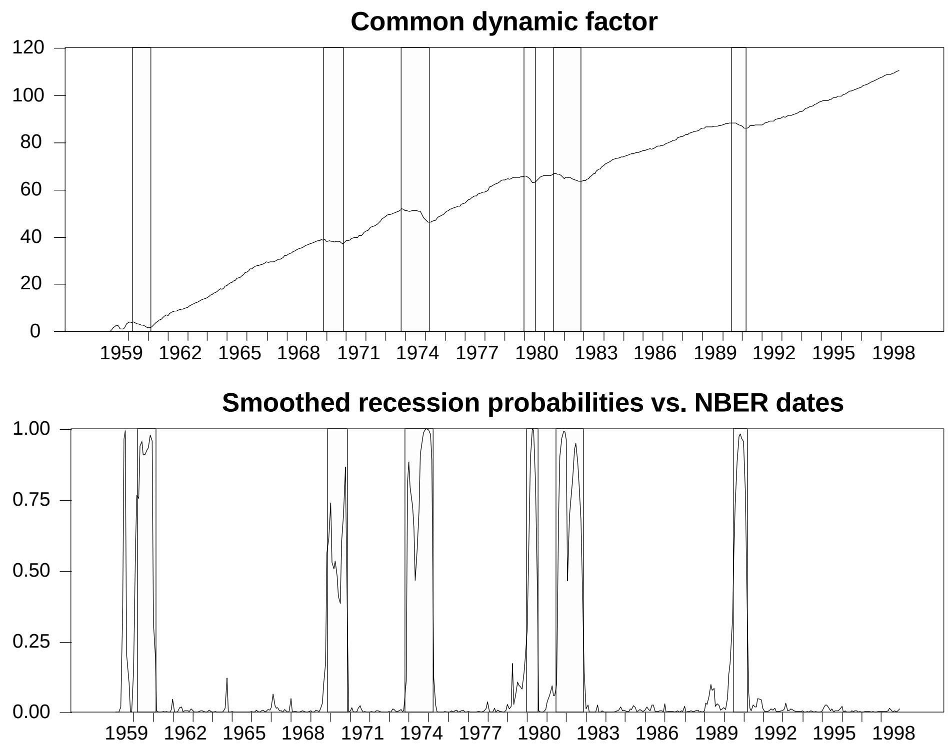

Figure 2: