Learning by Doing, Trade in Capital Goods and Growth

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Abstract

This paper aims at reconciling theoretical models of endogenous growth with the empirical evidence on trade and growth. In particular, we show that the conventional wisdom according to which trade is growth-impairing for a country with comparative advantage in goods with limited opportunities for learning fails to hold when the imported good is a capital good. The intuition is that the country gains access to cheaper capital goods, which raises investment, output per worker and learning by doing.

Key words: Trade, capital goods, growth

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I) Introduction

The last few decades have seen outward-oriented economies, most notably those in East Asia, grow at a more rapid pace than the rest of the world. Consequently, a huge body of empirical literature (see, among them, Levine and Renelt 1992, Harrison 1996, Frankel, Romer and Cyrus 1996 and Edwards 1998) has looked at the link between openness to trade and output growth. Yet, theoretical models proposed so far all have major features that do not stand up well against the data. On the one hand, earlier models, such as Rivera-Batiz and Romer (1991a) and (1991b), or Grossman and Helpman (1991) can explain the observed link between trade and growth but rely on the existence of a scale effect. A larger stock of scientists in an integrated world generates more knowledge and hence faster growth.[1] Unfortunately, recent empirical work based on time series strongly rejects the existence of such a scale effect (Jones 1995 and Backus, Kehoe and Kehoe 1992). On the other hand, models based on human capital (Stokey, 1991) or on learning by doing (Lucas 1988, and Young 1991) predict that an initially backward economy will grow at a slower rate than the rest of the world after opening up to trade, which is inconsistent with the Asian experience and, more generally, with the empirical evidence on trade and growth. The objective of this paper is to present a simple theoretical model that is consistent with the empirical evidence on trade and growth and to use the model to investigate how trade and trade policies affect long-run output growth.

We propose a much simplified model of trade and endogenous growth driven by learning by doing. The crucial difference with the existing literature is that we allow for capital goods

[1] See also Taylor (1994) where dynamic gains from trade arise not only through a scale effect but also through specialization in R&D when the cost of doing R&D differs across countries.

to be traded while the existing literature looks only at trade in consumption goods. This seemingly minor change is actually sufficient to reverse the conclusions of the existing literature. The conventional wisdom in models of open economies with learning by doing is that, "if the industries in which the less developed country has a static comparative advantage are industries in which there are limited opportunities for learning, then the effect of free trade is to speed up learning in the more developed country and to slow it down in the less developed one" (Stokey, 1991). In this paper however, we show that this conventional wisdom need not apply if the imported good is a capital good. Indeed, a country which has a comparative advantage in a low-learning consumption good but which gains access to cheaper capital goods under free-trade is able to accumulate more capital compared to autarky. This raises output per worker and thus learning by doing. Because of the interaction with learning by doing, the impact of capital accumulation on growth is permanent and can outweigh the negative effect of resources being reallocated toward a good with a lower potential for learning.

The model is a two-country (North and South) overlapping generations model. Consumption and capital goods are distinct. New good designs arrive (exogenously) each period. Newer designs are cheaper to produce. North and South differ only in that North is “one generation ahead” of South. In other words, a good design of vintage t in North is available in South only from period $t+1$ onwards. As workers get to use new technologies, they learn to become more efficient. Their knowledge is then passed on to their children. Hence, there are two components to growth in this economy: technological progress, which for simplicity we assume to be exogenous, and learning by doing, which depends on production in the different sectors of the economy.

We first solve for a steady-state equilibrium under autarky. We then allow both countries to trade. The pattern of specialization and trade follows the prediction of the Ricardian trade model whereby countries specialize according to technical comparative advantage. We give conditions under which a balanced growth path under free trade, where North and South grow at the same rate, exists. The world growth rate is always higher under free trade than under autarky. Depending on parameter values, two types of balanced growth path can arise. In both cases, North exports capital goods and imports South's newest consumption good. We show that the impact of an export subsidy (import tariff) depends on the specialization pattern at the equilibrium. If South only is specialized, the policy raises (lowers) the world growth rate. On the other hand, if both countries are specialized, there is no effect of the export subsidy on growth but there is a positive level effect for South.

We contend that these results reconcile theory and evidence on trade and growth. Our model does not rely on any scale effect which is rejected by the data nor does it predict that developing countries fall behind developed countries after opening up to trade. The latter point is important for two reasons. First, and as already mentioned, there is a huge empirical literature demonstrating a *positive* impact of trade on growth in cross countries or panel data growth regressions. [2] Second, an ever increasing gap between poor and rich countries is also inconsistent with, for instance, Parente and Prescott's (1993) observation that the gap between richest and poorest countries has remained constant in the last 25 years[3]. These observations are both consistent with our model where a balanced growth path exists both under autarky and under free trade, and where the world growth rate is higher under free trade.

[2] See however, Rodriguez and Rodrik (1999) for a recent and different perspective on the empirical evidence concerning the link between trade and growth.
[3] See also Jones (1997).

Furthermore, and maybe most importantly, there is direct empirical evidence supporting the importance of a capital accumulation effect in the link between trade and growth. Jones (1994) finds a strong significant positive correlation between price of machinery and growth. Levine and Renelt (1992) looking at a cross section of countries find evidence of a two-link chain between trade and growth through investment and conclude that "the relationship between trade and growth may be based on enhanced resource accumulation and not necessarily on the improved allocation of resources" (p. 954). This finding is also confirmed by Harrison (1996) using panel data and looking at only the developing countries. Together, these results support our contention that countries that are more open to trade grow faster as they gain access to cheaper imported capital goods, accumulate more capital and hence are able to produce more output and generate more learning by doing.

The rest of the paper is organized as follows: In section II, we present the basic model. In section III, we look at the effect of trade on growth when one of the traded good is a capital good. We show that there exists a balanced growth path equilibrium even if South exports a lower learning good under free-trade. The world growth rate is shown to be unambiguously higher under free-trade than under autarky. Section IV looks at the effect of an export subsidy. Section V concludes. Proofs are in the appendices.

II The Model

The model is a two-country (North and South), two-period overlapping generations model. Each agent works in the first period but consumes during both periods. Population of both countries is constant and the size of each generation is normalized to one.[4] We first

[4]This last assumption can easily be relaxed as there is no scale effect in the economy.

describe the production and consumption structures of the North and then proceed to describe the South.

There are two factors of production, human capital (or effective labor) and physical capital. Each period, a new design of the consumption good and of the capital good are invented in North. Agents find consumption goods of different designs to be imperfectly substitutable to each other. On the other hand, new designs of the capital good are perfectly substitutable to the old ones. Newer designs are cheaper to produce and learning by doing occurs only when agents engage in the production of goods of newly arrived designs. We allow for different learning potentials of different goods. Finally, we assume perfect competition.

A) North

Production technology

Production functions are Cobb-Douglas. Factor intensities are assumed to be the same across sectors.[5] Production of new good designs is (exogenously assumed to be) more efficient: a unit of human capital devoted to the production of a good of vintage t has the same productivity as ϕ ($\phi > 1$) units of human capital devoted to the production of a good of vintage *t-1*. Thus, we have:

$$Y_{k,t}^{N} = (\phi^{t} H_{k,t}^{N})^{\alpha} K_{k,t}^{N\,1-\alpha} \qquad (1)$$

$$Y_{t,t}^{N} = (\phi^{t} H_{t,t}^{N})^{\alpha} K_{t,t}^{N\,1-\alpha} \qquad (2)$$

$$Y_{t-1,t}^{N} = (\phi^{t-1} H_{t-1,t}^{N})^{\alpha} K_{t-1,t}^{N\,1-\alpha} \qquad (3)$$

[5] The main result of the paper remains unchanged with different factor intensities provided that human capital intensities in capital and consumption good sectors are not too far apart (proof available upon request).

The first subscript refers to the sector: *k* for capital, *t* for consumption good of vintage t. The second subscript refers to time. The superscript indicates the country, where N stands for North and S for South. We assume that physical capital depreciates fully after one period so that the capital stock at time $t+1$ is equal to the savings of agents at time t[6].

<u>Agent's problem</u>

Agents maximize their expected utility from consumption. Their utility function is assumed to be logarithmic in an index of consumption[7]. The index of consumption is assumed to be Cobb-Douglas in the best two consumption good designs available. Hence, agents choose consumption and savings so as to:

$$
\begin{aligned}
& Max \log {c_{t-1,t}^{N}}^{\varepsilon} {c_{t,t}^{N}}^{1-\varepsilon} + \beta \log {c_{t,t+1}^{N}}^{\varepsilon} {c_{t+1,t+1}^{N}}^{1-\varepsilon} \\
& s.t \\
& P_{t,t}^{N} c_{t,t}^{N} + P_{t-1,t}^{N} c_{t-1,t}^{N} \leq w_t^N h_t^N - P_{k,t}^{N} s_t^N \\
& P_{t+1,t+1}^{N} c_{t+1,t+1}^{N} + P_{t,t+1}^{N} c_{t,t+1}^{N} \leq P_{k,t}^{N} s_t^N r_{t+1}^{N}
\end{aligned}
$$

$c_{i,t}^{N}$ denotes the consumption of the good design which arrives in period *i*, s_t saving and h_t human capital. ε is a positive constant less than 1 which measures the share of the old good design in the consumption of agents.

Note that these preferences are related to the preferences used in Stokey (1988) and Young (1991). Stokey and Young define preferences on the whole set of consumption good designs and derive conditions under which agents optimally choose to consume a finite

[6] The main result of the paper can be shown not to depend on this assumption (proof available upon request), which allows us however to find closed form expressions.

[7] This assumption simplifies the analysis considerably by shutting off the impact of interest rates on savings decisions. Nevertheless, it can be shown that the main results of the paper still hold for as long as the elasticity of inter-temporal substitution is not too different from the logarithmic case.

number of designs. On the other hand, we exogenously impose the condition that agents consume only the two most recent designs of consumption goods. While this exogenous specification is much less appealing than Stokey's and Young's[8], it still captures the idea that there may be some substitution between existing designs but that ultimately older designs become obsolete and disappear[9]. It also buys us two things. First, it allows us to work in an economy with only two traded goods while keeping the main feature of Young's model, namely that a new good in the South may already be a mature product in the North. Second, it allows us to obtain simple closed form solutions for the balanced growth path. We hope that these two features make the intuition for our results more transparent.

Learning and the accumulation of human capital

We make four assumptions as far as learning by doing is concerned:

Assumption 1.:

Young agents inherit the human capital of their parents.

Assumption 2.:

Agents' human capital increases only when they produce a good of a design newly arrived in their country.

Assumption 3.:

The rate at which an agent’s human capital increases depends on the output he produces in

[8] The main problem with the specification is that a necessary condition for agents to consume a finite number of goods is a finite marginal utility at zero, which is obviously not satisfied by logarithmic utilities.

[9] A good example of this is personal computers. Even if you use, say, only word processors and would prefer to buy a cheap 386 to a more expensive Pentium, you will have a hard time finding a 386 in any computer store. A similar example are clothes that you may like but that have gone out of fashion.

the high-learning sectors. The learning function is assumed to be of the form:

$$\frac{H_{t+1}^{N}-H_{t}^{N}}{H_{t}^{N}}=\left(a_{c}\,\frac{Y_{t,t}^{N}}{\phi^{t}H_{t}^{N}}+a_{k}\,\frac{Y_{k,t}^{N}}{\phi^{t}H_{t}^{N}}\right)^{\delta} \tag{4}$$

where H_{t+1}^{N} is the total stock of human capital inherited by young agents born at time t+1 and H_{t}^{N} is the initial human capital of their parents. a_c, a_k, and δ (≤ 1) are positive constants. Note that the rate at which human capital grows does not depend on the size of the economy as we have normalized the RHS of equation (4) by the total quantity of human capital in the economy, $\phi^t H_t$. Such a specification is necessary to eliminate the scale effects present in many learning-by-doing models (see, for instance, Jovanovic 1995).

A last but important remark about Equation (4) is that we have implicitely assumed that human capital is *generic*, that is can be used in any sector of the economy regardless of whether it has been acquired through learning in the consumption good sector or through learning in the capital good sector. This assumption is unlike Lucas (1988) but the same as in Stokey (1988) and Young (1991). Since our model is closest to Young's in spirit, it is a natural assumption for us to make. It is also backed by some indirect empirical evidence[10] and makes the analysis significantly easier since we only need to keep track of one growth rate per country rather than one growth rate per sector and per country. Nevertherless, the same economic forces we emphasize in the paper would also be at play in a model with specific human capital.

Assumption 4.:

The learning coefficient in the capital good sector is at least as high as that in the

[10] See Glaeser, Kallal, Scheinkman and Scheifer (1992) for evidence that knowledge spillovers occur more across than within industries.

consumption good sector, that is, $a_k \geq a_c$.

Assumption 4 means that there is more learning taking place in the production of capital goods than in the production of consumption goods. It is justified by the observation that the capital goods industry is more skilled labor intensive than the consumption good industry and that learning is more important for skill-intensive production processes (Amsden, 1986). It also corresponds to the case generally studied by the existing theoretical literature, where the developing country has a comparative advantage in a good with limited opportunities for learning (cf. *Lemma 1* below).

B) South

The developing country, South, is similar to the developed country, North, in every aspect except that it is one generation behind, in the sense that new good designs are available in the South only one period after they arrived in the North. In other words, Southern producers can produce a good design of vintage *t* only from period *t+1* onwards and Southern consumers can consume a good design of vintage *t* only from period *t+1* onwards. The production lag captures the time required for the technology of production of the new designs to be transfered from North to South.[11] The consumption lag captures the need for infrastructures to be built and the necessity for Northern producers to standardize the good to suit the needs of Southern consumers. For simplicity, we assume that production and consumption lags are the same and equal to one period.[12]

[11] See Vernon (1996) for empirical evidence, Krugman (1979) for a model of North-South technology transfer.
[12] The assumption of the existence of a consumption lag can easily be released without affecting the results of the paper (proof available upon request). The existence of a production lag gives room for trade to arise between North and South. See *Lemma 1* below.

At time t, South produces the capital good[13] and consumption good designs of vintages t-1 and t-2. The production function is the same as in North. Hence:

$$Y_{k,t}^{S} = (\phi^{t-1} H_{k,t}^{S})^{\alpha} K_{k,t}^{S\ 1-\alpha} \quad (5)$$

$$Y_{t-1,t}^{S} = (\phi^{t-1} H_{t-1,t}^{S})^{\alpha} K_{t-1,t}^{S\ 1-\alpha} \quad (6)$$

$$Y_{t-2,t}^{S} = (\phi^{t-2} H_{t-2,t}^{S})^{\alpha} K_{t-2,t}^{S\ 1-\alpha} \quad (7)$$

At time t, agents in South consume good designs of vintages t-1 and t-2. Preferences are the same as in North. Hence, agents solve:

$$\begin{aligned}
&Max \log c_{t-2,t}^{S\ \varepsilon} c_{t-1,t}^{S\ 1-\varepsilon} + \beta \log c_{t-1,t+1}^{S\ \varepsilon} c_{t,t+1}^{S\ 1-\varepsilon} \\
&s.t \\
&P_{t-1,t}^{S}\ c_{t-1,t}^{S} + P_{t-2,t}^{S}\ c_{t-2,t}^{S} \leq \quad w_t^S h_t^S - P_{k,t}^{S}\ s_t^S \\
&P_{t,t+1}^{S}\ c_{t,t+1}^{S} + P_{t-1,t+1}^{S}\ c_{t-1,t+1}^{S} \leq \quad P_{k,t}^{S}\ s_t^S\ r_{t+1}^S
\end{aligned}$$

The learning function in South is the same as in North:

$$\frac{H_{t+1}^S - H_t^S}{H_t^S} = \left(a_c \frac{Y_{t-1,t}^S}{\phi^{t-1} H_t^S} + a_k \frac{Y_{k,t}^S}{\phi^{t-1} H_t^S} \right)^{\delta} \quad (8)$$

C) Comparative Advantage and North-South Trade

To summarize, we have the following production and consumption patterns in the two countries under autarky:

[13] Since the capital goods of different designs are perfectly substitutable, we shall never specify the vintage for capital goods and we always use the subscript k to refer to them, regardless of their vintage.

time-----------------t	------------------------t+1	------------------------
N:	k, t, t-1	k, t+1, t
S:	k, t-1, t-2	k, t, t-1

At any point in time *t*, there are five good designs being produced in the world economy with North and South producing only one consumption good design in common, namely, vintage *t-1*.[14] Nevertheless, as far as trade patterns between North and South are concerned, our model works exactly like a Ricardian model with only two goods, namely, a capital good, k, and a consumption good, t-1.[15] To see this, first note that the marginal rate of transformation between the any pair of goods in each country is a constant independent of time and factor costs. This result arises as a consequence of the assumption of same factor intensities across goods. Furthermore, given the assumptions about preferences, the high quality design produced by North and the low quality design produced by South remain non-traded.[16] Constant marginal rate of transformation and Cobb Douglas preferences together imply that the two non-traded goods absorb a constant fraction of a country's resource endowment regardless of prices.[17] Lastly, the two capital good designs are perfect substitutes and hence can be considered as the same good with the South being less efficient in producing it. Hence, trade between North and South involves the exchange of a capital good and of a (medium quality) consumption good with the pattern of trade being solely determined by technical comparative advantage, just as it is in a 2 good Ricardian model.

[14] To save on notations, we will from now on refer to the two consumption good designs consumed by Northern agents respectively as the high- and medium- quality designs, and to the two good designs consumed by Southern agents as the medium- and low- quality designs. Recall however, that the high quality design at time *t* becomes the medium quality design at time *t+1*.
[15] We are grateful to the anonymous referee for pointing out this interpretation of the model to us.
[16] This feature corresponds to empirical evidence as documented by Keesing and Lall (1992) and Egan and Mody (1992) that developing countries export only their highest quality goods.
[17] cf. equations (A16) and (A23) in the Appendix.

<u>Lemma 1</u>: North has a comparative advantage in the production of capital goods while South has a comparative advantage in the production of the medium quality consumption good design.

<u>Proof:</u> Let $C_{i,t}^{j}(w^{j}, r^{j})$ *be the unit cost of producing good i in country j. Then,*

(1) and (3) $\Rightarrow \dfrac{C_{k,t}^{N}(w^{N}, r^{N})}{C_{t-1,t}^{N}(w^{N}, r^{N})} = \dfrac{1}{\phi^{\alpha}} < 1$

(5) and (6) $\Rightarrow \dfrac{C_{k,t}^{S}(w^{S}, r^{S})}{C_{t-1,t}^{S}(w^{S}, r^{S})} = 1$

An important final remark is that, given *Assumption 4*, *Lemma 1* implies that South has a comparative advantage in the low learning good. According to the existing literature, this should imply that South necessary grows slower under free trade than under autarky since it is importing a high learning good and exporting a low learning good. The contention of this paper is that such a conclusion is not warranted when the high learning good is a capital good. By engaging in trade, South gains access to cheaper capital goods and is able to enjoy a higher level of investment, output per worker and hence, learning by doing, in the consumption good sector.

III Effects of Trade on Growth: Direct and Capital Accumulation Effects

In this section we study the effect of trade on growth when one of the traded goods is a capital good. We first solve for the autarky equilibrium and then compare the autarky eqilibrium to the free trade equilibrium.

A) Autarky Equilibrium

Since the autarky equilibrium can be solved using straightforward techniques, we have restricted all details of the proof to the appendix. We first define a balanced growth path equilibrium under autarky as an equilibrium where interest rate as well as growth rates of output, capital and wages are constant over time:

Definition 1:

A balanced growth path equilibrium under autarky is a competitive equilibrium such that:

(i) The interest rate is constant.

(ii) H_t grows at constant rate g_H^A, output and capital grow at constant rate $\phi + g_H^A$, wages grow at constant rate ϕ

In Appendix A, we show the existence of and characterize the balanced growth path under autarky of this economy. The results are summarized in *Proposition 1* below:

Proposition 1:

There exists a balanced growth path of both economies under autarky. The equilibrium growth rate of human capital is the same in both economies and is implicitly defined by:

$$(g_H^A)^{\frac{1}{\delta}}(1+g_H^A)^{\frac{1-\alpha}{\alpha}} = \left(\frac{\alpha}{\phi}\frac{\beta}{1+\beta}\right)^{\frac{(1-\alpha)}{\alpha}}\left\{a_c\left[\frac{(1-\varepsilon)\alpha}{1+\beta}+(1-\varepsilon)(1-\alpha)\right]+a_k\frac{\alpha\beta}{1+\beta}\right\} \tag{9}$$

Proof: See Appendix A.

Along the steady state autarky growth path, output and capital grow at rate $\phi + g_H^A$. There are two components to the growth rate: ϕ is the exogenous rate of technological progress while g_H^A is an endogenous component coming from learning by doing. The higher is the output of the newest goods, the more learning there is, and the faster the economy grows. Equation (9) uniquely characterizes this endogenous component.

We are now ready to analyze the equilibrium under free trade. Two main questions need to be answered. First, whether there still exists a balanced growth path equilibrium after trade or whether the output of both countries must necessarily diverge. Second, given that there exists a balanced growth path, whether the world growth rate is smaller or larger than under autarky. We address these questions in the next section.

B) Free trade equilibrium

We now allow for trade between the two countries. The high quality design is not available for consumption in South while the low-quality design is not desired by Northern agents. Since the relative cost of production of consumption goods of medium quality design with respect to capital good is lower in South, South exports consumption goods of medium quality design in exchange for capital goods.

Note that while this structure of trade is simplistic, it captures some of the main stylized facts concerning trade patterns between developing and developed countries. First, it is observed that developing countries exchange consumption goods for capital goods. Between 1980 and 1989, the share of machinery and transport equipment in total exports from developed to developing countries was constantly over 40%. By contrast, the same share in the exports from developing to developed countries has been around 5 - 10% for many years and only recently reached 20% (Source: GATT, 1990). Second, developing countries tend to export their highest quality consumption goods (see Keesing and Lall, 1992 or Egan and

Mody, 1992). The pattern of trade in our economy, unlike that of many existing models of trade and growth, captures both these observations.

We first define a balanced growth path equilibrium of an open economy.

Definition 2:

A balanced growth path equilibrium under free trade is a competitive equilibrium such that:

(i) Interest rates are constant in both countries.

(ii) Trade is balanced

(iii) In both North and South, human capital grows at the same constant rate g_H^T, output and capital grow at the constant rate $\phi + g_H^T$, wages grow at the constant rate ϕ.

In *Appendix A*, we show that two types of balanced growth path can arise. They are distinguished by the specialization pattern at the equilibrium. We first define what we mean by complete specialization in our framework and then state the main result of the paper:

Definition 3

South is said to be completely specialized if $Y_{k,t}^S$ is equal to zero, and North is completely specialized if $Y_{t-1,t}^N$ is equal to zero, at the trading equilibrium.

Proposition 2

(i): There exists a balanced growth path equilibrium under free trade iff:

$$\frac{a_k}{a_c} < 1 + \frac{(1-\varepsilon + \frac{\alpha\beta}{1+\beta}\varepsilon)(\phi^{1-\alpha} - 1)}{\frac{\alpha\beta}{1+\beta}}$$

(ii): Furthermore:

South only is completely specialized at the equilibrium iff:

$$1+\frac{\left[1-\varepsilon\left(1-\frac{\alpha\beta}{1+\beta}\right)\right]\phi^{1-\alpha}-1}{\frac{\alpha\beta}{1+\beta}+\left(1-\frac{\alpha\beta}{1+\beta}\right)\varepsilon} < \frac{a_k}{a_c} < 1+\frac{\left(1-\varepsilon+\frac{\alpha\beta}{1+\beta}\varepsilon\right)(\phi^{1-\alpha}-1)}{\frac{\alpha\beta}{1+\beta}}$$

Both countries are specialized iff:

$$\frac{a_k}{a_c} \leq 1+\frac{\left[1-\varepsilon\left(1-\frac{\alpha\beta}{1+\beta}\right)\right]\phi^{1-\alpha}-1}{\frac{\alpha\beta}{1+\beta}+\left(1-\frac{\alpha\beta}{1+\beta}\right)\varepsilon}$$

(iii): The world growth rate is higher than under autarky in both cases.

Proof: *See Appendix A*

Proposition 2 states a few important results. First, it says that even though South is specialized in a good with lower learning opportunities, there exists a balanced growth path as long as a_k and a_c are not too far apart. The result can best be understood by looking at Figure 1. Figure 1 shows the impact of trade on both Northern and Southern growth rates, as a function of the equilibrium world price of the medium quality consumption good $P_{t-1,t}^{W}$. Points A and D in Figure 1 correspond to the autarky equilibria of, respectively, North and South. The segments AB (North) and DE (South) depict a first effect of trade on growth, which we call the *direct effect*. The direct effect does not depend on $P_{t-1,t}^{W}$ and arises solely from the reallocation of resources across sectors with different learning externalities. The direct effect is positive for North since North imports medium quality consumption good and exports capital goods. The larger is the volume of trade, the more resources are being reallocated from the low learning consumption good sector to the high learning capital good sector. As a consequence, the larger is the volume of trade, the higher is North's growth rate, and the more we move to the right of the AB segment. Point B corresponds to the situation where North is completely specialized and where the direct effect has therefore reached its maximum. On the other hand, the direct effect is negative for South as resources are being reallocated from the high learning capital good (South's imports) sector to the low learning consumption good (South's exports) sector. Again, the magnitude of the direct effect increases with the volume of trade and reaches a maximum at the point E when South is completely specialized.

[INSERT FIGURE 1 AROUND HERE]

If the direct effect were the only effect present, the EF portion of the curve for South in Figure 1 would be vertical and there would be no balanced growth path under free trade. However, another effect comes into the picture through capital accumulation. When the terms of trade increases from South's autarky relative price, the real income (and hence savings) of South in terms of capital goods increases. Southern agents accumulate more capital, and capital/labor ratio and output per worker increase. Hence, more learning by doing occurs, which pushes up the growth rate of South. We call this second effect the *capital accumulation effect*. It explains why the EF segment of the curve for South in Figure 1 is upward sloping but not vertical. The higher the price of its exports $P_{t-1,t}^{W}$, the higher is the real income in South in terms of capital goods and therefore the larger is the capital accumulation effect. For the North, since its real income in terms of its export (capital) good does not change as the terms of trade changes, capital accumulation and growth rate remains unchanged as the relative price changes. This explains the vertical segment BC of the curve for North. As long as the direct effect is not too strong, the existence of the positive capital accumulation effect for the South is sufficient to guarantee the existence of a balanced growth path under free trade.[18] In terms of Figure 1, a balanced growth path exists if the two curves intersect.

The second important result in *Proposition 2* is the fact that the world growth rate is unambiguously higher under free trade than under autarky. This result can clearly be visualized from Figure 1 since the curve for North lies to the right of the autarky growth rate and hence an intersection of the two curves must necessarily take place at a growth rate higher than autarky. The intuition for the result is simple: North always benefits from trade because of the direct effect. Trade enables North to reallocate resources away from a sector where all possibilities of learning have been exhausted and into a new sector. Hence, if a balanced growth path under free trade exists at all, it must necessarily be such that South is also growing at a faster rate than under autarky.

[INSERT FIGURE 2 AROUND HERE]

The final result in *Proposition 2* is that two types of equilibria can exist, depending on the specialization pattern at the equilibrium. The two types of equilibria are displayed in Figures 1 and 2, respectively. Figure 1 corresponds to the case when a_k/a_c is large, which implies a strong direct effect for a given volume of trade and therefore relatively long segments AB and DE. Figure 2 corresponds to the case when a_k/a_c is smaller so that the two segments A'B' and D'E' are shorter. Keeping a_c, and hence the slope of the segment DE (D'E') fixed[19], we

[18] Note that capital accumulation affects the growth rate of output on the balanced growth path only because of the interaction with learning by doing. Otherwise, decreasing returns would kick in and there would be no permanent growth effect.
[19] For a proof of this statement, see equation (A20) of the appendix.

observe that the two curves intersect at North's autarky price when a_k is large (Figure 1), thus implying that South needs to be completely specialized, while they intersect at a $P^{W}_{t-1,t}$ strictly between the two autarky prices for a lower a_k (Figure 2), thus implying that both countries need to be completely specialized.

The intuition for the result is as follows. When a_k is large, the negative direct effect of trade on Southern growth is strong. In order to have a balanced growth path, the terms of trade needs to be most favourable to South so that the capital accumulation effect is also at its strongest. At the same time, North needs to be incompletely specialized so that the volume of trade is lower and the positive direct effect on Northern growth weaker. On the other hand, when a_k is smaller, the negative direct effect for South is weaker and a balanced growth path where North is completely specialized as well can arise.

An important remark is that the dependence of specialization patterns on the learning coefficients is only a *long run* property. In the short run, the specialization pattern depends solely on the relative size of the two countries and on productivity coefficients, just like in any standard Ricardian model. As a consequence, any specialization pattern, including one where North is specialized but South is not, is possible, irrespective of the learning coefficients. However, *Proposition 2* deals with balanced growth paths, that is long-run equilibria, where the relative size of the two countries is endogenously determined and is a function (among other things) of the learning coefficients a_c and a_k. The way relative size is determined at the balanced growth path puts a constraint on the possible specialization patterns in the long run. In particular, it can be seen from Figures 1 and 2 that the two curves cannot intersect at a point corresponding to South's autarky price and thus we cannot have an equilibrium where North only is specialized..

[INSERT FIGURES 3 &4 AROUND HERE]

To understand how the equilibrium relative size is endogenously determined in the long run, it can be helpful to start with a situation whereby both the North and South are completely specialized and such that growth rate of North is higher than South, for instance at the initial terms of trade given by P1 in Figure 2. As North grows larger relative to South, the production frontier of North shifts out further relative to South as shown in Figure 3. This leads to an improvement in the terms of trade for South as can be seen from the offer curve analysis in Figure 4. [20] As the terms of trade improves for South, its growth rate increases due to the capital accumulation effect. As long as North's growth rate is higher than South, the relative size of North to South continues to increase and the terms of trade continue to improve for South. If the conditions in *Proposition 2* are satisfied, a terms of trade smaller than the autarky relative price of North can be found which equates the growth rates of the two countries. Once this terms of trade is reached, the relative size of the two countries remains constant. A similar argument explains how the relative size is also endogenously determined in the case of Figure 1, where South only is specialized.

Turning now to empirical evidence, we contend that the capital accumulation effect which is at play in *Proposition 2* is a very natural candidate for explaining the observed relationship between trade and growth. Jones (1994) finds a significant negative relationship between output growth and relative price of machinery. Levine and Renelt (1992) point out that while exports, imports and total trade variables all come significant in growth regressions, the same trade indicators become non significant once one controls for the share

[20] A country's offer curve shows its willingness to trade at various relative prices. The equilibrium terms of trade is determined by the intersection of the two countries' offer curves.

of investment in GDP. Furthermore, they also show that the share of trade in GDP is significantly positively correlated with the share of investment. They conclude from these three observations that their results "indicate that the relationship between trade and growth may be based on enhanced resource accumulation and not necessarily on the improved allocation of resources" (p. 954) while "interestingly, however, the theoretical ties between growth and trade seem to run through improved resource allocation and not through a higher physical investment share" (p. 955). Levine and Renelt's findings are also confirmed by Harrison (1996) who finds that the same positive relationship between trade shares and investment share in GDP[21] in a panel data study based on developing countries' data.

A good example illustrating the importance of the impact of trade on resource accumulation is that of India. India has put a heavy emphasis on the development of a local capital good industry under protection. As shown by Lall (1985), significant learning took place in this industry. According to traditional models of learning by doing, protecting a high-learning industry should have led to faster growth. Yet, India's industrial performance has fallen sharply behind that of the East-Asian NIEs that imported considerable quantities of equipment from abroad. We suggest that one of the major economic forces explaining these two different growth experiences is the capital accumulation effect: because of free-trade, the East-Asian economies had access to cheaper capital goods than India and were thus able to grow faster.

[21] Note however that while Harrison shows that the relationship between trade share and investment is extremely robust, it is less so for other indicators of openness.

IV Trade Policies

In this section, we discuss the effects of an export subsidy policy by South. Under this scenario, for each dollar worth (local prices) of consumption good exported, South gives firms a *s* dollars subsidy. This subsidy is financed by income taxes. As we wish to abstract from redistributive considerations, we assume that a proportional income tax d(s) is levied on agents. This tax leaves the relative distribution of income unchanged. The same exercise applies to export taxes (the case where *s* is negative) which are used to finance a proportional transfer. It also applies to import tariffs since, in our model, imposing an import tariff has the same effect as imposing an export tax.[22]

We first look at the case where South only is specialized. In this case, we show that an export subsidy unambiguously raises the world growth rate. Indeed, North grows faster because of the direct effect as the export subsidy increases the volume of trade. South grows faster as well because it imports more capital goods, thus increasing worker's productivity and learning. On the other hand, the effect on the relative size of North and South is ambiguous. In *Appendix B*, we prove the following proposition:

Proposition 3:

(i): There exists a balanced growth path equilibrium where:

- South only is specialized

- South adopts a proportional export subsidy rate s financed by a proportional income tax

[22] This is because factor intensities are the same in all sectors, which implies that the relative price of traded versus non-traded consumption good is constant and equal to the ratio of total factor productivities.

iff: $\frac{a_k}{a_c} > 1 + \frac{\left(\frac{\phi^{\alpha}(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}\left[1-\varepsilon\,\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\left(1-\frac{\alpha\beta}{1+\beta}\right)\right]-1}{\frac{\alpha\beta}{1+\beta}+\varepsilon\left(1-\frac{\alpha\beta}{1+\beta}\right)}$

and:

$$\frac{a_k}{a_c} < 1 + \frac{\left(\frac{\phi^{\alpha}(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}\left[1-\varepsilon\,\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\left(1-\frac{\alpha\beta}{1+\beta}\right)\right]-\left[1-\varepsilon\left(1-\frac{\alpha\beta}{1+\beta}\right)\right]}{\frac{\alpha\beta}{1+\beta}}$$

(ii): The world growth rate is higher than under free-trade.

(iii): Furthermore, if we define: $R(s) \equiv \frac{H^S}{H^N}(s)$, *then:*

a) R'(0) > 0 iff: $\phi^{1-\alpha} < \frac{a_c(1-\varepsilon)(1-\frac{\alpha\beta}{1+\beta}) + a_k\,\frac{\alpha\beta}{1+\beta}}{a_c\alpha(1-\varepsilon)}$

b) R''(s) < 0

Proof: See Appendix B

A subsidy raises the world growth rate because resources are not allocated efficiently[23] across countries at the free trade equilibrium. North keeps producing the intermediate consumption good even though learning has already been exhausted. An export subsidy increases the volume of trade and reallocates resources more efficiently across countries: North produces less of the intermediate consumption good and more of the capital good. Hence an export subsidy by South is growth enhancing when South only is completely specialized. On the other hand, in terms of levels, which country benefits the most from the

[23] "Efficiently" is used here in the sense of "growth maximizing" and not as "welfare maximizing".

subsidy is unclear. Part (iii) of *Proposition 3* gives a condition on parameter values for which the size of South relative to North goes up with the introduction of a small export subsidy.

It is important to point out that *Proposition 3* has drastically different policy implications from earlier work on models of learning by doing and trade. The policy recommendation of Lucas (1988) is to "pick winners" (namely goods with a high potential for learning) and subsidize their production. What *Proposition 3* tells us is that subdizing the export of a low learning good may nevertheless raise the country's growth rate if by so doing the country's capital/ labor ratio increases. Obviously, such an effect can not be captured in a model where labor is the only factor of production and capital goods do not exist.

We now turn to the equilibrium where both North and South are specialized. In *Appendix B*, we prove the following:

<u>Proposition 4:</u>

(i): There exists a balanced growth path where

- both North and South are specialized

- South adopts a proportional export subsidy rate s financed by a proportional income tax

iff: $$\frac{a_k}{a_c} \leq 1 + \frac{\left(\frac{(1+\beta)}{\alpha s\beta + (1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}} \phi^{1-\alpha}[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})]-1}{[\frac{\alpha\beta}{1+\beta}+\varepsilon(1-\frac{\alpha\beta}{1+\beta})]}$$

(ii): The world growth rate is the same as under free trade.

(iii): The relative size of South is larger than under free trade.

What happens in the case where both countries are specialized is quite different from the case where South only is specialized. South's export subsidy now has a price effect that it did not have before. When South tries to implement an export subsidy policy to benefit from the capital accumulation effect, world prices adjust. In the steady-state North's exports are the same as under free-trade. Hence, North allocates resources in the same way and grows at the same rate. On the other hand, since the income tax lowers the demand for the old consumption good while the subsidy raises the demand for exports, South reallocates resources from the (low-learning) non-traded good sector to the (high-learning) export sector. At first South grows faster than North and hence becomes larger. This leads to a change in the terms of trade against South thus reducing the capital accumulation effect and brings back South's growth to North's level at the steady-state.

To summarize *Propositions 3 and 4*, the effect of an export subsidy by the developing country largely depends on the specialization pattern at the equilibrium. The export subsidy has a growth effect only if South is a small country in the sense that it is completely specialized while North is not. In the case where both countries are specialized, the export subsidy does not have any impact on the long-run growth rate but has a positive level effect. In both cases, the outcome of the export subsidy is different from that in Lucas (1988) and Young (1991), where subsidizing the good with the most (least) learning externalities always has a positive (negative) growth impact.

We end this section with two last comments. First, it is important to recall that we are conducting a steady-state analysis. Hence, a trade policy that may be growth maximizing in the long-run is not necessarily welfare enhancing. Second, the set of policies we have analyzed in this section is very limited. We are not making any claim that export subsidies are the optimal policy instrument, even in the reduced sense of being growth maximizing. In particular, one can show that in some cases a small production subsidy has a larger impact on the long-run growth rate than an export subsidy of the same size[24].

V Conclusion

In this paper, we constructed a simple two-country model that is consistent with basic stylized facts about growth and trade patterns. We have shown that the conventional wisdom according to which exporting goods with limited learning potential and importing high learning goods is growth-impairing fails to hold when the imported good is a capital good. We contend that this result reconciles open economy endogenous growth models with the empirical evidence on trade and growth and, especially, with the growth experience of the outward-oriented East-Asian NIE's of the last few decades. The main lesson of the paper is that having access to cheap imported capital goods may be sufficiently important for a developing country as to dominate any other negative impact of trade on growth.

[24] Proof available upon request.

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Appendix A

Proof of Proposition 1:

In what follows, the capital good is taken as the numeraire good. At time *t*, goods *t, k* and *t-1* are produced in the North under autarky. With perfectly competitive factor markets, factor rewards are equal to their marginal products:

$$w_t^N = P_{t,t}^N \alpha(\phi^t)^\alpha (\frac{K_{t,t}^N}{H_{t,t}^N})^{1-\alpha} = \alpha(\phi^t)^\alpha (\frac{K_{k,t}^N}{H_{k,t}^N})^{1-\alpha} = P_{t-1,t}^N \alpha(\phi^{t-1})^\alpha (\frac{K_{t-1,t}^N}{H_{t-1,t}^N})^{1-\alpha} \qquad \text{(A1)}$$

$$r_t^N = P_{t,t}^N(1-\alpha)(\phi^t)^\alpha (\frac{K_{t,t}^N}{H_{t,t}^N})^{-\alpha} = (1-\alpha)(\phi^t)^\alpha (\frac{K_{k,t}^N}{H_{k,t}^N})^{-\alpha} = P_{t-1,t}^N(1-\alpha)(\phi^{t-1})^\alpha (\frac{K_{t-1,t}^N}{H_{t-1,t}^N})^{-\alpha}$$

(A2)

w_t and r_t are the wage rate and rental rate, respectively. $P_{i,t}$ denotes the prices of good i at time t.

From the agents' first order conditions, we get:

$$P_{t-1,t}^N c_{t-1,t}^N = \frac{\varepsilon}{1+\beta} w_t^N h_t^N \quad ; \quad P_{t,t}^N c_{t,t}^N = \frac{1-\varepsilon}{1+\beta} w_t^N h_t^N \qquad \text{(A3)}$$

$$P_{t,t+1}^N c_{t,t+1}^N = \frac{\varepsilon\beta}{1+\beta} w_t^N h_t^N r_{t+1}^N \quad ; \quad P_{t+1,t+1}^N c_{t+1,t+1}^N = \frac{(1-\varepsilon)\beta}{1+\beta} w_t^N h_t^N r_{t+1}^N \qquad \text{(A4)}$$

$$s_t^N = \frac{\beta}{1+\beta} w_t^N h_t^N \qquad \text{(A5)}$$

Substituting the agents' f.o.c into the law of motion of capital and the good markets clearing conditions yields:

$$K_{t+1}^N = \frac{\beta w_t^N H_t^N}{(1+\beta)} \qquad \text{(A6)}$$

$$Y_{t-1,t}^N = \frac{\varepsilon\, w_t^N H_t^N}{(1+\beta)P_{t-1,t}^N} + \frac{\varepsilon\, r_t^N K_t^N}{P_{t-1,t}^N} \qquad \text{(A7)}$$

$$Y_{t,t}^{N} = \frac{(1-\varepsilon)w_t^N H_t^N}{(1+\beta)P_{t,t}^N} + \frac{(1-\varepsilon)r_t^N K_t^N}{P_{t,t}^N} \quad \text{(A8)}$$

$$Y_{k,t}^{N} = \frac{\beta w_t^N H_t^N}{(1+\beta)} \quad \text{(A9)}$$

Substituting equations (A1) and (A2) into equations (A8) and (A9), we get the fraction of human capital used in the production of, respectively, goods *t* and *k:*

$$\frac{H_{t,t}^N}{H_t^N} = \frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha) \quad \text{(A10)}$$

$$\frac{H_{k,t}^N}{H_t^N} = \frac{\beta\alpha}{1+\beta} \quad \text{(A11)}$$

Since capital/labor ratios are the same across sectors, substituting (A1) into (A6) and dividing the resulting equation by H_{t+1}^N we get:

$$\frac{K_{t+1}^N}{H_{t+1}^N} = \frac{\beta}{1+\beta}\alpha(\phi^t)^\alpha \left(\frac{K_t^N}{H_t^N}\right)^{1-\alpha}\left(\frac{H_t^N}{H_{t+1}^N}\right) \quad \text{(A12)}$$

Along the steady state growth path, the capital per (effective) worker ratio is constant over time. Thus, we have:

$\frac{K_{t+1}^N}{\phi^{t+1}H_{t+1}^N} = \frac{K_t^N}{\phi^t H_t^N}$, and with $H_{t+1}^N = H_t^N(1+g_H^N)$, (A12) yields:

$$\frac{K_{t+1}^N}{\phi^{t+1}H_{t+1}^N} = \left(\frac{\beta}{1+\beta}\frac{\alpha}{\phi}\frac{1}{1+g_H^N}\right)^{\frac{1}{\alpha}} \quad \text{(A13)}$$

The learning equation (4) can be rewritten as:

$$\frac{H_{t+1}^N - H_t^N}{H_t^N} = [a_c \frac{H_{t,t}^N}{H_t^N} (\frac{K_{t,t}^N}{\phi^t H_{t,t}^N})^{1-\alpha} + a_k \frac{H_{k,t}^N}{H_t^N} (\frac{K_{k,t}^N}{\phi^t H_{k,t}^N})^{1-\alpha}]^{\delta} \tag{A14}$$

Substituting (A10), (A11) and (A13) into (A14), we get:

$$(g_H^A)^{\frac{1}{\delta}} (1+g_H^A)^{\frac{1-\alpha}{\alpha}} = (\frac{\alpha}{\phi} \frac{\beta}{1+\beta})^{\frac{(1-\alpha)}{\alpha}} \{a_c [\frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha)] + a_k \frac{\alpha\beta}{1+\beta}\} \tag{A15}$$

Note that the LHS is a monotonic function of g_H, is equal to zero when the growth rate is equal to zero and goes to infinity when the growth rate goes to infinity. Since the RHS is a positive constant, g_H is uniquely defined by (A15). As South is identical to North except for the one period lag in the arrival of new designs, it follows that the autarky growth rate is the same as North. **QED.**

Proof of Proposition 2:

Case I: Free trade equilibrium with complete specialization in the South only

South

Substituting agents' f.o.c into the goods market clearing conditions we obtain as before:

$$\frac{H_{t-2,t}^S}{H_t^S} = \frac{\varepsilon\alpha}{1+\beta} + \varepsilon(1-\alpha) \tag{A16}$$

$$\frac{H_{t-1,t}^S}{H_t^S} = 1 - \left[\frac{\varepsilon\alpha}{1+\beta} + \varepsilon(1-\alpha)\right] \tag{A17}$$

Equating the savings of agents to the next period capital stock yields:

$$K_{t+1}^S = \frac{\beta w_t^S H_t^S}{(1+\beta)} \tag{A18}$$

Substituting $w_t^S = P_{t-1,t}^N \alpha (\phi^{t-1})^{\alpha} (\frac{K_{t-1,t}^S}{H_{t-1,t}^S})^{1-\alpha}$ into (A18) and dividing the resulting equation by

H_{t+1}^{S} gives:

$$\frac{K_{t+1}^{S}}{\phi^{t} H_{t+1}^{S}} = (P_{t-1,t}^{N} \frac{\beta}{1+\beta} \frac{\alpha}{\phi} \frac{1}{1+g_{H}^{S,T}})^{\frac{1}{\alpha}} \tag{A19}$$

Substituting (A17) and (A19) into the learning equation (8) of the South, we get:

$$\left(g_{H}^{S,T}\right)^{\frac{1}{\delta}} = a_{c}[1-\frac{\varepsilon\alpha}{1+\beta} - \varepsilon(1-\alpha)](P_{t-1,t}^{N} \frac{\alpha}{\phi} \frac{\beta}{1+\beta} \frac{1}{1+g_{H}^{S,T}})^{\frac{1-\alpha}{\alpha}} \tag{A20}$$

or:

$$(g_{H}^{S,T})^{\frac{1}{\delta}} (1+g_{H}^{S,T})^{\frac{(1-\alpha)}{\alpha}} = a_{c}[1-\varepsilon(1-\frac{\alpha\beta}{1+\beta})](\frac{\alpha}{\phi} \frac{\beta}{1+\beta})^{\frac{(1-\alpha)}{\alpha}} \phi^{1-\alpha} \tag{A21}$$

<u>North</u>

Substituting (A1) into the capital accumulation equation (A6) and dividing the resulting equation by H_{t+1}^{N} yields:

$$\frac{K_{t+1}^{N}}{\phi^{t+1} H_{t+1}^{N}} = (\frac{\beta}{1+\beta} \frac{\alpha}{\phi} \frac{1}{1+g_{H}^{N,T}})^{\frac{1}{\alpha}} \tag{A22}$$

As in the autarky case we obtain from the goods market clearing conditions:

$$\frac{H_{t,t}^{N}}{H_{t}^{N}} = \frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha) \tag{A23}$$

$$Y_{k,t}^{N} = \frac{\beta w_{t}^{N} H_{t}^{N}}{(1+\beta)} + \frac{\beta w_{t}^{S} H_{t}^{S}}{(1+\beta)}$$

$$\Leftrightarrow (\phi^{t})^{\alpha} (\frac{K_{k,t}^{N}}{H_{k,t}^{N}})^{1-\alpha} H_{k,t}^{N} = \frac{\beta}{1+\beta} \alpha (\phi^{t})^{\alpha} (\frac{K_{k,t}^{N}}{H_{k,t}^{N}})^{1-\alpha} H_{t}^{N} + \frac{\beta}{1+\beta} P_{t-1,t}^{N} \alpha (\phi^{t-1})^{\alpha} (\frac{K_{t-1,t}^{S}}{H_{t-1,t}^{S}})^{1-\alpha} H_{t}^{S}$$

$$\Leftrightarrow \frac{H_{k,t}^{N}}{H_{t}^{N}} = \frac{\alpha\beta}{1+\beta}(1+\frac{H_{t}^{S}}{H_{t}^{N}}) \tag{A24}$$

Substituting (A22), (A23) and (A24) into the learning equation (4), we have:

$$(g_H^{N,T})^{\frac{1}{\delta}}(1+g_H^{N,T})^{\frac{1-\alpha}{\alpha}} = (\frac{\alpha\beta}{\phi(1+\beta)})^{\frac{1-\alpha}{\alpha}}\{a_c[\frac{(1-\varepsilon)\alpha}{1+\beta}+(1-\varepsilon)(1-\alpha)]+a_k\frac{\alpha\beta}{1+\beta}(1+\frac{H_t^S}{H_t^N})\} \quad (A25)$$

We note that the LHS of (A25) is an increasing function of g_H. Comparing (A25) and (A15), we see that the RHS of (A25) is strictly larger than that of (A15). Therefore, if a balanced growth path exists, growth is strictly higher under free trade.

We equate the growth rate of the two countries to obtain the relative size of the economies along the balanced growth path:

$$\frac{H_t^S}{H_t^N} = \frac{a_c[1-\varepsilon(1-\frac{\alpha\beta}{1+\beta})](\phi^{(1-\alpha)}-1)+(a_c-a_k)\frac{\alpha\beta}{1+\beta}}{a_k\frac{\alpha\beta}{1+\beta}} \quad (A26)$$

We note that a balanced growth path with only South specializing completely exists iff:

$$\text{i)}\ \frac{H_t^S}{H_t^N} > 0 \iff \frac{a_k}{a_c} < 1+\frac{[1-\varepsilon(1-\frac{\alpha\beta}{1+\beta})](\phi^{(1-\alpha)}-1)}{\frac{\alpha\beta}{1+\beta}} \quad (A27)$$

and:

$$\text{ii)}\ \frac{H_{t-1,t}^N}{H_t^N} > 0 \iff 1-\frac{(1-\varepsilon)\alpha}{1+\beta}-(1-\varepsilon)(1-\alpha)-\frac{\alpha\beta}{1+\beta}(1+\frac{H_t^S}{H_t^N}) > 0$$

$$\iff \frac{a_k}{a_c} > 1+\frac{[1-\varepsilon(1-\frac{\alpha\beta}{1+\beta})]\phi^{1-\alpha}-1}{\frac{\alpha\beta}{1+\beta}+(1-\frac{\alpha\beta}{1+\beta})\varepsilon} \quad (A28)$$

QED

Case II: Free trade equilibrium with both North and South completely specialized.

i) As in the autarky case, we obtain from the goods market clearing conditions:

$$\frac{H_{t,t}^{N}}{H_{t}^{N}} = \frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha) \tag{A29}$$

Since North is completely specialized, we get:

$$\frac{H_{k,t}^{N}}{H_{t}^{N}} = 1 - \frac{(1-\varepsilon)\alpha}{1+\beta} - (1-\varepsilon)(1-\alpha) = \frac{\alpha\beta}{1+\beta} + \varepsilon(1 - \frac{\alpha\beta}{1+\beta}) \tag{A30}$$

Substituting (A22), (A29) and (A30) into the learning equation (4) for the North, we get:

$$(g_{H}^{N,T})^{\frac{1}{\delta}}(1+g_{H}^{N,T})^{\frac{1-\alpha}{\alpha}} = (\frac{\alpha\beta}{\phi(1+\beta)})^{\frac{1-\alpha}{\alpha}}\{a_{c}[\frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha)] + a_{k}[\frac{\alpha\beta}{1+\beta} + \varepsilon(1-\frac{\alpha\beta}{1+\beta})]\} \tag{A31}$$

Note that the RHS of (A31) is strictly greater than (A15) and hence the growth rate of human capital is strictly higher under free trade.

Denoting the terms of trade for the traded consumption good by $P_{t-1,t}^{W}$, we get from (A20):

$$(g_{H}^{S,T})^{\frac{1}{\delta}}(1+g_{H}^{S,T})^{\frac{(1-\alpha)}{\alpha}} = a_{c}[(1-\varepsilon) + \frac{\varepsilon\alpha\beta}{1+\beta}](P_{t-1,t}^{W}\frac{\alpha}{\phi}\frac{\beta}{1+\beta})^{\frac{(1-\alpha)}{\alpha}} \tag{A32}$$

We next find the terms of trade such that the two economies grow at the same rate. Equating (A31) and (A32), we get:

$$\left(P_{t-1,t}^{W}\right)^{\frac{1-\alpha}{\alpha}} = \frac{a_{c}[\frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha)] + a_{k}[\frac{\alpha\beta}{1+\beta} + \varepsilon(1-\frac{\alpha\beta}{1+\beta})]}{a_{c}[(1-\varepsilon) + \frac{\varepsilon\alpha\beta}{1+\beta}]} \tag{A33}$$

Note that the terms of trade must lie between the autarky relative prices of the two economies. This implies that for such a balanced growth path to exist we need:

$$1 \le P_{t-1,t}^{W} \le \phi^{\alpha}$$

or:

$$\frac{1}{1+(\frac{1+\beta}{\alpha\beta}-1)\varepsilon} \leq \frac{a_k}{a_c} \leq 1+\frac{[1-\varepsilon(1-\frac{\alpha\beta}{1+\beta})]\phi^{1-\alpha}-1}{\frac{\alpha\beta}{1+\beta}+(1-\frac{\alpha\beta}{1+\beta})\varepsilon} \tag{A34}$$

Note that the LHS inequality is automatically satisfied by *Assumption 4*.

Finally, we solve for the relative size of North versus South at the equilibrium. From the market clearing condition for the capital good in North we have:

$$(\phi^t)(\frac{K_{k,t}^N}{\phi^t H_{k,t}^N})^{1-\alpha} H_{k,t}^N = \frac{\beta}{1+\beta}\alpha\phi^t(\frac{K_{k,t}^N}{\phi^t H_{k,t}^N})^{1-\alpha} H_t^N + \frac{\beta}{1+\beta}P_{t-1,t}^W \alpha\phi^{t-1}(\frac{K_{t-1,t}^N}{\phi^{t-1}H_{t-1,t}^N})^{1-\alpha} H_t^S$$

$$\Leftrightarrow \frac{H_{k,t}^N}{H_t^N} = \frac{\alpha\beta}{1+\beta}(1+\frac{H_t^S}{H_t^N}\frac{P_{t-1,t}^{W\ \frac{1}{\alpha}}}{\phi}) \tag{A35}$$

Substituting (A30) into (A35) we have:

$$\frac{H_t^S}{H_t^N} = \frac{\phi\varepsilon(1-\frac{\alpha\beta}{1+\beta})}{\frac{\alpha\beta}{1+\beta}P_{t-1,t}^{w\ \frac{1}{\alpha}}} \tag{A36}$$

Substituting (A33) into (A36), we get:

$$\frac{H_t^S}{H_t^N} = \frac{\phi\varepsilon\left(1-\frac{\alpha\beta}{1+\beta}\right)\left\{a_c\left[1-\varepsilon\left(1-\frac{\alpha\beta}{1+\beta}\right)\right]\right\}^{\frac{1}{1-\alpha}}}{\frac{\alpha\beta}{1+\beta}\left\{a_c\left[\frac{(1-\varepsilon)\alpha}{1+\beta}+(1-\varepsilon)(1-\alpha)\right]+a_k\left[\frac{\alpha\beta}{1+\beta}+\varepsilon\left(1-\frac{\alpha\beta}{1+\beta}\right)\right]\right\}^{\frac{1}{1-\alpha}}} \tag{A37}$$

Appendix B

<u>Proof of Proposition 3</u>

Note first that:

$$P_{t-1,t}^{N} I_{t}^{N} = P_{t-1,t}^{S} E_{t}^{S} (1-s) \Rightarrow P_{t-1,t}^{S} = \frac{P_{t-1,t}^{N}}{1-s} \tag{B1}$$

where $P_{t-1,t}^{N}$ is the price faced by consumers in North and $P_{t-1,t}^{S}$ is the price faced by consumers in South which also equals to the unit cost of production under perfect competition.

Goods market clearing conditions:

$$Y_{t-1,t}^{S} = \frac{(1-\varepsilon) w_{t}^{S} H_{t}^{S} (1-d)}{(1+\beta) P_{t-1,t}^{S}} + \frac{(1-\varepsilon) r_{t}^{S} K_{t}^{S} (1-d)}{P_{t-1,t}^{S}} + E_{t-1,t}^{S} \tag{B2}$$

$$I_{k,t}^{S} = \frac{\beta w_{t}^{S} H_{t}^{S} (1-d)}{(1+\beta)} \tag{B3}$$

$$Y_{t-2,t}^{S} = \frac{\varepsilon w_{t}^{S} H_{t}^{S} (1-d)}{(1+\beta) P_{t-2,t}^{S}} + \frac{\varepsilon r_{t}^{S} K_{t}^{S} (1-d)}{P_{t-2,t}^{S}} \tag{B4}$$

As in the proofs of *Propositions 1 and 2*, we get from the goods market clearing conditions:

$$\frac{H_{t-2,t}^{S}}{H_{t}^{S}} = \frac{\varepsilon \alpha (1-d)}{1+\beta} + \varepsilon (1-\alpha)(1-d) \tag{B5}$$

$$\frac{H_{t-1,t}^{S}}{H_{t}^{S}} = 1 - \frac{\varepsilon \alpha (1-d)}{1+\beta} - \varepsilon (1-\alpha)(1-d) \tag{B6}$$

Equating agents' savings to next period capital stock and dividing both sides of the equation by H_{t+1}^{S}:

$$\frac{K_{t+1}^S}{H_{t+1}^S} = \frac{\beta w_t^S H_t^S (1-d)}{(1+\beta) H_{t+1}^S}$$

Substituting $w_t^S = P_{t-1,t}^S \alpha (\phi^{t-1})^{\alpha} (\frac{K_{t-1,t}^S}{H_{t-1,t}^S})^{1-\alpha}$ we get:

$$\frac{K_{t+1}^S}{H_{t+1}^S} = \frac{\beta \alpha P_{t-1,t}^S (\phi^{t-1})^{\alpha} \left(\frac{K_{t-1,t}^S}{H_{t-1,t}^S} \right)^{1-\alpha} (1-d)}{(1+\beta)(1+g_H)} \qquad \text{(B7)}$$

Since the capital-labor ratios are equalized across sectors, and by definition of a balanced growth path, (B7) is equivalent to:

$$\frac{K_{t+1}^S}{\phi^t H_{t+1}^S} = \left(\frac{\alpha \beta (1-d)}{\phi^{1-\alpha} (1+\beta)(1-s)(1+g_H)} \right)^{\frac{1}{\alpha}} \qquad \text{(B8)}$$

By the definition of d, we have:

$$d(r_t^S K_t^S + w_t^S H_t^S) = s P_{t-1,t}^S E_{t-1,t}^S = \frac{s}{1-s} I_{k,t}^S$$

$$\Leftrightarrow d\ (\frac{r_t^S K_t^S}{w_t^S H_t^S} + 1) = \frac{s\beta(1-d)}{(1-s)(1+\beta)}$$

$$\Leftrightarrow 1-d = \frac{(1-s)(1+\beta)}{\alpha s \beta + (1-s)(1+\beta)} \qquad \text{(B9)}$$

Substituting (B5) and (B8) into the learning equation (8), we get:

$$(g_H^S)^{\frac{1}{\delta}} = a_c \left(\frac{K_{t-1,t}^S}{\phi^{t-1} H_{t-1,t}^S} \right)^{1-\alpha} \frac{H_{t-1,t}^S}{H_t^S} = a_c \left(\frac{\alpha \beta (1-d)}{\phi^{1-\alpha} (1+\beta)(1-s)(1+g_H)} \right)^{\frac{1-\alpha}{\alpha}} [1-\varepsilon(1-d)(1-\frac{\alpha\beta}{1+\beta})]$$

or:

$$(g_H^S)^{\frac{1}{\delta}}(1+g_H^S)^{\frac{1-\alpha}{\alpha}} = a_c\left(\frac{\phi^{\alpha-1}\alpha\beta}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})] \quad \text{(B10)}$$

Comparing (B10) to (A21) above yields that South's growth rate is strictly higher when South gives an export subsidy than under free trade.

Similarly for North, the goods markets clearing conditions imply:

$$\frac{H_{t,t}^N}{H_t^N} = \frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha) \quad \text{(B11)}$$

$$Y_{k,t}^N = \frac{\beta w_t^N H_t^N}{(1+\beta)} + E_{k,t}^N$$

$$\Leftrightarrow Y_{k,t}^N = \frac{\beta w_t^N H_t^N}{(1+\beta)} + \frac{\beta w_t^S H_t^S(1-d)}{(1+\beta)}$$

$$\Leftrightarrow (\phi^t)^\alpha (\frac{K_{k,t}^N}{H_{k,t}^N})^{1-\alpha} H_{k,t}^N = \frac{\beta}{1+\beta}\alpha(\phi^t)^\alpha(\frac{K_{k,t}^N}{H_{k,t}^N})^{1-\alpha} H_t^N + \frac{\beta(1-d)}{1+\beta} P_{t-1,t}^S \alpha(\phi^{t-1})^\alpha(\frac{K_{t-1,t}^S}{H_{t-1,t}^S})^{1-\alpha} H_t^S$$

$$\Leftrightarrow \frac{H_{k,t}^N}{H_t^N} = \frac{\alpha\beta}{1+\beta}(1+\frac{H_t^S}{H_t^N}\left(1-s+\frac{\alpha\beta}{1+\beta}\ s\right)^{-\frac{1}{\alpha}}) \quad \text{(B12)}$$

Substituting (A21), (B11) and (B12) into the learning equation (4) for North, we have:

$$(g_H^N)^{\frac{1}{\delta}}(1+g_H^N)^{\frac{1-\alpha}{\alpha}} = (\frac{\alpha\beta}{\phi(1+\beta)})^{\frac{1-\alpha}{\alpha}}\{a_c[\frac{(1-\varepsilon)\alpha}{1+\beta}+(1-\varepsilon)(1-\alpha)]+a_k\frac{\alpha\beta}{1+\beta}(1+\frac{H_t^S}{H_t^N}\left(1-s+\frac{\alpha\beta}{1+\beta}\ s\right)^{-\frac{1}{\alpha}})\} \quad \text{(B13)}$$

Equating the growth rates of North and South given by (B13) and (B10) respectively, yields:

$$\frac{H_t^S}{H_t^N} = \frac{a_c\left(\frac{\phi^\alpha(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})] - a_c(1-\varepsilon)(1-\frac{\alpha\beta}{1+\beta}) - a_k\frac{\alpha\beta}{1+\beta}}{a_k\frac{\alpha\beta}{1+\beta}[\frac{(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}]^{\frac{1}{\alpha}}} \quad \text{(B14)}$$

For a balanced growth path to exist we need:

i) $\frac{H_t^S}{H_t^N} > 0$

$$\Leftrightarrow \frac{a_k}{a_c} < 1 + \frac{\left(\frac{\phi^{\alpha}(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})]-[1-\varepsilon(1-\frac{\alpha\beta}{1+\beta})]}{\frac{\alpha\beta}{1+\beta}} \quad \text{(B15)}$$

and:

ii) $\frac{H_{t-1,t}^N}{H_t^N} > 0 \quad \Leftrightarrow \frac{H_{t-1,t}^N}{H_t^N} = 1 - \frac{\alpha\beta}{1+\beta}(1+\frac{H_t^S}{H_t^N}[\frac{(1-d)}{(1-s)}]^{\frac{1}{\alpha}}) - \frac{(1-\varepsilon)\alpha}{1+\beta} - (1-\varepsilon)(1-\alpha) > 0$

$$\Leftrightarrow \frac{a_k}{a_c} > 1 + \frac{\left(\frac{\phi^{\alpha}(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})]-1}{\frac{\alpha\beta}{1+\beta}+\varepsilon(1-\frac{\alpha\beta}{1+\beta})} \quad \text{(B16)}$$

iii) Let $R(s) \equiv \frac{H^S}{H^N}(s, d(s))$

$$R(s) = \frac{a_c\left(\frac{(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}\phi^{1-\alpha}[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})]-a_c(1-\varepsilon)(1-\frac{\alpha\beta}{1+\beta})-a_k\frac{\alpha\beta}{1+\beta}}{a_k\frac{\alpha\beta}{1+\beta}[\frac{(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}]^{\frac{1}{\alpha}}}$$

Straightforward differentiation yields:

R'(0)>0 iff $\phi^{1-\alpha} < \frac{a_c(1-\varepsilon)(1-\frac{\alpha\beta}{1+\beta})+a_k\frac{\alpha\beta}{1+\beta}}{a_c\alpha(1-\varepsilon)}$

and *R''(s) <0* $\forall s \in (0,1)$. **QED.**

Proof of Proposition 4

As above, the growth rate of South is given by:

$$(g_H^S)^{\frac{1}{\delta}}(1+g_H^S)^{\frac{1-\alpha}{\alpha}} = a_c\left(\frac{P_{t-1,t}^N \alpha\beta(1-d)}{\phi(1+\beta)(1-s)}\right)^{\frac{1-\alpha}{\alpha}}[1-\varepsilon(1-d)(1-\frac{\alpha\beta}{1+\beta})] \qquad \text{(B17)}$$

When North is completely specialized the subsidy policy of South does not change the allocation of human capital to the production of the capital good and of the consumption good *t+1* in the North. Hence, the growth rate of North is the same as under free trade:

$$(g_H^{N,T})^{\frac{1}{\delta}}(1+g_H^{N,T})^{\frac{1-\alpha}{\alpha}} = (\frac{\alpha\beta}{\phi(1+\beta)})^{\frac{1-\alpha}{\alpha}}\{a_c[\frac{(1-\varepsilon)\alpha}{1+\beta}+(1-\varepsilon)(1-\alpha)]+a_k[\frac{\alpha\beta}{1+\beta}+\varepsilon(1-\frac{\alpha\beta}{1+\beta})]\} \qquad \text{(B18)}$$

Equalizing both growth rates yields:

$$\left(\frac{1-d}{1-s}P_{t-1,t}^N\right)^{\frac{1-\alpha}{\alpha}} = \frac{a_c[\frac{(1-\varepsilon)\alpha}{1+\beta}+(1-\varepsilon)(1-\alpha)]+a_k[\frac{\alpha\beta}{1+\beta}+\varepsilon(1-\frac{\alpha\beta}{1+\beta})]}{a_c[1-\varepsilon(1-d)(1-\frac{\alpha\beta}{1+\beta})]} \qquad \text{(B19)}$$

For balanced growth to exist we need:

i) $P_{t-1,t}^N \leq \phi^\alpha$

$$\Leftrightarrow \frac{a_k}{a_c} \leq \frac{\left(\frac{1-d}{1-s}\right)^{\frac{1-\alpha}{\alpha}}\phi^{1-\alpha}[1-\varepsilon(1-d)(1-\frac{\alpha\beta}{1+\beta})]-(1-\varepsilon)(1-\frac{\alpha\beta}{1+\beta})}{[\frac{\alpha\beta}{1+\beta}+\varepsilon(1-\frac{\alpha\beta}{1+\beta})]}$$

or substituting *d(s)* using (B9)

$$\Leftrightarrow \frac{a_k}{a_c} \leq 1+\frac{\left(\frac{(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}\phi^{1-\alpha}[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})]-1}{[\frac{\alpha\beta}{1+\beta}+\varepsilon(1-\frac{\alpha\beta}{1+\beta})]} \qquad \text{(B20)}$$

and:

ii) $P^S_{t-1,t} \geq 1$

$$\Leftrightarrow \frac{a_k}{a_c} \geq 1 + \frac{\left[\frac{(1-s)(1+\beta)}{\alpha s\beta + (1-s)(1+\beta)}\right]^{\frac{1-\alpha}{\alpha}} [1-\varepsilon \frac{(1-s)(1+\beta)}{\alpha s\beta + (1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})] - 1}{[\frac{\alpha\beta}{1+\beta} + \varepsilon(1-\frac{\alpha\beta}{1+\beta})]} \qquad \text{(B21)}$$

Note that this inequality is automatically satisfied by *Assumption 4.*

Finally, we get from the goods market clearing condition for capital goods in North:

$$(\phi^t)(\frac{K^N_{k,t}}{\phi^t H^N_{k,t}})^{1-\alpha} H^N_{k,t} = \frac{\beta}{1+\beta}\alpha\phi^t(\frac{K^N_{k,t}}{\phi^t H^N_{k,t}})^{1-\alpha} H^N_t + \frac{\beta(1-d)}{1+\beta}\frac{P^N_{t-1,t}}{1-s}\alpha\phi^{t-1}(\frac{K^S_{t-1,t}}{\phi^{t-1}H^S_{t-1,t}})^{1-\alpha} H^S_t$$

$$\Leftrightarrow \frac{H^N_{k,t}}{H^N_t} = \frac{\alpha\beta}{1+\beta}(1 + \frac{H^S_t}{H^N_t}\frac{P^{N\,\frac{1}{\alpha}}_{t-1,t}}{\phi}[\frac{(1-d)}{(1-s)}]^{\frac{1}{\alpha}})$$

$$\Leftrightarrow \frac{H^S_t}{H^N_t} = \frac{\phi\varepsilon(1-\frac{\alpha\beta}{1+\beta})\{a_c[1-\varepsilon\frac{(1-s)(1+\beta)}{\alpha s\beta+(1-s)(1+\beta)}(1-\frac{\alpha\beta}{1+\beta})]\}^{\frac{1}{1-\alpha}}}{\frac{\alpha\beta}{1+\beta}\{a_c[\frac{(1-\varepsilon)\alpha}{1+\beta} + (1-\varepsilon)(1-\alpha)] + a_k[\frac{\alpha\beta}{1+\beta} + \varepsilon(1-\frac{\alpha\beta}{1+\beta})]\}^{\frac{1}{1-\alpha}}} \qquad \text{(B22)}$$

Comparing (B22) and (A36), we observe that the relative size of South increases in the presence of the export subsidy compared to the free trade equilibrium. **QED**

Figure 1

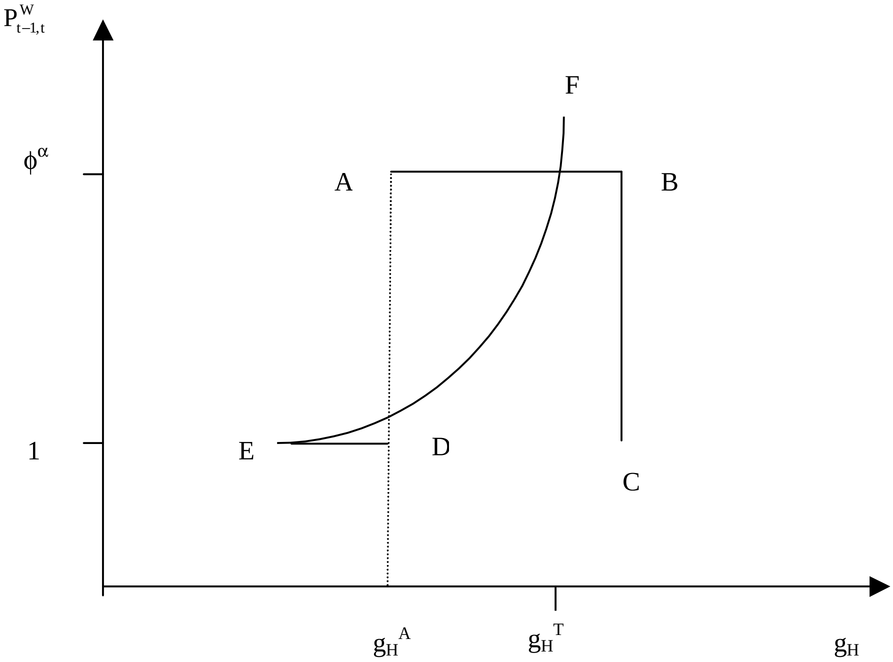

Figure 2

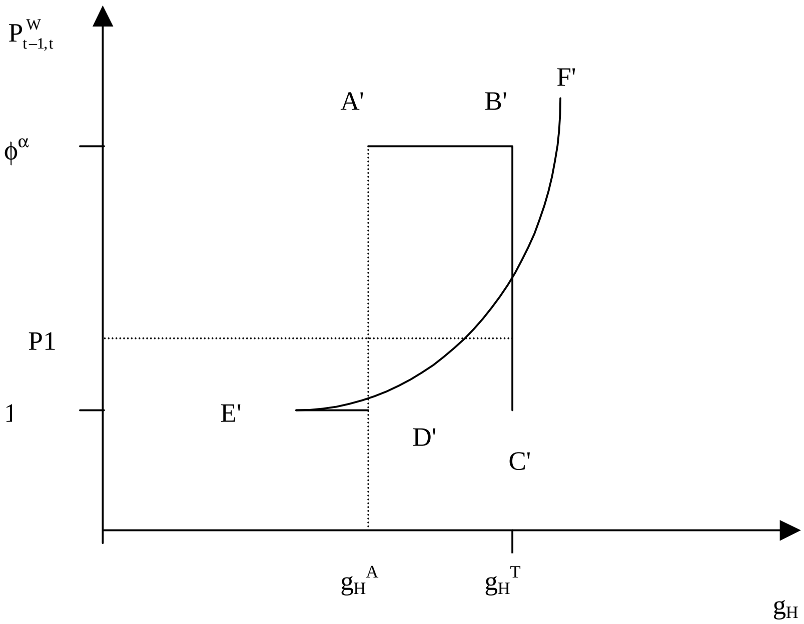

Figure 3 Production Possibility Frontiers

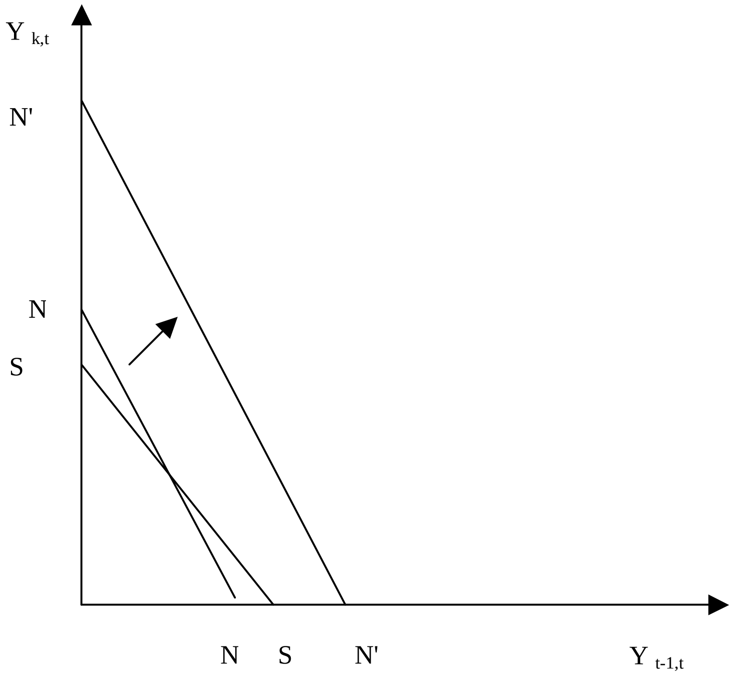

Figure 4 Offer curves