Preferences over Capital Income versus Labor Income Taxation*

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Abstract

Empirical papers show that labor income and capital income are differently taxed all over the world. We investigate whether this may correspond to individual preferences. We tackle this question in an overlapping generations general equilibrium model with heterogeneous agents: young versus old and low skilled versus high skilled individuals. Taxes finance unemployment benefits and government consumption. High skilled agents prefer capital income taxes, while young unskilled and old agents prefer labor income taxation.

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1 Introduction

Why does labor income tax differ from capital income tax? And why is this difference not similar across countries? Indeed, labor income is more taxed than capital income in Europe, while it is the contrary in the US. Recent empirical papers (for instance, Tabellini and Daveri (1997)) show that in 1976-91, the effective tax rate on labor reached 40.5% in Europe (and 27.7% in US), while the effective tax rate on capital was only 32.9% in Europe (and 42.2% in US) over the same period.

The questions concerning the level of income taxation and their effects have been treated in so far in various ways by the literature. Some authors try to determine the growth-maximizing fiscal policy and conclude to the optimality of a zero capital income tax in the long run (see Judd (1985), Chamley (1994), Bertola (1993)). Another stream of literature has adopted the point of view of a central planner whose objective is to maximize the social welfare such as Ordover and Phelps (1975, 1976, 1979) and Ranzagas (1989). The optimal taxation mix is the one that minimizes the marginal welfare cost of redistribution.

Several authors have endogenized taxation through political mechanism. The models related to this theoretical stream differ by their initial motivation, the characteristics of the fiscal policy and by the type of voters considered. Krusell and Rios-Rull (1994) study how fiscal and political constitutions influence the equilibrium tax rate, economic growth and welfare. For this purpose, they develop a neoclassical growth model where infinitely-lived agents voting for a unique tax applying to both capital and labor income. Fiaschi (1999) focusing on the correlation between inequality and growth considers infinitely lived agents, heterogeneous in terms of their initial capital and labor endowments. They vote for taxes on capital income and labor income that are linked through an optimality condition at steady state. Persson and Tabellini (1994) examine a political model of taxation where agents differing by their time endowment vote for labor income tax and capital tax linked by the budget constraint. They mainly focus on the way political institutions can solve the 'capital levy problem'. Renström (1996) develops an overlapping generation framework of endogenous taxation where young individuals have different skill level and bequest motives. However, they vote today for a tax rate that will be levied only tomorrow. They conclude that the level of tax depends on the median bequest motive. In Bassetto (1999), the heterogeneity is related to the age of the agents and the political determination of fiscal policy takes the form of a bargaining between generations.

This paper fits in the political economy approach. Our purpose is to focus on the preferences of individuals and the potential inter- and intragenerational conflicts concerning the optimal taxation mix of capital income tax and labor income tax. There are two main sources of observable heterogeneity across individuals in modern societies: differences in age and in income. As a consequence of this double heterogeneity, disparities of preferences are likely to arise regarding labor and capital income tax, the former hitting mostly the young and middle-age, the latter hitting relatively more the old. Old individuals do not work, which makes them a relatively homogeneous voting group that may have a relevant political power. Young individuals differ by their skill level, hence their wage. This heterogeneity and the existence of an unemployment benefit may generate various optimal taxation mixes. On one hand, tax rates will affect differently the labor supply decision of each individual. On the other hand, each individual may value differently the change in capital income taxation that is necessary to balance the government budget constraint.

We develop a general equilibrium model in a steady state settings. We consider a double heterogeneity: individuals differ by the generations they belong to and by their economic skills. Labor supply is endogenized in a way we believe to be close to the actual individual behavior: when young, agents must decide, not the amount of time they will work but rather whether they will work or get an unemployment allocation. The government taxes labor and capital income in order to finance its consumption and a social security system with a transfer to the unemployed and a transfer to the old. The level of these expenditures is assumed to be fixed so that the labor income and capital income tax are linked through the government budget constraint. Young and old individuals choose a fiscal policy, which will be valid today and tomorrow. After having expressed their optimal preferences for a tax combination, individuals will decide to work or to remain unemployed. In a third stage, they will smooth their consumption over their remaining lifetime.

This model allows us to investigate the preferences of agents (young versus old, low skilled versus high skilled) over the taxation method to finance a given social security system. Hence, the preferences of agents do not depend on the level of the effects of the taxation, on the level of social transfers, nor on the price effects. They rather depend on the direct income effects of each taxation method and on the indirect effects on the labor supply and hence on the government budget. Despite the heterogeneity of young individuals, we found that they are normally divided in only two categories according to their preferences, independently of their employment status. High-skilled young agents will mostly prefer a zero labor income tax, while low skilled will favor a very low capital income tax. On the other hand, old individuals are homogeneous in their choice and all favor the same taxation policy.

The paper is organized as follows. In section 2, we present the economic model and solve the consumer's problem. In section 3, we analyze the preferences over taxation of old and young individuals with various skill levels. In section 4, we illustrate the theoretical predictions with specific realistic values. Section 5 concludes.

2 The Economic Environment

We develop a dynamic general equilibrium model with overlapping generations of heterogeneous individuals, who live for two periods[1]. Time t is discrete and

[1] Each period representing approximately 30 years.

goes from 0 to 1 .

2.1 The Production Side

Firms in economy produce one good using a linear production technology, assumed to be the same in each period:

$$F(K_t; L_t) = RK_t + wL_t \tag{1}$$

with K_t and L_t the input of physical and human capital at time t,

$R = 1 + r$ the gross interest rate assuming full depreciation of capital each period,

and w the wage per efficiency unit of labor.

This linear production function is chosen on simplicity grounds. It leads to constant factor prices $(w; 1 + r)$ and allows to focus on the political preferences regarding to two different taxing methods, while keeping the economic structure simple.

2.2 The Consumer Side

Individuals are heterogeneous: they belong to two generations and have different innate production skills e: Skill level e is a continuous variable distributed on the support $(0; 1)$ according to a uniform probability density function f(e), which is constant over time.

The population (N_t) is assumed to grow at a constant rate n:

$$N_{t+1} = N_t(1 + n) \tag{2}$$

Each individual e lives for two periods, only works in the first period and consumes in both periods. He is assumed to have the following homothetic lifetime utility function defined over consumption when young (c_e) and old (d_e):

$$V_e = \ln c_e + \beta \ln d_e \tag{3}$$

where $\beta < 1$ is the subjective discount factor. With this specification, the intertemporal elasticity of substitution is equal to one.

In the first period, each individual can be employed or unemployed. If employed, he earns wage income net of taxes ($ew(1-\tau^L)$) with τ^L the labor income tax rate, identical in every period. Individual labor supply is set equal to one. If unemployed, he earns an unemployment subsidy b, which is not taxable. In the second period, each individual is retired. When old, he earns a return net of taxes of $(1 + r(1-\tau^K))$ on savings made when young (s_e), with τ^K the capital income tax rate, not changing over time. He consumes his entire income and does not leave bequests.

Under these assumptions, the individual budget constraints of a young and old individual e respectively are:

$$c_e + s_e = ew(1-\tau^L)\mu + b(1-\mu) \tag{4}$$

$$d_e = s_e(1 + r(1-\tau^K)) \tag{5}$$

with μ a dummy variable equal to one when employed and to zero when unemployed.

Each individual will smooth his consumption over time in order to maximize his lifetime utility:

$$MAX_{s_e} \ln[ew(1-\tau^L)\mu + b(1-\mu) - s_e] + \beta \ln[s_e(1 + r(1-\tau^K))]$$

s.t. (4) and (5).

Using the first order condition, we can compute the optimal savings of the worker with skill e:

$$s_e^* = \frac{\beta}{1+\beta}\left[ew(1-\tau^L)\right] \tag{6}$$

Savings depend positively on education level e, on the subjective discount factor β, but negatively on the labor income tax rate. With a log-linear utility function, the marginal propensity to save is independent on the interest rate, hence on capital income tax.

The optimal savings of unemployed individuals are given by:

$$s_u^* = \frac{\beta b}{1+\beta} \tag{7}$$

Savings are always positive with $w, b > 0$ and $0 \leq \tau^L \leq 1$.

To determine if he will work or not, individual with skill e compares the indirect utility in each situation. Individual e will work if:

$$\begin{aligned} & \ln\left[\frac{e\hat{w}}{1+\beta}\right] + \beta \ln\left[\frac{\beta}{1+\beta}(e\hat{w}R^*)\right] \\ > \quad & \ln\left[\frac{b}{1+\beta}\right] + \beta \ln\left[\frac{\beta}{1+\beta}(bR^*)\right] \end{aligned} \tag{8}$$

$$\text{with } R^* = 1 + r(1-\tau^K)$$
$$\hat{w} = w(1-\tau^L)$$

Equation (8) implies that individual e is employed if his after-tax wage is above the unemployment subsidy (b), namely if his skill is above some minimum level: $e > e_{min}$ with

$$e_{min} = \frac{b}{w(1-\tau^L)} \tag{9}$$

We notice that $\frac{de_{min}}{d\tau^L} > 0$ and $\frac{d^2 e_{min}}{d(\tau^L)^2} > 0$ for every $\tau^L > 0$.

Aggregate labor supply in efficiency unit is given by:

$$L_t^s(\tau) = N_t \int_{e_{min}}^{1} e\, f(e)\, de = N_t\left(\frac{1}{2} - \frac{(e_{min})^2}{2}\right) \tag{10}$$

Human capital is provided by a fraction of the young. It depends negatively on the labor income taxation: the higher the labor income tax rate, the higher e_{min}, and the lower the labor supply.

Definition 1 Since $e_{min} \leq 1$, there exists an upper bound for τ^L above which all young individuals will be unemployed: $\tau^L_{max} = 1 - \frac{b}{w}$.

2.3 Market equilibrium and dynamics

The equilibrium path of the economy is characterized by the market clearing conditions for labor and capital:

$$L_t = L_t^s(\tau^L) \tag{11}$$

and

$$K_{t+1} = S_t^*(\tau^L; b; w; r) = N_t\left[\frac{\beta}{1+\beta}\left(w(1-\tau^L)\left(\frac{1}{2} - \frac{(e_{min})^2}{2}\right) + b e_{min}\right)\right] \tag{12}$$

The physical capital stock in $t+1$ is determined by aggregate savings (S_t^*) of the previous period:

$$\frac{K_{t+1}}{N_{t+1}} = \frac{K_t}{N_t} = k = \frac{1}{1+n}\frac{\beta}{1+\beta}\left[\frac{w}{2}(1-\tau^L)\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}\right]$$

Note that the growth rate of aggregate savings only depends on the growth rate of population, because the linear production function excludes price effects. When labor income taxation is changed, the economy jumps from one steady-state to another in the period after the modification. The steady state capital stock negatively depends on τ^L because it reduces the net income of workers:

$$\frac{\partial k}{\partial \tau^L} = \frac{1}{1+n}\frac{\beta}{1+\beta}\left[-\frac{w}{2}(1-e_{min}^2)\right] < 0 \tag{13}$$

2.4 Government

Government insures a minimum income guarantee to every individual. In order to finance the unemployment subsidies and its consumption (g denoting the constant per capita government consumption), the government levies taxes on labor and capital income. The government has four instruments ($\tau^L; \tau^K; b; g$) and must balance the budget every period. Consequently, the two tax rates must satisfy in each period the following government budget constraint:

$$be_{min} + g - \tau^L w\left(\frac{1}{2} - \frac{(e_{min})^2}{2}\right) - \tau^K r k_t = 0 \qquad (14)$$

with k_t denoting the per capita capital stock in period t. We constrain the tax rates to be positive and smaller than one ($0 \le \tau^L \le 1$ and $0 \le \tau^K \le 1$).

Capital income tax can be expressed as a function of τ^L:

$$\tau^K \equiv \Phi(\tau^L) = \frac{1+n}{r}\frac{1+\beta}{\beta}\left[1 - \frac{\frac{w}{2}\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) - g}{\frac{w}{2}(1-\tau^L)\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}}\right] \qquad (15)$$

where capital stock is replaced by its steady state value.

This equation (15) shows that to every τ^L corresponds one unique τ^K, for given prices $(r; w)$ and given government expenditures (g, b):

An increase in labor income tax will produce an ambiguous effect on the capital income tax necessary to finance given government expenditures:

$$\frac{d\Phi(\tau^L)}{d\tau^L} = \frac{(1+n)(1+\beta)}{r\beta\left[\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}\right]} \times \left[\begin{array}{l} \left(-\frac{w}{2}(1-e_{min}{}^2) + b\left(1+\frac{\tau^L}{1-\tau^L}\right)\right)\frac{de_{min}}{d\tau^L} + \\ \left(1 - \frac{\frac{w}{2}\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right) - g}{\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}}\right)\frac{w}{2}(1-e_{min}{}^2) \end{array}\right] \qquad (16)$$

First, the rise in labor income tax increases the revenue raised from each worker, which allows to reduce the taxation on capital income ($-\frac{w}{2}(1-e_{min}{}^2)$). However, increasing the taxation of labor income lowers the incentives to work, decreasing by this way the number of contributors to the system and increasing the number of unemployed to finance. This labor supply effect ($b(1+\frac{\tau^L}{1-\tau^L})\frac{de_{min}}{d\tau^L}$)

requires an increase of the capital income taxation. Third, a higher labor income tax reduces the per capita capital stock, hence the tax basis for capital taxation. This capital accumulation effect is reflected in the third term of the brackets in the above expression. The sign of the total effect depends on the relative importance of the three components, which in turn depends on the parameters values (b, w, g) and on the value of τ^L. Since $\frac{d^2 e_{min}}{d(\tau^L)^2} > 0$, the higher is τ^L, the larger will be the positive labor supply effect. When τ^L is equal to τ^L_{max} ($= 1 - \frac{b}{w}$), only the positive labor supply effect remains. In addition, the higher the ratio $\frac{b}{w}$, the larger will be the labor supply effect relatively to the revenue and capital accumulation effects. On the other hand, the larger the government expenditures, the larger the capital accumulation effect will be.

The revenue effect dominates for τ^L close to zero iff the unemployment subsidies and the government consumption are not too large:

$$\frac{d\Omega(\tau^L)}{d\tau^L}\Big|_{\tau^L=0} < 0 \;,\quad 0 < g < \left(\frac{1}{4} - \frac{b^2}{w^2} - \frac{b^4}{4w^4}\right)\frac{2w^3}{w^2 - b^2}$$
$$\text{with} \quad \frac{b}{w} < 0.486 \tag{17}$$

The condition on $\frac{b}{w}$ insures that g is not negative. See appendix 1 for a proof.

We will suppose that this condition is always satisfied. For very small τ^L, the labor supply and capital accumulation effects do not overcome the revenue effect, so that an increase in the labor income tax allows a reduction in the capital income tax.

Condition (17) together with $\frac{d\Omega(\tau^L)}{d\tau^L}\big|_{\tau^L=\tau^L_{max}} > 0$ and the continuity of $\Omega(\tau^L)$ for $0 < \tau^L < 1 - \frac{b}{w}$ imply:

Proposition 2 Iff $g < \frac{w}{2}$; there exists a unique labor income tax τ^L_* that minimizes the capital income tax. It is defined by the following condition:

$$\frac{d\Omega(\tau^L_*)}{d\tau^L} = 0 \text{ and } 0 < \tau^L_* < 1 - \frac{b}{w}:$$

Hence,

$$\tau^L_i = 1 - \frac{b}{w}\sqrt{\frac{1 - \frac{g}{2w} + \sqrt{\frac{5}{4} - \frac{3g}{2w} + \frac{g^2}{4w^2}}}{\frac{1}{2} - \frac{g}{w}}} \tag{18}$$

See appendix 1 for a proof. The condition on g insures that the government consumption is not too large in comparison to the tax basis so that it can be financed with a combination of tax between 0 and 1.

3 Individual preferences over taxation

This section analyzes the preferences of individuals regarding to their optimal combination of capital income tax and labor income tax to finance such a security system, abstracting from any strategic consideration. We investigate whether the actual breakdown of taxation burden between labor income and capital income reflects the preferences of the majority of individuals or the choices of some of them.

To be able to define the preferences of individuals over a combination of two tax rates, we link them via the budget constraint. We suppose the unemployment benefit b as well as the current government consumption g fixed. We do not look at the level of the social security allocations but we investigate the preferences of individuals over the taxation mix aimed at financing this given social security system. Their preferences will then depend on the effects of taxation on the labor supply (hence on the government budget) and on the redistributive characteristics of each taxation method.

Two different categories of individuals must be considered: old and young individuals.

Regarding their taxation preferences, old individuals are homogeneous. They will choose the combination of tax rates that maximizes their utility in their last period of life.

When examining the preferences of young agents over the breakdown of the taxation burden, we assume that agents expect this breakdown to prevail next period as well. Young individuals differ by their skill level, which affects their decision to work or not, and may therefore have different optimal choices. Since the tax combination influences the labor supply decision, the determination of the optimal tax combination for each young individual requires two steps. First, he determines the optimal tax combination in case he works and in case he is unemployed. Second, he compares his utility in each of these situations. The optimal tax rate for this young individual will be the tax combination that gives him the highest utility.

3.1 The optimal tax choice of old individuals $(\tau_o^{L*}; \tau_o^{K*})$

Old individuals care only about current consumption and maximize their utility in period zero:

$$V_e^o = \ln d_e = \ln[s_e(1 + r(1 - \tau^K))]$$

s.t.

$$\tau^K \equiv \Phi(\tau^L) = \frac{1+n}{r}\frac{1+\beta}{\beta}\left[1 - \frac{\frac{w}{2}\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) - g}{\frac{w}{2}(1-\tau^L)\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}}\right]$$

and

$0 \le \tau^K \le 1$ and $0 \le \tau^L \le 1$.

Old do not support labor taxation anymore and their utility is strictly decreasing in τ^K. Hence, the tax combination that maximizes their welfare will be the one that minimizes the capital income taxation, while respecting the government budget constraint. They choose:

If $0 \le \Phi(\tau_i^L) \le 1$:

$\tau_o^{L*} = \tau_i^L$ and $\tau_o^{K*} = \Phi(\tau_i^L)$.

If $\Phi(\tau_i^L) < 0$:

$\tau_o^{K*} = 0$ and $\tau_o^{L*} \equiv \mathrm{Min}\{\tau^L : \Phi(\tau^L) = 0\}$[2].

[2] We assume that if an individual is indifferent between two tax rates, he chooses the lowest one.

If $\Phi(\tau^L_i) > 1$, we would have so large government expenditures (very high g, b) that it could not be financed with any tax combination between 0 and 1. We exclude this case.

3.2 The preferred tax choice of young individuals

Young individual with skill level e has two types of preferences: when employed or unemployed. The optimal taxes for individual e will be either the optimal tax rates of a worker with skill e, either the optimal choice of an unemployed, depending on the one that gives him the highest utility.

3.2.1 Preferences of young unemployed individuals $(\tau_u^{L*}; \tau_u^{K*})$

Unemployed individuals choose the combination of labor and capital income taxes that maximizes their utility over their lifetime:

$$V^u = \ln\left[\frac{b}{1+\rho}\right] + \beta \ln\left[\frac{\beta}{1+\rho}\left(b(1 + r(1 - \tau^K))\right)\right] \tag{19}$$

s.t.

$$\tau^K \equiv \Phi(\tau^L) = \frac{1+n}{r}\frac{1+\beta}{\beta}\left[1 - \frac{\frac{w}{2}\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) - g}{\frac{w}{2}(1-\tau^L)\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}}\right]$$

and $0 \le \tau^K \le 1$ and $0 \le \tau^L \le 1$

The utility of unemployed only depends on τ^K. Hence, we can derive it with respect to the capital income tax:

$$\frac{\partial V^u}{\partial \tau^K} = -\frac{1}{c_u^*}\frac{r}{1 + r(1 - \tau^K)} s_u^* < 0 \tag{20}$$

with s_u^* defined by (7) and $c_u^* = \frac{b}{1+\rho}$.

The utility of unemployed is decreasing in τ^K and will be maximized with the minimal capital income tax compatible with the budget constraint: $\tau_u^{K*} = \max\{0; \Phi(\tau^L_i)\}$. The corresponding labor income tax will be: $\tau_u^{L*} \equiv \mathrm{Min}\{\tau^L : \Phi(\tau^L) = 0\}$ or τ^L_i. Their preferences are in this case similar to the one of the retirees.

3.2.2 Preferences of young employed individuals ($\tau^{L*}_{we}; \tau^{K*}_{we}$)

Young employed individuals care about the present and the future and they are subject to both labor income and capital income taxation.

Young working individual e will prefer the tax combination that maximizes his indirect lifetime utility V_e^w:

$$V_e^w = \ln\left[(ew(1-\tau^L) - s_e^*)\right] + \beta \ln\left[s_e^*(1 + r(1-\tau^K))\right] \tag{21}$$

s.t.

$$\tau^K \equiv \Phi(\tau^L) = \frac{1+n}{r}\frac{1+\beta}{\beta}\left[1 - \frac{\frac{w}{2}\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right) - g}{\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}}\right]$$

$$0 \leq \tau^K \leq 1 \text{ and } 0 \leq \tau^L \leq 1$$

$$\text{and } s_e^* = \frac{\beta}{1+\beta} ew(1-\tau^L)$$

An increase of labor income taxation has two effects on the utility of the worker: a direct negative revenue effect by reducing the net wage income and an ambiguous indirect effect by changing the capital income tax:

$$\begin{aligned}\frac{dV_e^w}{d\tau^L} &\equiv \left[-ew - s_e^* \frac{r}{1+r(1-\Phi(\tau^L))}\frac{d\Phi(\tau^L)}{d\tau^L}\right]\frac{1}{c_e} \\ &= -\frac{1+\beta}{1-\tau^L} - \frac{\beta}{1+\beta}\frac{r}{1+r(1-\Phi(\tau^L))}\frac{d\Phi(\tau^L)}{d\tau^L}\end{aligned} \tag{22}$$

with $\frac{d\Phi(\tau^L)}{d\tau^L}$ defined by (16).

The sign of the indirect effect depends on the sign of $\frac{d\Phi(\tau^L)}{d\tau^L}$ since savings for employed worker (s_e^*) are always positive. When $\frac{d\Phi(\tau^L)}{d\tau^L} < 0$, the indirect effect is positive because the savings revenue are less taxed (and inversely when $\frac{d\Phi(\tau^L)}{d\tau^L} > 0$). Hence, a young employed individual will wish a positive tax rate on labor income if an increase in the labor income tax allows a sufficiently large reduction in the capital income tax, namely if $\frac{d\Phi(\tau^L)}{d\tau^L}$ is very negative. However, the magnitude of $\frac{d\Phi(\tau^L)}{d\tau^L}$ decreases with the ratio $\frac{1+r}{1+n}$: the smaller n and the larger r, the smaller the labor income tax basis, the larger the capital income tax basis and the smaller will be the variation of τ^K following a change in τ^L.

If the ratio $\frac{1+r}{1+n}$ is sufficiently large, the utility of worker is decreasing in τ^L and is maximized in $\tau^{L*}_{we} = 0$. Proposition 3 gives a sufficient condition for the utility of worker to decrease in τ^L:

Proposition 3 The utility of worker decreases with labor income tax if $\frac{1+n}{1+r}$ is sufficiently large:

$$\frac{dV_e^w}{d\tau^L} < 0 \quad \text{for all } \frac{b}{w} < 1 \text{ and for all } 0 < \tau^L < 1 - \frac{b}{w}$$

$$\text{if } \frac{1+r}{1+n} > \frac{1}{\beta} + \frac{1+\beta}{\beta}\frac{g}{w} \quad \text{when } \beta > \frac{\frac{g}{w}}{1-\frac{g}{w}}$$

$$\text{and if } \frac{1+r}{1+n} > \frac{w}{\beta b} + \frac{1+\beta}{\beta}\frac{g+b-w}{b} \quad \text{when } \beta < \frac{\frac{g}{w}}{1-\frac{g}{w}}$$

In this case, the taxation mix maximizing the utility of every worker is: $\tau^{L*}_{we} = 0$ and $\tau^{K*}_{we} = \Phi(0)$ if $\Phi(0) \leq 1$. If $\Phi(0) > 1$, $\tau^{K*}_{we} = 1$ and $\tau^{L*}_{we} = M$ in $\{\tau^L : \Phi(\tau^L) = 1\}$

See appendix 2 for a proof.

3.2.3 Preferences of young individuals

The optimal tax rate for a young individual is the one that provides him the highest utility. Young individual with skill level e will compare the utility with the optimal tax when employed and unemployed, namely the indirect utility obtained in section 3.2.1 and 3.2.2.

Using (6), (19) and (21), it is easy to see that individual with skill level e will prefer the taxation mix $(\tau_u^{K*}, \tau_u^{L*})$ iff:

$$V^u > V_e^w$$

or equivalently

$$\beta \ln[\frac{\beta}{1+\beta} bR\kappa_u] - \beta \ln[\frac{\beta}{1+\beta} R\kappa_e e\hat{w}] > \ln[\frac{1}{1+\beta} e\hat{w}] - \ln[\frac{b}{1+\beta}] \tag{23}$$

with

$\tilde{R}_e = 1 + r(1 - \tau_{we}^{K*})$

$\tilde{R}_u = 1 + r(1 - \tau_u^{K*})$

$\hat{w} = w(1 - \tau_{we}^{L*})$

$\tilde{R}_u \geq \tilde{R}_e$ with the strict inequality if the sufficient condition of proposition 3 is satisfied (since $\tau_u^{K*} = \max\{0; \phi(\tau_i^L)\}$ and $\tau_{we}^{K*} = \min\{1; \phi(0)\}$).

The left-hand side of (23) expresses the gain in the second period consumption if he is an unemployed rather than a worker. The term on the right-hand side reflects the potential gain in the first period if he works rather than being unemployed. Since τ_u^{K*} is the minimal capital income tax compatible with the budget constraint, the return on capital is higher if he decides to be unemployed ($\tilde{R}_u \geq \tilde{R}_e$). On the other hand, young individual with skill level e must also take into account the difference in net income in the first period (namely $b - ew(1 - \tau_{we}^{L*})$), which may be positive or negative, depending on his skill level and on his optimal tax rate in case he works (τ_{we}^{L*}). Hence, the optimal mix of taxation for individual e will be the one of an unemployed iff the gain in term of capital income in the second period outweighs the eventual loss in terms of net wage.

Using (23), one can identify the individuals with such preferences. Individual e will choose the optimal tax of an unemployed if his skill level e is lower than a threshold:

$$e < \frac{b}{\hat{w}}\left[\left(\frac{\tilde{R}_u}{\tilde{R}_e}\right)^{\frac{\beta}{1+\beta}}\right] \qquad (24)$$

Relatively low-skilled individuals would choose a low capital tax and would prefer to be unemployed. The threshold depends on τ_{we}^{L*} and may be different for each individual. If $\frac{1+r}{1+n}$ is sufficiently high (proposition 3 satisfied), this threshold is identical for all individuals.

This threshold may be higher or lower than $e_{min} = \frac{b}{w(1-\tau^L)}$, with τ^L the effective labor income tax resulting from the political process (not necessarily equal to τ_{we}^{L*}, the tax rate preferred by individual e). If e_{min} is higher than this threshold, some low-skilled individuals (with e larger than this threshold but lower than e_{min}) would prefer to work with a labor tax rate τ_{we}^{L*}, while they will be effectively unemployed. On the other hand, if the effective labor income tax rate is low (for example $\tau^L = 0$ with $e_{min} = \frac{b}{w}$), the threshold is higher than e_{min}[3]: some low-skilled workers would prefer to be unemployed and would vote for the taxation mix (τ_u^{K*}, τ_u^{L*}), even if the unemployment benefit is lower than their expected net wage in case they work.

Note also that the higher the unemployment benefit b, the higher will be the threshold. A higher unemployment benefit increases therefore the number of individuals that will prefer the mix of taxation (τ_u^{K*}, τ_u^{L*}).

3.3 The majority voting solution

Under proposition 3, we obtain corner solutions for the preferences of every agent. Old individuals prefer the lowest possible capital income tax. Very low-skilled young agents will have the same preferences. On the other hand, high-skilled agents will favor the lowest possible labor income tax. According to their preferences, population can be divided in two groups. The largest group could impose his preferred taxation mix in case of majority voting. The lower the population growth and the higher the unemployment benefits, the more labor income taxation will be favored.

4 Illustrations

In section 3, we have seen that there exist three categories of individuals regarding to their preferences over the financing method of a given social security system:

[3] The threshold is indeed always higher than $\frac{b}{w}$, the minimal skill level under which individuals will always be unemployed. See appendix 3 for a proof.

- Old individuals who wish the lowest possible capital income tax that respects the government budget constraint.

- Young individuals who prefer the optimal taxes for employed, with a skill level such that the wage income gain is sufficiently large to overcome an potential loss in term of capital revenue.

- Young individuals who prefer the optimal taxes for unemployed.

Since the definition of these categories as well as the optimal tax combination in each case are complex expressions of parameters that obscure economic intuition, we illustrate them by taking some specific realistic values for the parameters.

w	1	r	3.13
b	0.1	n	0.16
g	0.28	β	0.4

Table 1: Parameters values

We normalize the wage rate w to 1. The value of the unemployment allocation b satisfies (17). b influences the unemployment rate since all individuals with $e < \frac{b}{w}$ will always be unemployed. With a uniform distribution, this would imply a minimal unemployment rate of 10 percent[4]. The values of r and β correspond respectively to annual values of 4.84% and 0.97. With these values, the capital income/ labor income ratio close to $3/7$. The population growth corresponds to the annual growth rate of the EU in the nineties. The value of g corresponds to a ratio of public consumption over output of almost 20% and implies that the labor income tax will never be sufficient to finance the government expenditures[5]. To test the sensitivity of our results, we made the same

[4] A more realistic distribution (implying a lower density at the extrema) would lead to a smaller unemployment rate.

[5] The minimal capital income taxation compatible with the budget constraint is positive and close to zero.

exercise with higher and lower values of unemployment benefits ($b = 0.2$ and $b = 0.05$)[6]. This did not affect the conclusion.

With the numbers of table 1, we obtain the following results.

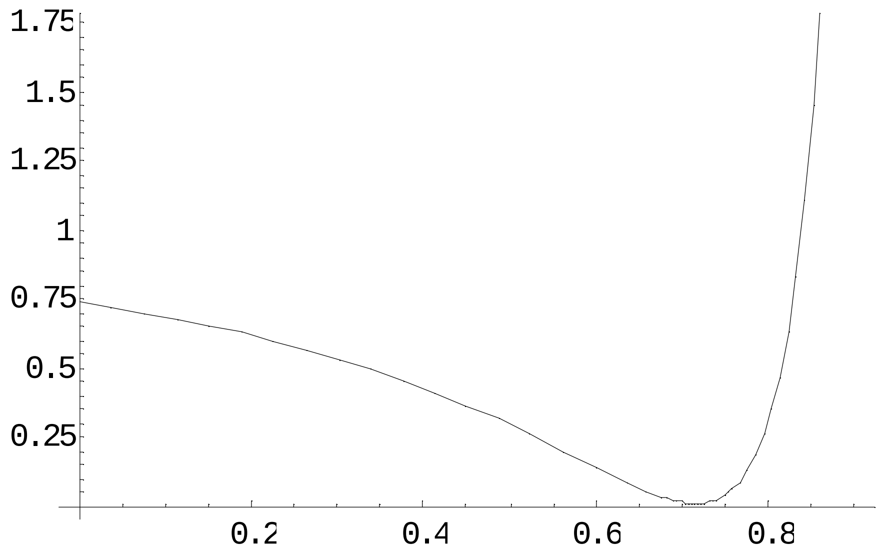

Figure 1: The government budget constraint (capital income tax in Y-Axis and labor income tax in X-Axis)

As stressed by equation (18), the government budget constraint is strictly convex between 0 and τ^L_{max} and reaches a unique minimum in $\tau^L_i = 71.54\%$. For lower values of τ^L, an increase in the labor income tax allows a reduction the capital income tax. For $\tau^L > \tau^L_i$, the labor supply effect dominates the revenue effect and an increase in the labor income tax requires an increase in the capital income tax to finance the larger number of beneficiaries (see equation (16)).

Old individuals prefer the lowest possible capital income taxation rate as seen in section 3.1. Their optimal tax combination is $\tau^L_i = 71.54\%$ and its corresponding capital income tax $\tau^{K*}_o = 1.30\%$.

[6] The values of g was adapted in each case to keep the minimal income close to zero (respectively $g = 0.14$ and $g = 0.37$).

Young individuals will compare their utility with their optimal tax rate when working and when unemployed (see section 3.2).

When working, their utility is always decreasing in $¿^L$ for all e (proof in appendix 2). The reduction in net wage always overcomes the gain due to the decrease in the taxation of savings that occurs when $¿^L < ¿^L_i$ (which is the largest when $¿^L = 0$). They all maximize their utility by choosing $¿^{L\,¤}_{we} = 0$ and $¿^{K\,¤}_{we} = 74{:}49\%$. Moreover, the higher their skill level e, the higher their indirect utility V^w_e for $¿^L = 0$ (see appendix 2).

For instance with $e = 0{:}5$; the utility depends in this way on $¿^L$ when working:

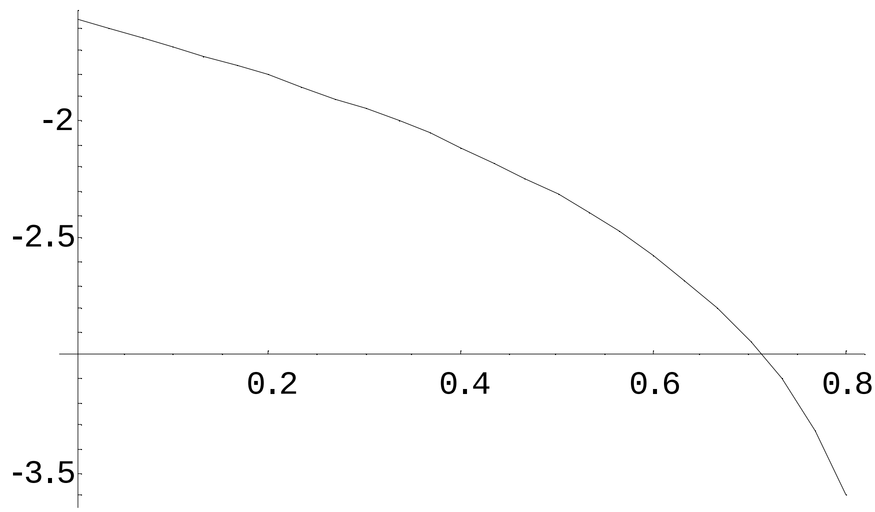

Figure 2: Indirect utility of a young employed as a function of labor income tax

Young individuals with skill level $e = 0{:}5$ will only work when labor income taxation rate is lower than 80%: Otherwise, his net wage is smaller than employment bene..t b. He will always prefer a zero labor income tax rate and a corresponding capital income tax of 74:49%. With this tax combination, he obtains a utility of -1.5732.

When they are unemployed, their utility depends in this way on τ^K:

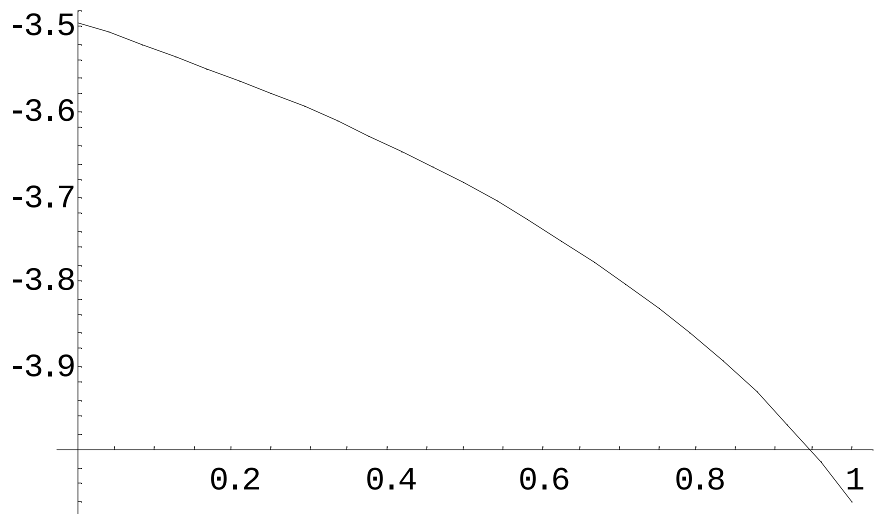

Figure 3: Indirect utility of a young unemployed as a function of capital income tax

Their utility is always decreasing in τ^K as indicated in (20) since their savings are positive. They will choose the lowest possible capital income tax : $\tau_u^{K*} = \phi(\tau_i^L) = 1.30\%$ and $\tau_u^{L*} = \tau_i^L = 71.54\%$. They will get a utility of -3.497.

Comparing the utility of individual when working and when unemployed, one can see that every individual with a skill level higher than the threshold $e = 0.1265$ will prefer τ_{we}^{L*} and τ_{we}^{K*}. This threshold is unique since all individuals choose a zero labor income tax when they work. As can be seen in figure 4, individual $e = 0.1265$ gets the same utility when working ($\tau^L = 0$) and when unemployed ($\tau^L = 71.54\%$).

Hence, low-skilled individuals will prefer τ_u^{L*} and a very low capital income tax, even if the unemployment benefit is lower than their expected net wage in

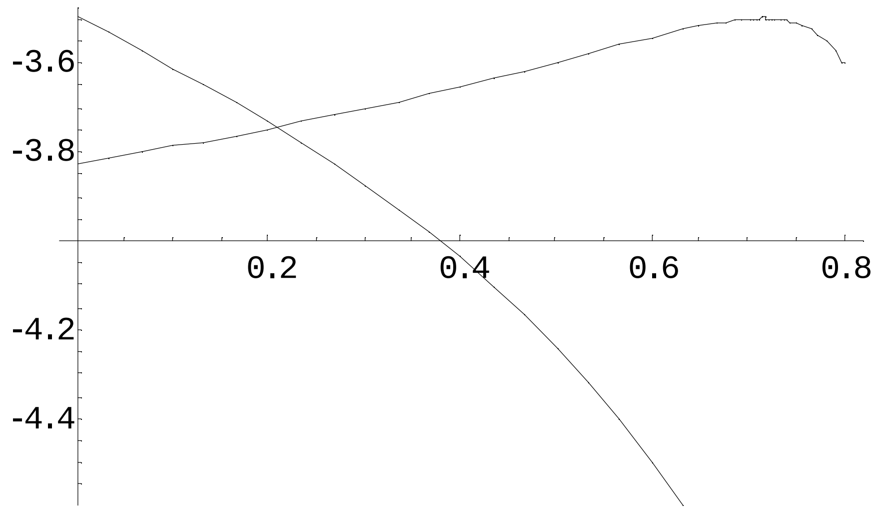

Figure 4: Indirect utility of a young individual with $e = 0.1265$ when working and when unemployed as a function of labor income tax

case they work. With $\tau^{L*}_{we} = 0$; this is the case for individuals with skill levels between $[0.1, 0.1265]$. This happens because the gain due to a higher net capital income more than outweighs this eventual loss in term of labor income.

With the parameter values of table 1, we obtain single-peaked preferences and corner solutions for all individuals, a conclusion that holds for the alternative values as well. There exist three types of preferences: old individuals, low-skilled individuals (with $e < 0.1265$) and high-skilled individuals. Two features must be stressed. First, we notice that despite the heterogeneity of young individuals, one can divide them in only two categories according to their preferences, independently of their effective employment status. High-skilled young agents will prefer a zero labor income tax, while low-skilled will favor a very low capital income tax. Second, we notice that low-skilled young individuals have the same preferences as the old. They could form a political majority in this case, sustaining a very low capital income tax. With a lower b and less unemployed individuals, a majority (composed of high skilled agents) could sustain a low labor income tax as in the United States.

5 Conclusion

The political support of capital income and labor income taxes has often been studied separately. In this paper we examine the preferences of voters regarding both taxes. We look at the way government consumption and a given social security system with unemployment allocations can be financed. We consider a double heterogeneity of voters : individuals belong to two generations and there exists a continuity of skill level. We analyze the preferred taxation policy mix of each agent. Every agent must take into account how each taxation may diminish his revenue and to what extent a change of one tax requires a change of the other tax. A tax modification also influences the endogenous labor supply.

Old agents always choose the lowest possible capital income tax rate and the corresponding labor income tax rate. Despite the continuity of skillness of young agents, young agents vote in only two ways. High-skilled voters prefer the lowest possible labor tax to minimize its dominant negative revenue effect. Low-skilled agents on the other hand prefer a low capital income tax : they vote as if they were unemployed and want to maximize capital income. With a high unemployment allocation, a majority of voters (a coalition of old and low-skilled young voters) sustain the lowest possible capital tax. With a low unemployment benefit, we obtain a majority (high-skilled agents) in favor of low labor taxes.

Future research could compare the preferences of agents over labor versus consumption taxes, a question often raised by politicians in recent years. We could also endogenize the level of unemployment benefits.

6 Appendix

6.1 The shape of the budget constraint

The budget constraint can be written as:

$$\tau^K \equiv \Phi(\tau^L) = \frac{1+n}{r}\frac{1+\beta}{\beta}\left[1 - \frac{\frac{w}{2}\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right) - g}{\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right)+\frac{b^2}{w(1-\tau^L)}}\right]$$

The slope of this function $\Phi(\tau^L)$ is given by:

$$\frac{d\Phi(\tau^L)}{d\tau^L} = \frac{(1+n)(1+\beta)}{r\beta\left[\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right)+\frac{b^2}{w(1-\tau^L)}\right]} \ast \left[\left(-\frac{w}{2}(1-e_{min}{}^2)+b\left(1+\frac{\tau^L}{1-\tau^L}\right)\right)\frac{de_{min}}{d\tau^L} + \left(1-\frac{\frac{w}{2}\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right)-g}{\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right)+\frac{b^2}{w(1-\tau^L)}}\right)\frac{w}{2}(1-e_{min}{}^2)\right]$$

Replacing e_{min} and manipulating the expression, it can also be written as:

$$\frac{d\Phi(\tau^L)}{d\tau^L} = \frac{(1+n)(1+\beta)}{r\beta\left[\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right)+\frac{b^2}{w(1-\tau^L)}\right]^2} \ast \left(\frac{b^2}{(1-\tau^L)^2}+\frac{b^4}{4w^2(1-\tau^L)^4}-\frac{w^2}{4}+\frac{gw}{2}\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right)\right) \quad (25)$$

To determine the shape of the function $\Phi(\tau^L)$, we compute the value of its slope at the two extrema of the labor income tax ($\tau^L = 0$ and $\tau^L = \tau^L_{max} = 1 - \frac{b}{w}$).

When $\tau^L = 0$:

$$\left.\frac{d\Phi(\tau^L)}{d\tau^L}\right|_{\tau^L=0} = \frac{(1+n)(1+\beta)}{r\beta\left[\frac{w}{2}\left(1-\frac{b^2}{w^2}\right)+\frac{b^2}{w}\right]} \ast \left(b^2+\frac{b^4}{4w^2}-\frac{w^2}{4}+\frac{gw}{2}\left(1-\frac{b^2}{w^2}\right)\right)$$

The first term in the multiplication is always positive. The second term is negative if the per capita public consumption is not too large: $-\ast\; g < \left(\frac{1}{4}-\frac{b^2}{w^2}-\frac{b^4}{4w^4}\right)\frac{2w^3}{w^2-b^2}$. Since $g > 0$, $\left(\frac{1}{4}-\frac{b^2}{w^2}-\frac{b^4}{4w^4}\right)\frac{2w^3}{w^2-b^2}$ must be positive. This will be the case if unemployment benefits are not too large: $\frac{b}{w} < \sqrt{5} - 2 = 0.486$. This leads to the condition of equation (17).

When $\tau^L = \tau^L_{max} = 1 - \frac{b}{w}$, the slope of $\Psi(\tau^L)$ is positive:

$$\frac{d\Psi(\tau^L)}{d\tau^L}\Big|_{\tau^L=\tau^L_{max}} = \frac{(1+n)}{r}\frac{(1+\beta)}{\beta}\frac{w^2}{b^2} > 0 \qquad (26)$$

Hence, the function $\Psi(\tau^L)$ is downwards sloping in $\tau^L = 0$ and upwards sloping in τ^L_{max}. To compute the minimum of this function (τ^L_i), we need to find the value of τ^L that satisfies:

$$\frac{d\Psi(\tau^L_i)}{d\tau^L} = 0 \text{ and } 0 < \tau^L_i < 1 - \frac{b}{w}$$

Since the first term of (25) is always positive, the minimum must satisfy:

$$\frac{b^2}{(1-\tau^L)^2} + \frac{b^4}{4w^2(1-\tau^L)^4} - \frac{w^2}{4} + \frac{gw}{2}\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right) = 0$$

This equation can be expressed as a second order equation in $(1-\tau^L)^2$:

$$a(1-\tau^L)^4 + b(1-\tau^L)^2 + c = 0$$

with $a = \left(\frac{gw}{2} - \frac{1}{4}\right)$
$b = \frac{b^2}{w^2}\left(1 - \frac{g}{2w}\right)$
$c = \frac{b^4}{4w^2}$

If $g < \frac{w}{2}$, this expression is concave ($a < 0$, $b > 0$, $c > 0$) and has only one positive root given by:

$$(1-\tau^L_i)^2 = \frac{1 - \frac{g}{2w} + \sqrt{\frac{5}{4} - \frac{3g}{2w} + \frac{g^2}{4w^2}}}{\frac{1}{2} - \frac{g}{w}}\frac{b^2}{w^2}$$

or equivalently by:

$$\tau^L = \tau^L_i = 1 - \frac{b}{w}\sqrt{\frac{1 - \frac{g}{2w} + \sqrt{\frac{5}{4} - \frac{3g}{2w} + \frac{g^2}{4w^2}}}{\frac{1}{2} - \frac{g}{w}}}$$

The labor income tax rate τ^L_i is therefore the unique labor income tax that minimizes the capital income tax.

6.2 The preferences of young employed individuals

The preferences of young working agent e with respect to the taxation mix can be found by computing the derivative of his indirect utility with respect to τ^L:

$$\frac{dV_e^w}{d\tau^L} \equiv \left[-ew - s_e^* \frac{r}{1+r(1-\phi(\tau^L))}\frac{d\phi(\tau^L)}{d\tau^L}\right]\frac{1}{c_e} \qquad (27)$$

with $\frac{d\phi(\tau^L)}{d\tau^L}$ defined by (16).

Since the savings of employed individuals s_e^* is positive for all τ^L, this expression can only become positive if $\frac{d\phi(\tau^L)}{d\tau^L}$ is sufficiently negative to make the second term $(-s_e^* \frac{r}{1+r(1-\phi(\tau^L))}\frac{d\phi(\tau^L)}{d\tau^L})$ positive and larger than (ew). This requires a small value of $\frac{1+r}{1+n}$. A sufficient condition for this expression to be unambiguously negative is that $\frac{1+r}{1+n}$ is large enough.

Substituting (16) and (15) in (27), we get:

$$\frac{dV_e^w}{d\tau^L} = -\frac{(1+\beta)}{1-\tau^L} - \frac{\beta}{D}\left(\frac{b^2}{(1-\tau^L)^2} + \frac{b^4}{4w^2(1-\tau^L)^4} - \frac{w^2}{4} + \frac{gw}{2}\left(1 - \frac{b^2}{w^2(1-\tau^L)^2}\right)\right)$$

$$\text{with } D = \left[\frac{w}{2}(1-\tau^L)\left(1-\frac{b^2}{w^2(1-\tau^L)^2}\right) + \frac{b^2}{w(1-\tau^L)}\right]^2 \left[1 + r(1-\phi(\tau^L))\right] > 0$$

$$= \left[\begin{array}{l} \left(\tau^L + (1-\tau^L)\frac{1+r}{1+n}\frac{\beta}{1+\beta}\right)\left(\frac{w^2}{4} - \frac{b^2}{2(1-\tau^L)^2} + \frac{b^4}{4w^2(1-\tau^L)^4}\right) \\ + \frac{b^2}{(1-\tau^L)}\left(\frac{1+r}{1+n}\frac{\beta}{1+\beta} - 1\right) + \frac{b^2}{2(1-\tau^L)^2}\left[1 - \frac{g}{w}\right] - \frac{b^4}{2w^2(1-\tau^L)^4} - \frac{gw}{2} \end{array}\right]$$

$$*(1-\tau^L)$$

After some manipulations, the above expression can be written as:

$$\frac{dV_e^w}{d\tau^L} = \frac{w^2}{2D} * \left[\begin{array}{l} \frac{b^4}{w^4(1-\tau^L)^4}\left(\frac{\tau^L}{4}\left(-(1+\beta)+\beta\frac{1+r}{1+n}\right) + \frac{1}{2} + \frac{\beta}{4} - \frac{\beta}{4}\frac{1+r}{1+n}\right) + \\ \frac{b^2}{w^2(1-\tau^L)^2}\left(\frac{\tau^L}{2}\left(-(1+\beta)+\beta\frac{1+r}{1+n}\right) + \frac{1}{2} - \frac{\beta}{2} - \frac{\beta}{2}\frac{1+r}{1+n} + \frac{g}{2w}(1+2\beta)\right) \\ + \frac{\tau^L}{4}\left(-(1+\beta)+\beta\frac{1+r}{1+n}\right) + \frac{\beta}{4} - \frac{\beta}{4}\frac{1+r}{1+n} + \frac{g}{2w} \end{array}\right]$$

Since the first ratio is positive, the sign of $\frac{dV_e^w}{d\tau^L}$ depends on the sign of the expression between brackets. This expression (called $*(\frac{b^2}{w^2(1-\tau^L)^2})$) constitutes a second order equation in $\frac{b^2}{w^2(1-\tau^L)^2}$:

$$\Phi\left(\frac{b^2}{w^2(1-\tau^L)^2}\right) = A\frac{b^4}{w^4(1-\tau^L)^4} + B\frac{b^2}{w^2(1-\tau^L)^2} + C$$

with
$$A = \frac{\tau^L}{4}\left(-(1+\beta) + \beta\frac{1+r}{1+n}\right) + \frac{1}{2} + \frac{\beta}{4} - \frac{\beta}{4}\frac{1+r}{1+n}$$
$$B = \frac{\tau^L}{2}\left(-(1+\beta) + \beta\frac{1+r}{1+n}\right) + \frac{1}{2} - \frac{\beta}{2} - \frac{\beta}{2}\frac{1+r}{1+n} + \frac{g}{2w}(1+2\beta)$$
$$C = \frac{\tau^L}{4}\left(-(1+\beta) + \beta\frac{1+r}{1+n}\right) + \frac{\beta}{4} - \frac{\beta}{4}\frac{1+r}{1+n} + \frac{g}{2w}$$

The sign of $\Phi(\frac{b^2}{w^2(1-\tau^L)^2})$ depends on the sign of A, B, C (determining the sign of the two roots X_1 and X_2), which in turn depends on the magnitude of the ratio $\frac{1+r}{1+n}$:

$C < 0$ if $\frac{1+r}{1+n} > 1 + \frac{2\frac{g}{w} - \tau^L}{\beta(1-\tau^L)}$

$B < 0$ if $\frac{1+r}{1+n} > \frac{1}{\beta} + \frac{g}{w}\frac{1+2\beta}{\beta(1-\tau^L)} - \frac{1+\tau^L}{1-\tau^L}$

$A < 0$ if $\frac{1+r}{1+n} > 1 + \frac{2-\tau^L}{\beta(1-\tau^L)}$

and $C < A$, $B < A$ (with $g < \frac{w}{2}$) and, if $\beta < \frac{1}{2}$, $C < B < A$.

A sufficient condition for $\frac{dV_e^w}{d\tau^L} < 0$ for all $0 < \frac{b^2}{w^2(1-\tau^L)^2} < 1$ (with $g < \frac{w}{2}$) is:

$$\frac{1+r}{1+n} > \frac{1 + (1+\beta)(\frac{g}{w} - \tau^L)}{\beta(1-\tau^L)}$$

Indeed, this condition guarantees that $\Phi(\frac{b^2}{w^2(1-\tau^L)^2})$ has at most one positive root (if $A > 0$) and that this root (X_2) is larger than one ($B < 0$, $C < 0$, and $A + B + C < 0$). If $\frac{1+(1+\beta)(\frac{g}{w}-\tau^L)}{\beta(1-\tau^L)} < \frac{1+r}{1+n} < 1 + \frac{2-\tau^L}{\beta(1-\tau^L)}$, the function $\Phi(.)$ is convex and is negative for all values of $X_1 < 0 < \frac{b^2}{w^2(1-\tau^L)^2} < X_2 > 1$. If $\frac{1+r}{1+n} > 1 + \frac{2-\tau^L}{\beta(1-\tau^L)}$, $\Phi(.)$ is concave and is negative for all values $\frac{b^2}{w^2(1-\tau^L)^2} > 0$ (the two roots are negative).

To determine a sufficient condition independent of τ^L, we notice that:

$$\partial\left(\frac{1+(1+\beta)(\frac{g}{w}-\tau^L)}{\beta(1-\tau^L)}\right)\Big/\partial\tau^L \gtrless 0 \iff \beta \lessgtr \frac{\frac{g}{w}}{1-\frac{g}{w}}$$

Hence, a sufficient condition for $\frac{dV_e^w}{d\tau^L} < 0$ for all $0 < \frac{b^2}{w^2(1-\tau^L)^2} < 1$ and for all $0 < \tau^L < 1 - \frac{b}{w}$ is:

when $\beta > \frac{\frac{g}{w}}{1-\frac{g}{w}}$:

$$\frac{dV_e^w}{d\tau^L}\Big|_{\tau^L=0} < 0 \iff \frac{1+r}{1+n} > \frac{1}{\beta} + \frac{(1+\beta)}{\beta}\frac{g}{w}$$

and when $\beta < \frac{\frac{g}{w}}{1-\frac{g}{w}}$:

$$\frac{dV_e^w}{d\tau^L}\Big|_{\tau^L=1-\frac{b}{w}} < 0 \iff \frac{1+r}{1+n} > \frac{w}{\beta b} + \frac{(1+\beta)}{\beta}\frac{g+b-w}{b}$$

Notice that these conditions are not necessary: if $\frac{1+r}{1+n} < \frac{1+(1+\beta)(\frac{g}{w}-\tau^L)}{\beta(1-\tau^L)}$, $\frac{dV_e^w}{d\tau^L} < 0$ if $\frac{b^2}{w^2(1-\tau^L)^2}$ is relatively small ($\frac{b^2}{w^2(1-\tau^L)^2} < \mathcal{X}_2 < 1$). In addition, if $\frac{1+r}{1+n} < 1 + \frac{2\frac{g}{w}-\tau^L}{\beta(1-\tau^L)}$, the two roots of $\Lambda(.)$ are negative and $\frac{dV_e^w}{d\tau^L} > 0$ for all $\frac{b^2}{w^2(1-\tau^L)^2} > 0$. This may however imply a negative value for $\frac{1+r}{1+n}$.

The illustrative values chosen satisfy this sufficient condition:

- For $\beta = 0.4$, $w = 1; b = 0.1$, $g = 0.28; r = 3.13; n = 0.16$:

  $\beta > \frac{\frac{g}{w}}{1-\frac{g}{w}}$

  $\frac{1+r}{1+n} = 3.56 > \frac{1}{\beta} + \frac{(1+\beta)}{\beta}\frac{g}{w} = 3.48$

  $\Rightarrow \frac{d\Phi(\tau^L)}{d\tau^L}\Big|_{\tau^L=0} < 0$ and $\frac{dV_e^w}{d\tau^L} < 0$ for all $0 < \tau^L < \tau^L_{max}$

Therefore, $\tau^{L*}_{we} = 0 \quad \forall e > e_{min}$.

In addition, the indirect utility of a young worker is increasing in e as can be seen in the following expression where we have substituted (6) in (21):

$$V_e^w = \ln\left[\frac{1}{1+\beta}(ew(1-\tau^L))\right] + \beta \ln\left[\frac{\beta}{1+\beta} ew(1-\tau^L)(1+r(1-\tau^K))\right]$$

Hence, the lower the skill level of a young individual, the lower will be his utility of working when $\tau^{L*}_{we} = 0$.

6.3 Limit value for the skill level determining the optimal tax rate of young individuals

The threshold is always higher than $\frac{b}{w}$:

$$\frac{b}{\hat{w}}\left[\left(\frac{R^*_u}{R^*_e}\right)^{\frac{\beta}{1+\beta}}\right] > \frac{b}{w}$$

Indeed, two cases can be distinguished:

if $\tau^{L*}_{we} = 0$:

$\frac{b}{\hat{w}} = \frac{b}{w}$ and $R^*_e < R^*_u$ (since $\tau^{K*}_{we} = \min\{1;\Phi(0)\}$ and $\tau^{K*}_u = \max\{0;\Phi(\tau^L_i)\}$);

if $\tau^{L*}_{we} > 0$:

$\frac{b}{\hat{w}} > \frac{b}{w}$ and $R^*_e \leq R^*_u$ (since $\tau^{K*}_u = \max\{0;\Phi(\tau^L_i)\}$).

Hence, this condition is satisfied in each case.

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