Endogenous Versus Exogenous Growth Facing a Fertility Shock

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Abstract

This paper simulates the impact of a permanent fertility shock on economic growth, using endogenous versus exogenous growth OLG models. An endogenous growth model, with education as the engine of growth, dampens the negative impact of a decline in fertility on growth when compared with an exogenous growth model. This result stems from the rise in education levels brought about by the expectation of an increase in the discounted value of labour earnings.

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1. INTRODUCTION

A vast literature exists on the economic consequences of ageing, although nearly all models assume exogenous growth (e.g. McMorrow-Roeger, 1999; and Turner et al., 1998). In particular, many economic studies on the solvency of social security faced with an ageing population treat growth, labour productivity, and other variables as exogenous. This paper raises serious doubts on that methodological approach. De la Croix (1998) had already pointed out the shortcomings of many analyses of the impact of fertility shocks due to the extensive use of exogeneity assumptions, and in particular, lack of an endogenous growth framework.

The ageing problem has been studied both in a context of (computable) general equilibrium OLG and of representative agent models. The former modelling approach is preferable to analyse the impact of demographic shocks. Examples of this approach include Auerbach-Kotlikoff (1987), Hviding-Merette (1998) and Miles (1999). The OLG approach is necessary to evaluate the intergenerational impact of demographic and/or economic policy changes. Moreover, issues related to the solvency and reform of social security systems (financed on a pay-as-you-go basis) are better addressed using the OLG model framework. However, OLG models have some drawbacks, namely they are generally more difficult to implement, and often have multiple equilibria that can be unstable. For these reasons, the EU (1999), and international organisations like the OECD (1998), and the IMF (1990), prefer to use their (large) macroeconomic representative agent models, which are adapted, instead of developing dedicated OLG models to analyse the ageing problem (McMorrow-Roeger, 1999).

To analyse the impact of demographic changes, this paper compares two computable OLG models, assuming exogenous versus endogenous growth, respectively ([1]). Demographic variables reproduce the observed decline in population growth rates registered since the early 1960s in Western Europe. The benchmark is provided by a modified version of the exogenous growth OLG model (EXO) developed in Auerbach-Kotlikoff (1987) ([2]). The endogenous growth OLG model (ENDO), with education as the engine of growth, is based on Medeiros (2000), which develops a model to study the role of altruism in reducing the poverty risk in old age.

In exogenous growth models, the impact of a decline in population growth rates works exclusively through capital deepening. This is a well-known result since the seminal work of Solow (1956). A (permanent) decline in fertility raises the capital intensity ratio, which in a competitive equilibrium increases the wage rate and lowers the interest rate ([3]). These changes in factor prices raise long-run welfare, because they allow for higher levels of consumption and leisure. Although economic growth remains exogenous in the long-run, in the transition period capital deepening partly offsets the impact of a decline in population growth rates on economic growth ([4]).

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[1] The major difference between the two computable OLG models is that in the exogenous model the margin of decision is between leisure and work; whereas, in the endogenous model time is allocated between education and the production of goods. Aside from this major difference, the two models have a similar structure: they consider many generations, individuals maximise lifetime utility subject to analogous financial constraints, economic agents have perfect foresight, there are perfect capital markets, and prices are determined in a perfect competition equilibrium.

[2] This modified version differs from the original model in two major respects: first, it assumes lifetime uncertainty -instead of a fixed life span; and, second, economic policy is not considered.

[3] In a lifecycle model, this reflects the fact that there are fewer young workers with limited financial assets relative to elderly individuals with sizeable retirement savings.

[4] Moreover, Auerbach-Kotlikoff (1987) argue that the long-run effect of demographic shocks on welfare analysis dominates that of various simulated social security policy changes, which have only important effects on the transition period. Even in the extreme case where payroll taxes raise dramatically, a decline in fertility continues to have a substantially favourable impact on long-run welfare due to capital deepening.

In the ENDO model, education is the engine of growth. An individual allocates human capital between production and education activities to maximise lifetime resources. There are spillover effects in human capital formation (Lucas, 1988), and the average stock of human capital is (partially) transmitted across generations (Azariadis-Drazen, 1990). In these circumstances, in addition to the direct effect of capital deepening, there is an induced effect on the education sector as the discounted value of labour earnings rises, stimulating education and raising long-run growth.

In the ENDO model, a negative fertility shock represents an "opportunity" to invest in education. In fact, a decline in fertility increases the future capital intensity ratio in the economy and the discounted value of labour earnings, raising the profitability of investments in education ([5]). Therefore, the negative impact of a decline in fertility on the active population will be partly offset through a rise in the per capita level of human capital, resulting from the increase in education. This rise in the level of human capital will be (partly) passed over to future generations due to the externality in education. Thereby, a fall in fertility raises permanently the level of labour productivity.

This paper is organised as follows. Section 2 presents the two models, and derives the optimal individual programmes. Section 3 assumes a competitive equilibrium, and presents the general equilibrium conditions. In section 4, the models are calibrated using some stylised facts that characterise the European economy. Section 5 uses the two computable OLG models to simulate the effect of a permanent fertility shock,

[5] Moreover, the market solution is not optimal, which implies that education should be subsidised.

which mirrors the decline in population growth rates registered in Europe since the early 1960s. Finally, concluding remarks are presented in section 6.

2. THE OLG MODELS

This section presents two computable OLG models. The first is a modified version of the exogenous growth model presented in Auerbach-Kotlikoff (1987), hereafter referred as the EXO model. The second is an endogenous growth model (ENDO), with education as the engine of growth, developed in Medeiros (2000). In the former, the margin of decision is between leisure and the production of goods; whereas, in the latter, it is between education and the production of goods. They are both lifecycle models, where an individual lives for a maximum of T years, there is lifetime uncertainty, and selfish behaviour.

In the EXO model, economic growth is the result of exogenous labour-augmenting technological progress. An individual maximises a nested CES utility function with consumption and leisure as arguments subject to a wealth constraint.

In the ENDO model, an individual maximises an intertemporal separable utility function with consumption as the only argument subject to a wealth constraint and a human capital equation. An individual allocates the total length of time between education and production activities. Because leisure is not an argument of the period utility function, the individual's maximisation problem can be divided in two stages ([6]): first, time is allocated between education and the production of goods to maximise

[6] Separability is necessary to obtain a closed form equation for the accumulation of human capital.

total resources ([7]); and, second, the profile of consumption is chosen for a given level of resources.

2.1 The insurance market

As it is well-known since Yaari (1965), the assumptions of lifetime uncertainty and perfect capital markets raise a difficulty, because an individual can die with negative financial wealth. Blanchard (1985) solves this problem introducing competitive insurance companies that earn a premium (on negative financial wealth positions) or pay an interest rate supplement (on positive positions) in exchange for the financial balance of the insured when they die.

A similar insurance mechanism is assumed in this paper. However, due to the age-variable probability of death, insurance companies pay a variable interest rate supplement, depending on the age structure of the population.

The interest rate supplement $\pi(t)$ is the ratio between the financial wealth of those recently deceased (the numerator) and aggregate financial wealth in the previous period (the denominator):

$$\pi(t) = \frac{\sum_{v=t-\mathrm{T}}^{t} f(t,v)B(v)d(t-v)\prod_{l=v}^{t-1}\left[1-d(l-v)\right]}{\sum_{v=t-1-\mathrm{T}}^{t-1} f(t-1,v)B(v)\prod_{l=v}^{t-2}\left[1-d(l-v)\right]} \qquad \textbf{(1)}$$

where $f(t,v)$ is financial wealth in period t of an individual of generation v; T is the maximum life span; $d(n)$ is the probability of death at age n, with $d(0)=0$, and $d(\mathrm{T})=1$; and, $B(v)$ is the number of individuals born in period v.

[7] Or equivalently, to maximise human wealth: the discounted value of future labour earnings net of

With constant probability of death and *B(v)*, as in Blanchard (1985), the interest rate supplement equals the (constant) probability of death (i.e. $\pi(t)=d$).

2.2 The wealth constraint

The flow of income of an individual depends on the amount of time spent at work and the stock of human capital. In the EXO model, human capital grows at an exogenous fixed rate; whereas, in the ENDO model, the stock of human capital is driven by education.

Let us denote *h(t,v)* as human capital, and $\lambda(t,v)$ as the fraction of time allocated to leisure (in the EXO model) or to education (in the ENDO model). Furthermore, and in order to facilitate the reproduction of the stylised facts regarding the age-profile of labour earnings, let us introduce *e(t-v)* as the exogenous human capital efficiency factor ([8]). Then, the budget constraint can be written as:

$$f(t,v) = R(t)\ f(t-1,v) + w(t)\left[1-\lambda(t,v)\right]e(t-v)\ h(t,v) - c(t,v) - \tau(t)$$

where **(2)**

$$R(t) = 1 + r(t) + \pi(t) \quad ; \quad 0 \le \lambda(t,v) \le 1$$

where *R(t)* is the interest rate factor between periods *t-1* and *t*; *r(t)* is the interest rate; *w(t)* is the wage rate; *c(t,v)* is consumption; and, $\tau(t)$ is a lump-sum tax.

Financial wealth increases with interest and labour income, and declines with consumption and lump-sum taxes. Labour income is the product of human capital used in production (in efficiency units) and the wage rate. Labour force participation

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lump-sum taxes.

[8] It is a well-known fact that the typical age profile of labour earnings rises until about 50–55 years of age, declining thereafter. The distinction between human capital (in physical units) and human capital in efficiency units facilitates the reproduction of the above-mentioned stylised fact.

begins/ends at the fixed ages $\underline{a}$ and $\bar{a}$, respectively; thereby, the previous expression applies in the interval $t \in \left[v+\underline{a}, v+\bar{a} \right]$ ([9]). In youth and in retirement, the dynamic equation of financial wealth is obtained from (2) by setting $\lambda(t,v)$=1.

A no-Ponzi-game (NPG) condition is used to prevent an individual from leaving assets after the maximum possible duration of life (T):

$$f(v+\mathrm{T}, v) = 0 \tag{3}$$

Using the NPG condition, the individual's budget constraint (2) can be integrated forwards to obtain the intertemporal budget constraint:

$$\omega(t,v) = \sum_{q=t+1}^{v+\mathrm{T}} \frac{c(q,v)}{\left\{ \prod_{n=t+1}^{q} R(n) \right\}} \tag{4}$$

Equation (4) states that total resources net of lump-sum taxes or total wealth -$\omega(t,v)$- equals the present value of lifetime consumption. Total wealth is the sum of financial and human wealth:

$$\omega(t,v) \equiv f(t,v) + l(t,v) \tag{5}$$

where $l(t,v)$ is human wealth -the discounted value of future labour earnings minus lump-sum taxes. The expression for human wealth is:

$$l(t,v) \equiv \sum_{q=t+1}^{v+\mathrm{T}} \frac{w(q)\left[1-\lambda(q,v)\right]e(q-v)\,h(q,v) - \tau(q)}{\left\{ \prod_{n=t+1}^{q} R(n) \right\}} \tag{6}$$

[9] The age of entry in the active population should be higher in the EXO model (where it marks full-time participation in the labour market) than in the ENDO model (where it includes education and training).

2.3 Human capital

In the EXO model there is exogenous labour-augmenting technological progress. Therefore, human capital grows at the steady state growth rate of per capita variables: κ_v.

$$h\,(t,v) = (1+\kappa_v)^t \tag{7}$$

In the ENDO model, the production function of human capital is a *Cobb-Douglas* function, combining a private and a social inputs. The private input is the fraction of human capital in efficiency units allocated to education activities: $\lambda(t,v)e(t-v)h(t,v)$ (Ben-Porath, 1967) ([10]). The social input is the per capita average stock of human capital in the active population in efficiency units: *H(t)*:

$$H(t) \equiv \frac{\sum_{v=t-\bar{a}}^{t-\underline{a}} e(t-v)\,h(t,v)B(v)\left\{\prod_{n=v}^{t-1}\left[1-d(n-v)\right]\right\}}{\sum_{v=t-\bar{a}}^{t-\underline{a}} B(v)\left\{\prod_{n=v}^{t-1}\left[1-d(n-v)\right]\right\}} \tag{8}$$

The production function of human capital involves the type of positive externality present in Lucas (1988). The dynamic equation of human capital is:

$$\begin{gathered} h\,(t+1,v) = (1-\delta_h)\,h\,(t,v) + \xi\,[\,\lambda(t,v)e(t-v)h(t,v)\,]^{\alpha}\;H(t)^{1-\alpha} \\ \text{where} \\ 0 \le \lambda(t,v) \le 1\;,\; 0 \le \delta_h < 1\;,\; 0 < \alpha < 1\;,\; \xi > 0 \end{gathered} \tag{9}$$

where δ_h is the depreciation rate of human capital; ξ is a multiplicative factor; and, α is the elasticity of the production function with respect to the private input.

[10] To simplify, and without changing the qualitative nature of the results, goods do not enter in the production function of human capital. For a production function of human capital that includes as arguments both the amount of time and goods used in education see Becker (1965) for the seminal paper.

2.4 The optimisation programmes

In the EXO model, an individual maximises a nested CES utility function with consumption and leisure as arguments subject to the financial wealth constraint (2). Recall that in this model human capital (*h*) grows at a fixed rate.

$$\max_{\{c,\lambda\}} \quad U(t,v) = \sum_{q=t}^{v+T} \left[\frac{\underline{u}(q,v)^{1-\sigma} - 1}{1-\sigma} \right] \beta^{q-t} \left\{ \prod_{l=t}^{q-1} \left[1 - d(l-v) \right] \right\} \qquad st \ (2)$$

where **(10)**

$$\underline{u}(q,v) = \left[c(q,v)^{1-\upsilon} \ + \ \theta \left[\lambda(q,v) e(q-v)\, h\,(q,v) \right]^{1-\upsilon} \ \right]^{\frac{1}{1-\upsilon}}$$

$$\sigma > 0 \ ; \ \upsilon > 0 \ ; \ \theta \geq 0$$

where $\underline{u}$ is the period utility function; $1/\sigma$ is the intertemporal elasticity of substitution in consumption; $1/\upsilon$ is the intratemporal elasticity of substitution between consumption and leisure; β is the time discount factor; and, θ measures the intensity of an individual's preferences for leisure in relation to consumption ([11]).

In the ENDO model, with leisure exogenous, the optimisation problem is recursive: first, time is allocated between education and the production of goods to maximise total resources; and, second, the profile of consumption is chosen (for a given level of total resources) to maximise lifetime utility (Medeiros, 2000). This is equivalent to maximising the intertemporal utility function (10), taking θ=0, subject to the budget constraint (2) and the production function of human capital (9).

In the first stage, an individual chooses the amount of time spent in education that maximises the discounted value of future labour earnings (minus lump-sum taxes) subject to the production function of human capital.

$$\max_{\{\lambda\}} \; l(v,v) \equiv \sum_{u=v+\underline{a}}^{v+\bar{a}} \frac{w(u)\left[1-\lambda(u,v)\right]e(u-v)\,h(u,v)-\tau(u)}{\left\{\prod_{n=v}^{u-1} R(n)\right\}} \quad s.t. \quad (9) \tag{11}$$

where $l(v,v)$ is human wealth at birth (i.e. lifetime earnings net of lump-sum taxes).

In the second stage, an individual chooses the consumption profile that maximises lifetime utility (i.e. (10) for θ=0) subject to the financial wealth constraint (2).

2.5 The solutions

In the EXO model, the optimal level of leisure -measured as a fraction of the total time endowment available to work- is:

$$\lambda(t,v) = \min\left\{ 1 \, , \, \frac{c(t,v)}{e(t-v)h(t,v)}\left[\frac{\theta}{w(t)e(t-v)}\right]^{\frac{1}{\upsilon}} \right\} \tag{12}$$

In the ENDO model, the optimal level of human capital evolves according to:

$$h(t,v) = h(v,v)\left(1-\delta_h\right)^{t-v} + \sum_{n=v+\underline{a}}^{t-1} \left(1-\delta_h\right)^{t-1-n}\,\Omega(n,v)\,H(n) \tag{13}$$

where $h(v,v)$ is the level of human capital at birth.

$\Omega(n,v)$ is the period n (private) optimal increment in the stock of human capital per unit of $H(n)$. It is given by:

$$\Omega\,(t,v) = \xi \,\min\left\{ \left[\frac{\xi\,\alpha\,\mu\,(t,v)}{w(t)}\right]^{\frac{\alpha}{1-\alpha}}, \left[\frac{e\,(t-v)\,h(t,v)}{H\,(t)}\right]^{\alpha} \right\} \tag{14}$$

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[11] Leisure is expressed in efficiency units of human capital to guarantee the existence of a steady state.

In an interior solution ([12]), the accumulation of human capital varies directly with the ratio of the shadow price of human capital -$\mu(t,v)$- to the opportunity cost of investing in education -$w(t)$.

The shadow price of human capital -the discounted value of future labour earnings- is given by:

$$\mu(t,v) = \sum_{n=t+1}^{v+\bar{a}} \left\{ w(n)\left[1-\lambda(n,v)\right]e(n-v)\frac{\prod_{m=t+1}^{n-1}\left\{\alpha\frac{h(m+1,v)}{h(m,v)}+(1-\alpha)(1-\delta_h)\right\}}{\prod_{p=t}^{n-1}\left\{R(p)\right\}} \right\}$$

with the terminal condition **(15)**

$$\mu(v+\bar{a},v)=0$$

The level of human capital at birth -$h(v,v)$- is assumed to be a strictly positive fraction (γ) of the per capita average stock of human capital in the active population ([13]):

$$h(v,v) = \gamma\, H(v) \qquad 0<\gamma\le 1 \tag{16}$$

Let $\Phi(t,v)$ denote the inverse of the propensity to consume out of total wealth. The optimal consumption profile is:

$$c(t,v) = \frac{\omega(t,v)}{\Phi(t,v)} \tag{17}$$

In the EXO model, $\Phi(t,v)$ is given by:

$$\Phi(t,v) = \sum_{l=t+1}^{v+\mathrm{T}} \left\{ \prod_{n=t}^{l-1} \frac{\kappa(n+1,v)}{R(n+1)} \right\}$$

with **(18)**

$$\kappa(n,v) = \left[\, R(n)\beta\left[1-d(n-1-v)\,\right]\right]^{\frac{1}{\upsilon}}\left[\frac{\underline{u}(n,v)}{\underline{u}(n-1,v)}\right]^{\frac{\upsilon-\sigma}{\upsilon}}$$

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[12] Corresponding to the first argument in the *min* function in (14).

[13] Sustained growth is possible because the production function (9) is a CRS function and human capital is transmitted across generations.

and in the ENDO model:

$$\Phi(t,v) = \sum_{l=t+1}^{v+T} \left[\prod_{n=t}^{l-1} \left\{ [\, R(n+1) \,]^{\frac{1-\sigma}{\sigma}} \{ \beta [1 - d(n-v)] \}^{\frac{1}{\sigma}} \right\} \right] \tag{18a}$$

3. THE COMPETITIVE EQUILIBRIUM

Firms produce a homogeneous good which can be used either for consumption or investment. Firms use a Cobb-Douglas production function *F(K,L)* using capital (*K*) and labour in efficiency units (*L*). The production function in its intensity form is $f(k(t)) \equiv F(k(t),1)$, where $k \equiv \frac{K}{L}$ is the capital intensity ratio. There are many firms operating in a competitive setting. Therefore, gross rates of return equal marginal productivities, and the capital intensity ratio is identical across firms:

$$\begin{aligned} w(t) &= f(k(t)) - k(t) f'(k(t)) \quad (i) \\ r(t) + \delta_k &= f'(k(t)) \quad (ii) \end{aligned} \tag{19}$$

where δ_k is the depreciation rate of physical capital.

The aggregate supply of labour (in efficiency units) used in production is:

$$L(t) = \sum_{v=t-\bar{a}}^{t-a} [1 - \lambda(t,v)]\, e(t-v)\, h(t,v)\, B(v) \left\{ \prod_{n=v}^{t-1} [1 - d(n-v)] \right\} \tag{20}$$

It is assumed that physical capital becomes available for production in the period after its purchase. Thereby, physical capital evolves according to:

$$\begin{gathered} K(t+1) = I(t) + (1 - \delta_k) K(t) \\ \text{where} \\ 0 \le \delta_k \le 1 \end{gathered} \tag{21}$$

where *I* is gross investment.

The equilibrium condition in the goods market equals gross output to gross investment plus aggregate private and public consumption. Government consumption equals aggregate lump-sum taxes in every period. The only role of public consumption is to facilitate model calibration.

$$F(K(t),L(t)) = C(t) + I(t) + G(t) \qquad (i)$$

$$G(t) = \Gamma(t) \qquad (ii)$$

where

$$C(t) = \sum_{v=t-\mathrm{T}}^{t} c(t,v)\, B(v) \left\{ \prod_{n=v}^{t-1} \left[1 - d(n-v) \right] \right\} \qquad \textbf{(22)}$$

$$\Gamma(t) = \tau(t) \sum_{v=t-\mathrm{T}}^{t} B(v) \left\{ \prod_{n=v}^{t-1} \left[1 - d(n-v) \right] \right\}$$

where C is aggregate private consumption; G is public consumption; and, Γ is aggregate lump-sum taxes.

The equilibrium condition in the financial assets market can be derived using the set of equations in the model.

4. THE MODELS' CALIBRATIONS

The process of model calibration consists of selecting a number of parameter values to describe: individual's preferences, production, and the age-profiles of a number of variables. The parameters are chosen in such a way as to guarantee the existence of balanced growth, and to reproduce a number of important stylised facts.

The forward-looking simulator in *Troll* (version 1.08) -which employs a stacked time algorithm- is used in the calculations ([14]).

Two calibrations are calculated: one for the EXO model and the other for the ENDO model.

The mainframe version of Troll runs in an Unix computer, using 1020 megabytes of memory. This limits the number of overlapping generations to 27 ([15]). Consequently, each period represents approximately a 4 years interval ([16]). However, and to facilitate the interpretation of the results, graphs will be presented using an yearly scale.

4.1 The mortality rates

Mortality rates are based on the death tables published by the Belgian *Institut National de Statistique* and the *Bureau Féderal du Plan* in “Perspectives de Population 1995-2050”, which are adjusted to obtain an average life expectancy at birth (equal for men and women) of 78 years.

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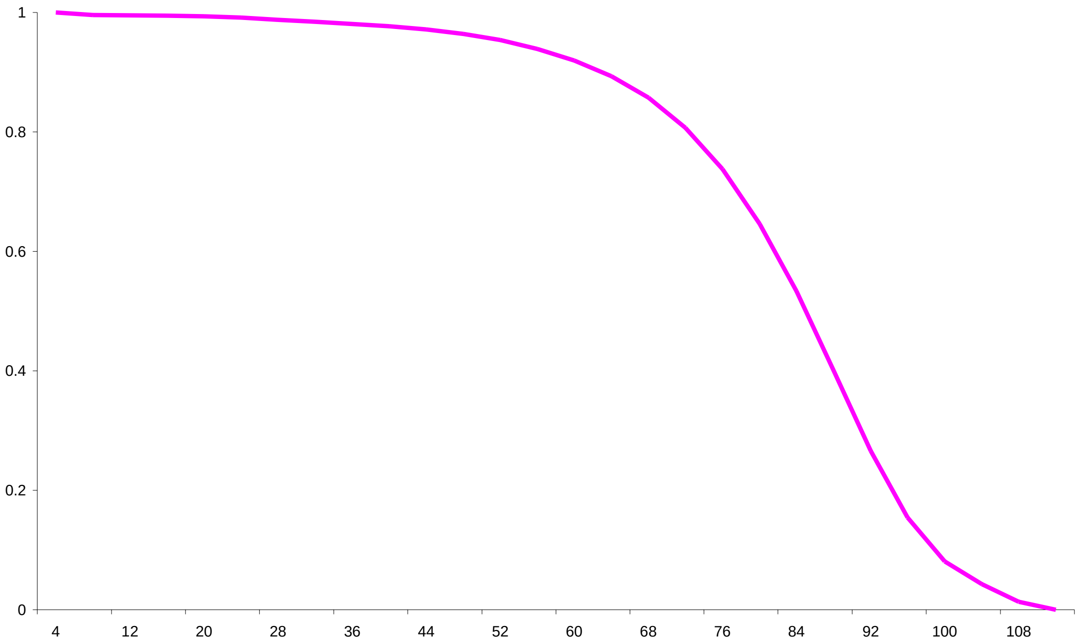
[14] The stacked-time algorithm stacks all time periods into one large system of equations and then solves them simultaneously using Newton-Raphson iterations. Newton-Raphson is usually an efficient and robust method. Each iteration in Newton-Raphson requires solving a matrix equation involving the Jacobian matrix. In a stacked system, the Jacobian matrix can be very large. However, the large Jacobian matrix has a repetitive structure of nonzero blocks along its diagonal, and each of these blocks is sparse. The stacked-time simulator in Troll uses methods developed by Laffargue (1990), Boucekkine (1995) and Juillard (1996) to take advantage of the repetitive block structure, and introduces computational techniques to explore the sparsity within the blocks.

The resulting algorithm is generally more accurate and usually faster than Fair-Taylor’s. However, it does require more memory than the latter, and that is possibly the limiting factor for large models with many lags and leads.

[15] In the original Auerbach-Kotlikoff model, an individual lives exactly 55 periods.

[16] According to the death tables used, an individual can live for a maximum of 107 years. Therefore, in a model with 27 life periods, each period represents a 4 years interval.

Graph 1: The survival curve



4.2 The age-profile of human capital efficiency

The exogenous factor of human capital efficiency evolves according to the parabola used in Auerbach-Kotlikoff (1987) (Graph 2), which is based on estimates obtained by Welch (1979) from a cross-section regression of weekly labour earnings of full-time workers on personal variables, including experience and experience squared ([17]) ([18]):

$$e(t) = \frac{\exp\left\{4.47 + 0.033\,[t - \hat{a} + 1] - 0.0007\,[t - \hat{a} + 1]^{2}\right\}}{\exp\{4.47 + 0.033 - 0.0007\}} \qquad t \geq \hat{a}$$

(23)

$$e(t) = 1 \qquad t < \hat{a}$$

where $\hat{a}$ is the starting age of the parabola describing the human capital efficiency factor.

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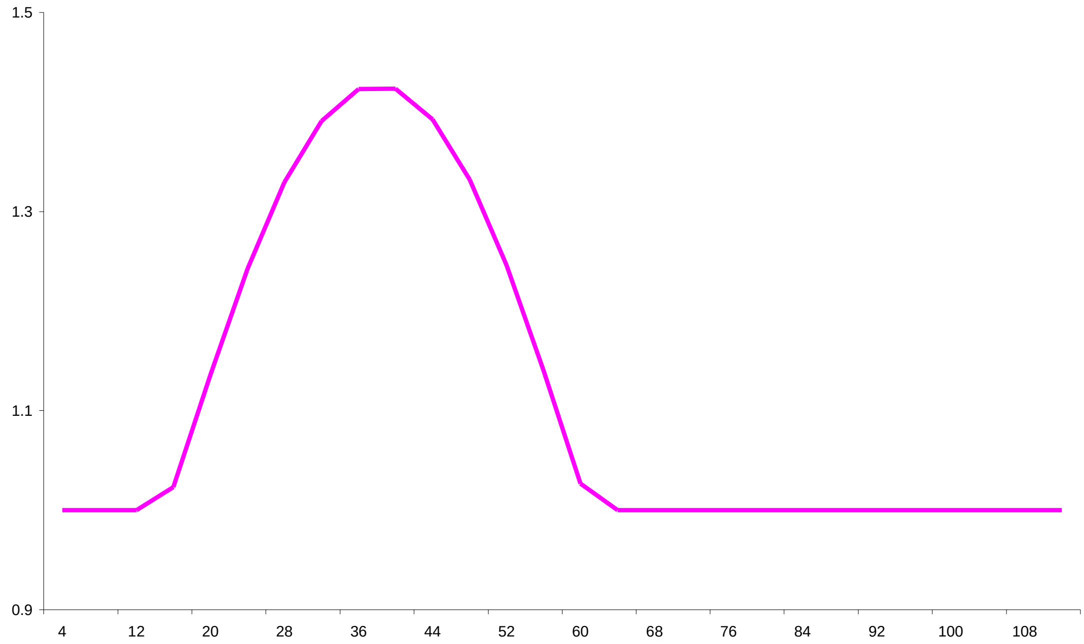
[17] In Auerbach-Kotlikoff (1987), *e* is referred to as the exogenous profile of household's human capital.

[18] Japelli-Pagano (1989) argue that age earning profiles vary considerably across countries, which could partly reflect differences in the age profile of the human capital efficiency factor (*e*).

Graph 2 The age-profile of the human capital efficiency factor (*e*)



4.3 The relevant ages in the calibrations

Recall that all relevant ages are parametrically fixed. A lifecycle of 27 periods implies that generations are separated by approximately a 4 years interval. The parameters corresponding to the 107 overlapping generations model (where generations are separated by a 1 year interval) are also presented for comparison. Parameters apply both to the EXO and ENDO models unless stated otherwise.

Table 1

Description	***Symbol***	***27 generations***	***107 generations***
The entry age in the labour market in the EXO model	$\underline{a}$	5	17
The starting age of education/active life in the ENDO model	$\underline{a}$	2	5
The maximum age used to define a "student" in the ENDO model	$\tilde{a}$	6	24
The retirement age	$\bar{a}$	16	65
The maximum life span	T	27	107
The starting age of the parabola describing the human capital efficiency factor	$\hat{a}$	4	18

In the EXO model calibration, the annual entry age in the labour market is set to 17 years, which roughly coincides with the legal minimum age for work in developed countries.

In the ENDO model and using annual data, the starting age of education/active life is parameterised to 5 years. However, in the steady-state of the ENDO model calibration, an individual uses all her time in school (i.e. chooses a corner solution) until about 16 years of age. Therefore, and despite the differences between the two models considered, an individual actually enters the labour market at roughly the same age (i.e. 17 years of age) in both calibrations used.

In the ENDO model, an individual is defined as a "student" in the age interval: $[\,\underline{a}, \tilde{a}\,]$, because education is her predominant activity (in the steady state).

4.4 Major conditions used in the calibrations

Table 2

Description	***Symbol***	***27 generations***	***107 generations***
The per capita/productivity growth rate	κ_v	0.16985856	0.04
The population growth rate	κ_a	0.04060401	0.01
The economic growth rate	κ_h	0.21735951	0.0504
The equilibrium interest rate	r	0.36048896	0.08
The time preference rate	$1-\beta$	0.07763184	0.02
The aggregate consumption over output ratio	$\frac{C}{F(K,L)}$	0.6	0.6
The depreciation rate of physical capital	δ_k	0.13282000	0.035
The labour share in output		2/3	2/3

The models are calibrated using data for the European Union (EU) in the period 1960-1965 ([19]). Note that the reference period chosen precedes the decline in fertility registered in Western Europe.

In the 1960-1965 period, output per capita grew at the (geometric) annual average of 4.2%. The calibrations assume an initial per capita annual growth rate of 4%. In the same period, the average annual growth rate of the population was 0.9%. The annual growth rate of the population is set to 1% (before the permanent fertility shock).

The dynamic efficiency condition (i.e. $r > \kappa_h$) requires an interest rate greater than 5%. The annual interest rate (r) is set to 8% ([20]). This variable has a major impact,

[19] Using the European Commission's AMECO database, average data for the period 1960-1965 are calculated for the EU-14 (the EU-15, excluding Germany due to the break in series caused by the unification).

affecting the profiles of consumption in both models; and, the allocation of human capital between education and production activities in the ENDO model, through the shadow price of human capital ([21]).

The time preference rate (1-β) is set to 2%. In order to obtain an aggregate private consumption over GDP ratio of 60%, it is necessary to set the intertemporal elasticity of substitution in consumption close to 1 (i.e. the logarithmic utility function), more specifically, at 0.894 and 1.335, respectively, in the EXO and ENDO models ([22]).

In developed economies, the ratio of aggregate consumption over GDP is between 0.6 and 2/3. The lower limit is closer to both European and Japanese data, whereas the upper limit describes better the U.S. situation. The ratio of aggregate consumption over output is set to 60% –the average of EU-14 in the 1960-1965 period is 59.8% ([23]).

The annual depreciation rate of physical capital is set to 3.5%. In the 1960-1965 period, the average annual depreciation rate in the EU-14 was 3.6%.

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[20] An equilibrium interest rate of 8% for riskless investments could be considered as being too high. However, this rate is also used to discount future labour earnings. Estimates of the return to human capital investments tend to find rates above 10% for all levels of education (Psacharopoulos, 1994).

[21] Other things being equal, a rise in the interest rate increases the steepness of the consumption profile (i.e. consumers become less impatient); and, reduces the fraction of human capital allocated to education in the ENDO model, because future labour earnings are discounted more.

[22] Estimates of the intertemporal elasticity of substitution in consumption suggest that it is likely to be lower than 0.5. Econometric work based on U.S. and U.K. time series data by Patterson-Pesaran (1992) suggest that it is between 0.1 and 0.3. Faruqee and al. (1996) argue that a more plausible range for the intertemporal elasticity of substitution is centred around ¼. Unfortunately, as pointed out in Evans (1991) and Faruqee and al. (1996), there is a lower limit for the intertemporal elasticity of substitution in the CRRA type of models, beyond which, a negative value for the pure time preference factor (1-β) is necessary -given a reasonable baseline for interest rates- in order to reproduce the actual ratio of aggregate consumption over output.

[23] The ratio of aggregate consumption over output increases when: the intertemporal elasticity of substitution in consumption rises; the interest rate increases; or, the time preference discount factor (1-β) decreases.

The labour share in output is 2/3, which corresponds to the average in the EU-14 (65.6% in the 1960-1965 period).

4.5 A parameter specific to the EXO model calibration

Table 3

Description	***Symbol***	***Value***
The intratemporal elasticity of substitution in consumption	$\frac{1}{\upsilon}$	0.8

The intratemporal elasticity of substitution in consumption ($\frac{1}{\upsilon}$) is 0.8, which is the same value used in Auerbach-Kotlikoff (1987).

4.6 Variables related to human capital formation in the ENDO model

Table 4

Description	***Symbol***	***27 generations***	***107 generations***
The elasticity of the production function of human capital w.r.t. the private input	α	2/3	2/3
The fraction of the average stock of human capital inherited at birth	γ	0.1	0.1
The depreciation rate of human capital	δ_h	0.01	0.0025

The parameters α and γ define the strength of the externality in education. When α approaches 1, the externality vanishes. In the absence of concrete information on estimates of α, α is set to 2/3. A lower (higher) value for α reduces (increases) the productivity of human capital in education.

The fraction of the average stock of human capital inherited at birth (γ) is set to 10%. Early in life, this parameter basically determines the allocation of human capital between education and production activities. A low (high) γ implies a(n) late (early) start with production activities (i.e. more (less) time spent studying initially). The value chosen brings about a corner solution in the allocation of time, with an individual spending all her time in school until about 17 years of age.

The annual depreciation rate of human capital (δ_h) is set to the low value of 0.25%.

4.7 Major results in the steady state of the two model calibrations

Table 5

Description	***Symbol***	**EXO**	**ENDO**
The interest rate supplement	π	0.0251	0.037
The physical capital over output ratio	$\frac{K}{F(K,L)}$	0.68	0.68
The intertemporal elasticity of substitution in consumption	$\frac{1}{\sigma}$	0.894	1.335
The gross investment over output ratio	$\frac{I}{F(K,L)}$	0.236	0.236
The public consumption over output ratio	$\frac{G}{F(K,L)}$	0.164	0.164
The fraction of aggregate labour used in production (in efficiency units)		0.4	--
The fraction of aggregate human capital used in education (in efficiency units)		--	0.313

According to (1), and on an annual basis, the interest rate supplement paid by insurance companies is 2.5% and 3.7% in the EXO and ENDO models, respectively.

The ratio of physical capital over output is only about 0.7, which is low when compared with actual ratios in EU economies of between 2 to 4.

The set of parameters in these calibrations, *inter alia*, an intertemporal elasticity of substitution in consumption of 0.9 and 1.3, respectively, in the EXO and ENDO models, for a time preference rate of 2%, secure a consumption-to-GDP ratio of 60%.

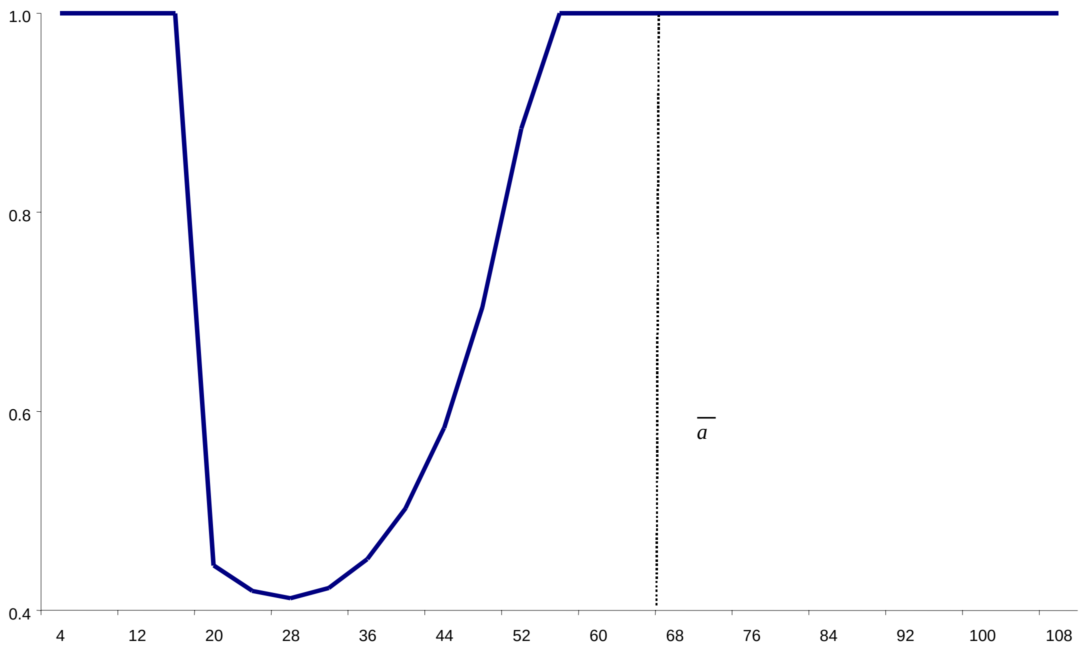
A gross investment over output ratio of 23.6% is very close to the average of 23.3% in the EU-14 in the 1960-1965 period.

Public consumption satisfies the equilibrium condition (22).

In the EXO model, aggregate labour supply used in production is 40% of the total labour endowment, as in the original Auerbach-Kotlikoff model. This ratio is arrived at by assuming: first, a daily labour endowment of 14 hours; and, second that an individual works around 40 hours per week.

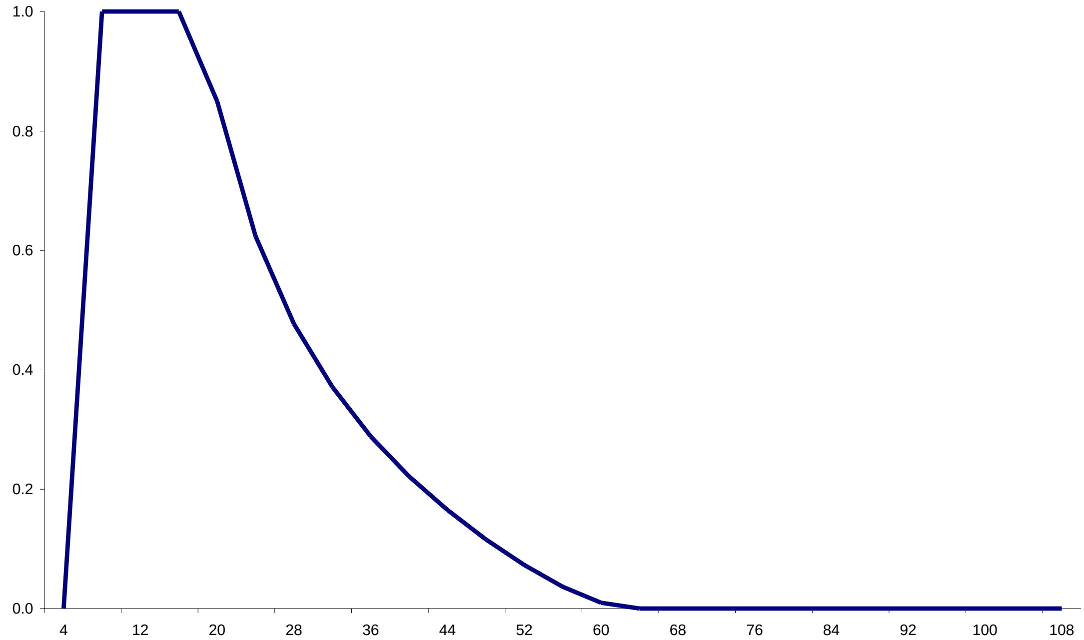
In the EXO model, an individual leaves the labour market before the retirement age ($\bar{a}$) (Graph 3).

Graph 3: The optimal age-profile of leisure in the steady state



In the ENDO model, about 1/3 of aggregate human capital (in efficiency units) is used in education. For an individual entering the labour market/education at 5 years of age and retiring at 65, she finds optimal to spend all her time in education activities from 5 to 16 years of age. This is close to actual values registered in developed countries (OECD, 2000).

Graph 4: The optimal age-profile of education in the steady state



Summarising, the results of the two calibrations are close in many respects. The two major differences are the intertemporal elasticity of substitution in consumption and the interest rate supplement, which are both higher in the ENDO model ([24]).

[24] The two models were also simulated using a single set of parameters to evaluate to what extent the difference in results depends on the structure of the model versus differences in parameters. It turns out that the former aspect is crucial in explaining the different outcomes, namely on per capita productivity, following a demographic shock.

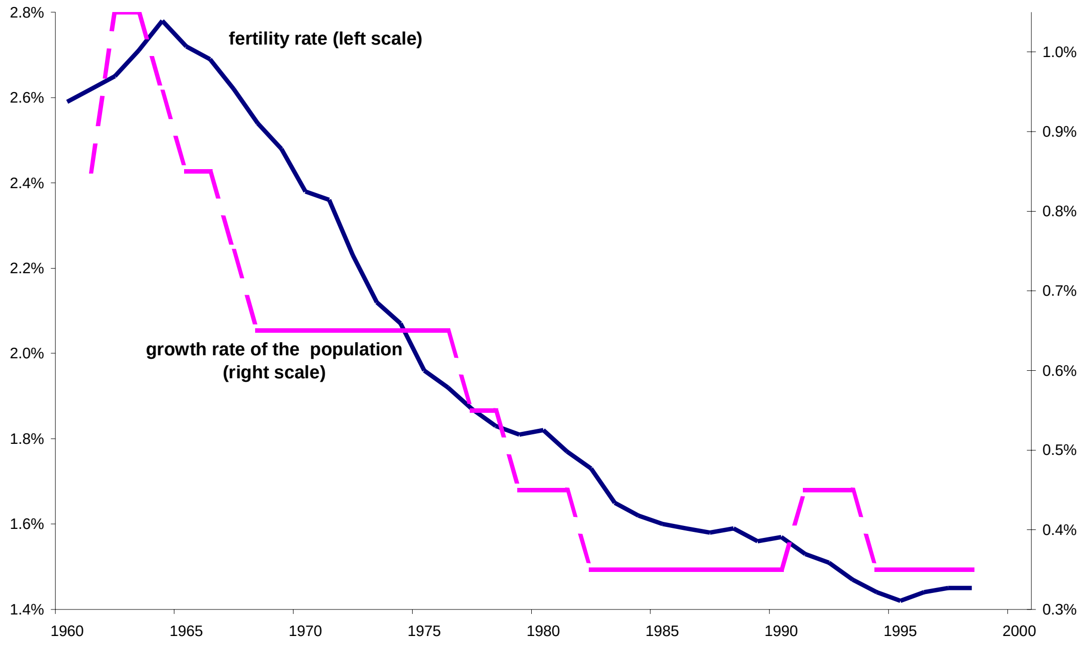
5. The impact of a permanent fertility shock

Using the two computable OLG models presented, this paper compares the impact of a permanent fertility shock on growth and on per capita productivity, both in terms of long-run values and of the transition dynamics to the new equilibrium.

5.1 The fertility shock

Since the second half of the 1960s, the fertility rate declined considerably in developed countries ([25]). Moreover, the current decline does not appear to be a cyclical but a permanent phenomenon (Auerbach-Kotlikoff, 1987, pp.162). In the EU-15 area, it fell from over 2½% in the early 1960's to less than 1½% in the mid 1990s, which represents a decline of nearly 50% (Graph 5).

Graph 5: The fertility and the population growth rates

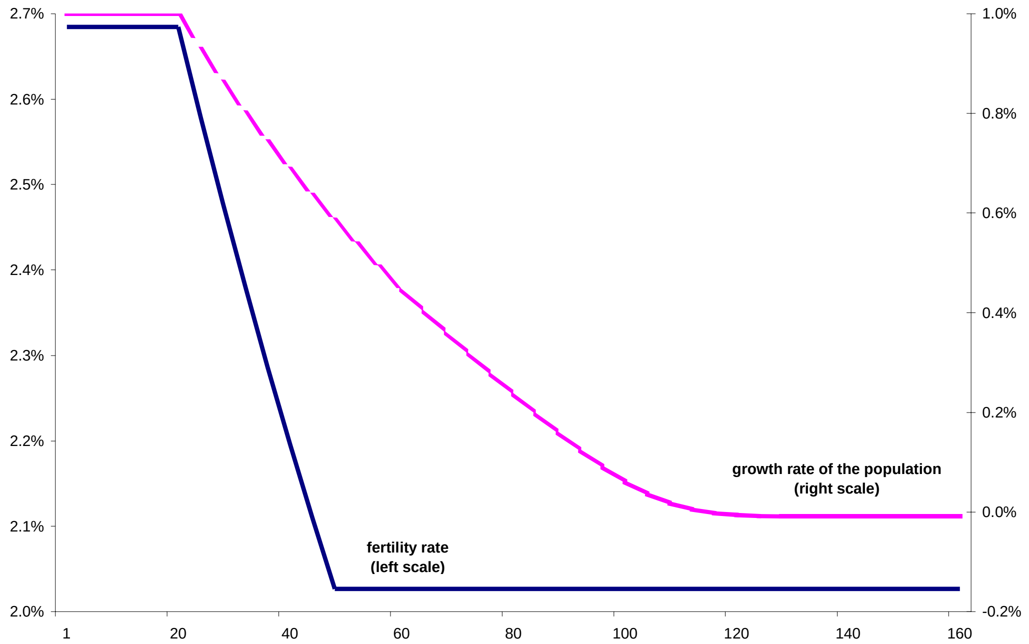


[25] The fertility rate is defined as the number of new-born children over women aged between 15 and 50 years of age.

The growth rate of the population in the EU-14 fell from some 1% in the first half of the 1960s to about ¼% in the 1990s, which is, however, above the value implied by the observed reduction in fertility due to favourable migration flows.

The following permanent fertility shock is considered. The growth rate of the population declines (linearly) from 1% to 0% during a 30 years period ([26]). Given that there are no migration flows, the reduction in the growth rate of the population is engineered by an appropriate (linear) decline in the fertility rate.

Graph 6: The demographic shock

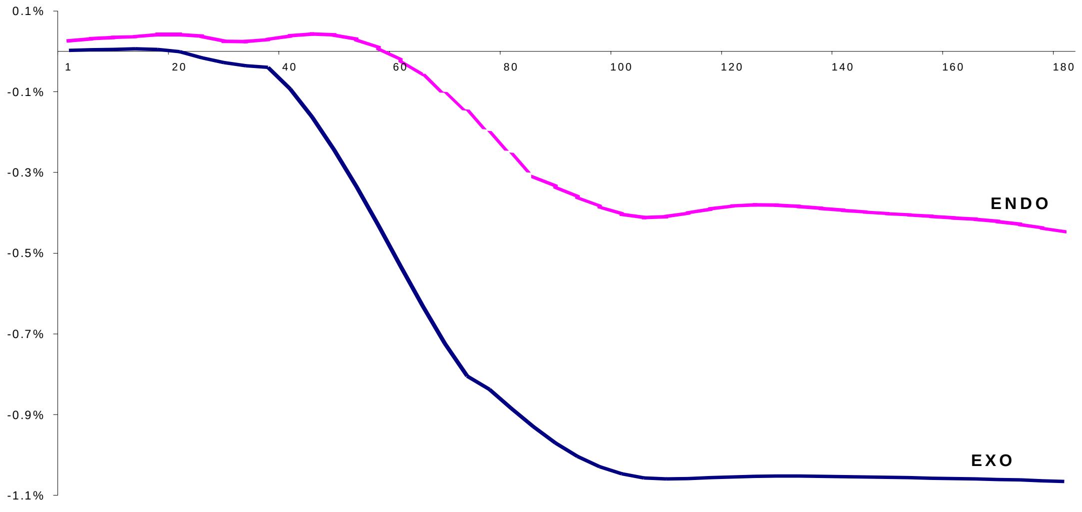


5.2 The impact on growth and on per capita productivity

The impact of a permanent fertility shock on growth and on per capita productivity is presented as a deviation from their initial steady state annualised values, respectively 5% and 4% in both calibrations.

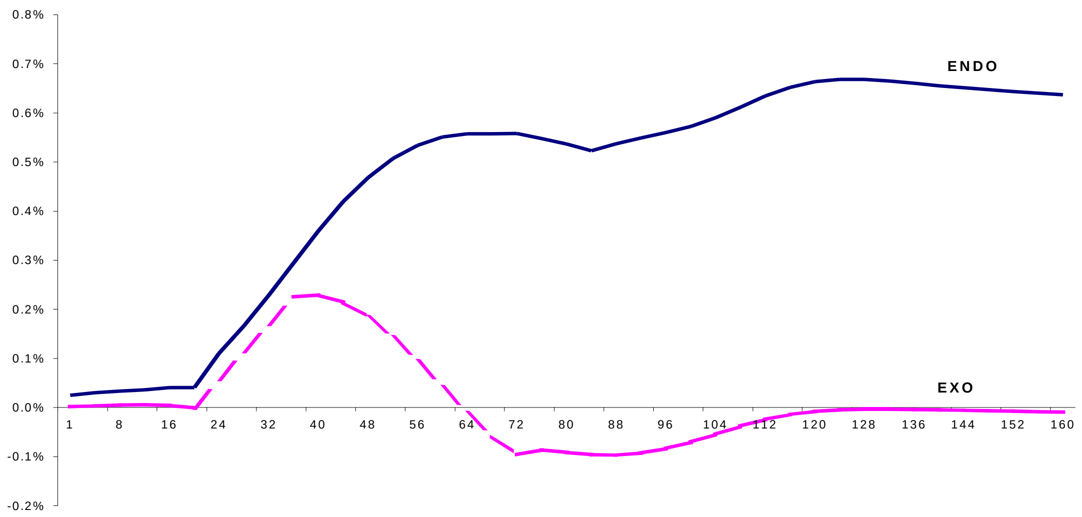
[26] Recall that in the present calibrations, a 30 years period corresponds to between 7 to 8 generations.

Graph 7: The growth rates



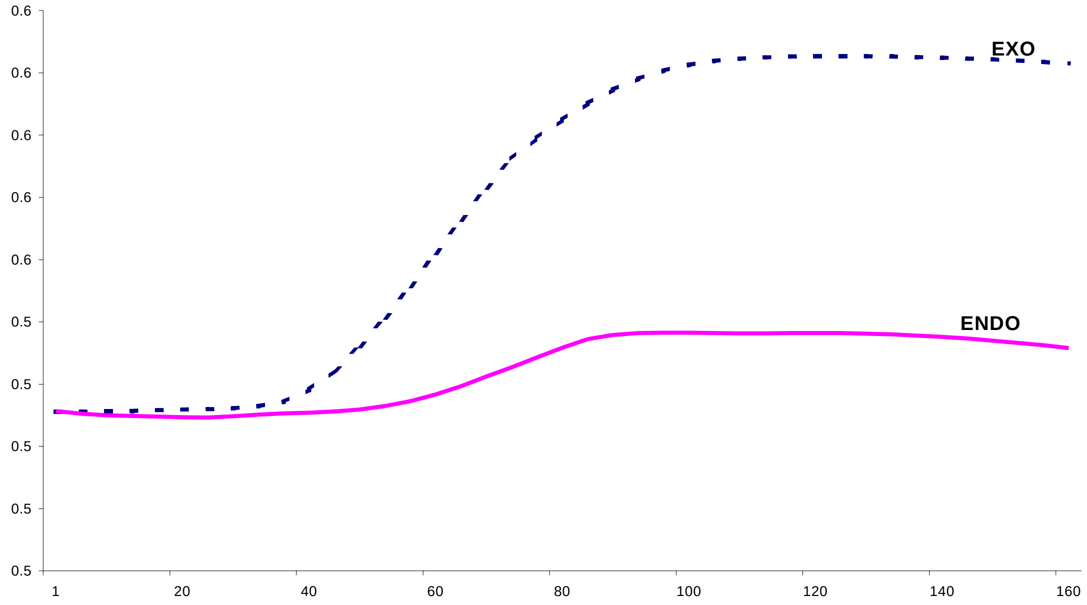
Growth declines by less in the ENDO model than in the EXO model (Graph 7), because of offsetting developments in per capita productivity (Graph 8). In the ENDO model, a (negative) fertility shock raises per capita productivity in the long-run by just over ½%; whereas, in the EXO model, it has only a temporary effect. This difference reflects basically the role of education in increasing (permanently) per capita human capital in the ENDO model, and is not due to the differences between the two models used ([27]).

Graph 8: The growth rates of per capita productivity



The transition dynamics of factor prices are determined by the evolution of the capital intensity ratio. Following a decline in fertility, the capital intensity ratio rises (Graph 9), because of the ageing of the population, coupled with the age-profile of financial holdings, which is humped in middle-age. This raises the discounted value of labour earnings ([28]), fostering investment in education, which coupled with the externality in education raises per capita productivity in the long-run.

Graph 9: The capital intensity ratio

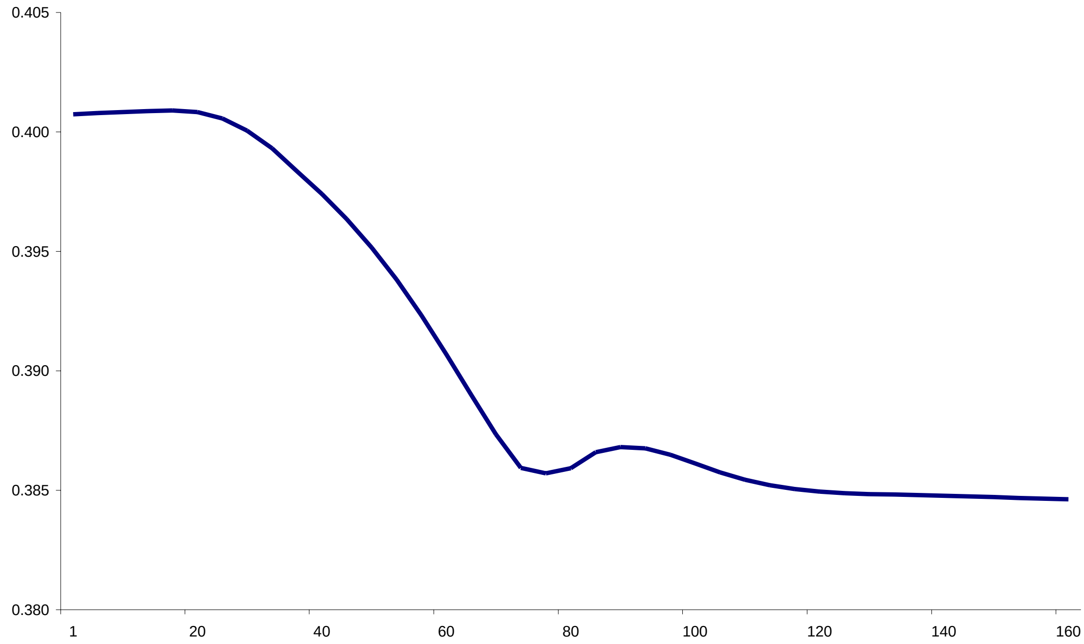


The rise in the capital intensity ratio corresponds to an increase in the individual level of welfare. Given that leisure is a normal good in the EXO model, the fraction of aggregate labour used in production declines with fertility (Graph 10) from 40% of the total labour endowment to 38.5% in the final long-run equilibrium (i.e. minus 3¾%).

[27] In particular, the EXO model was simulated assuming exogenous leisure (i.e. θ=0) as in the ENDO model. This modification did not change the results obtained in any significant way.

[28] Because both the wage rate increases and the interest rate declines.

Graph 10: The fraction of aggregate labour used in production in the EXO model



In the ENDO model, the changes in factor prices constitute a powerful incentive to invest in education. Let us define "students" (S) as all active individuals younger than $\tilde{a}$. The intensity of investment in education per "student" (ψ) is then:

$$\psi(t) \equiv \frac{\phi(t)}{S(t)}$$

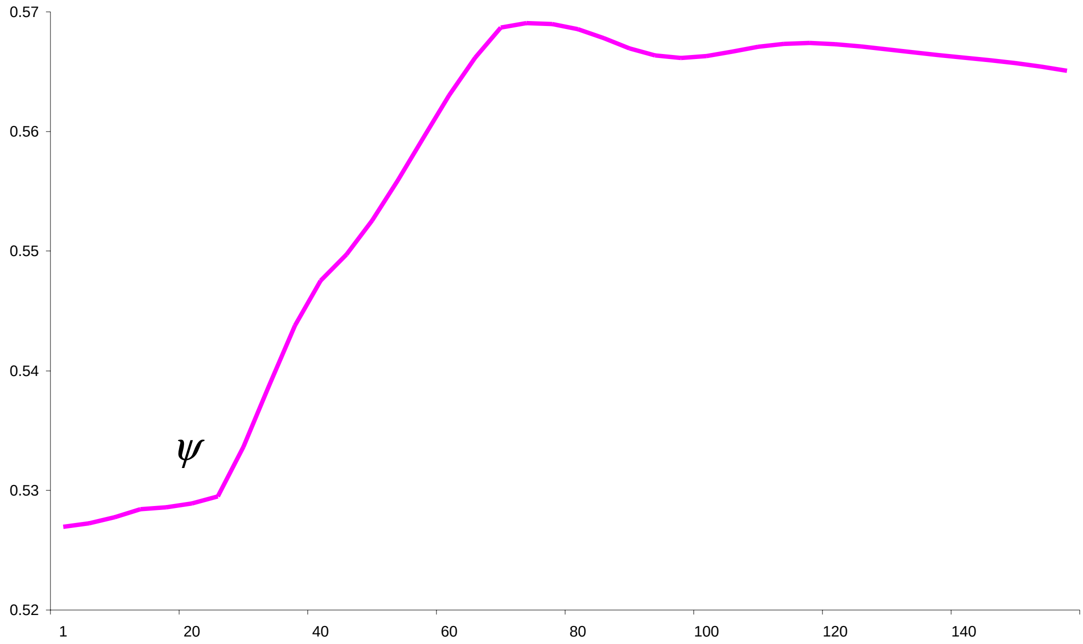
where

$$\phi(t) \equiv \sum_{v=t-\tilde{a}}^{t-\underline{a}} \lambda(t,v) e(t-v) h(t,v) B(v) \prod_{n=v}^{t-1} \left[1-d(n-v)\right] \quad \textbf{(24)}$$

$$S(t) = \sum_{v=t-\tilde{a}}^{t-\underline{a}} B(v) \prod_{n=v}^{t-1} \left[1-d(n-v)\right]$$

where ϕ is aggregate human capital employed in the education of "students" (in efficiency units).

Graph 11: The intensity of investment in education per "student" (ψ)



The decline in fertility induces a "student" to increase time spent in school by about 7½% in relation to the initial steady state value.

6. Conclusions

A negative fertility shock reduces growth. This result is not reversed when considering an ENDO model with education as the engine of growth. However, the negative impact on growth is considerably dampened, and a permanent positive effect on labour productivity occurs due to the rise in the fraction of time allocated to education.

Consequently, the ENDO model considered in this paper is considerably more optimistic than the EXO model (or other less "sophisticated" models [29]) on the ability of developed economies to withstand a negative demographic shock.

[29] See De la Croix (1998) for a classification of the models currently used to assess the solvency of social security systems facing the ageing problem, according to various model assumptions.

Using the computable OLG models calibrated for the period 1960-1965, and for the fertility shock considered, the simulated growth rates in the final long-run equilibria are 4% and 4.6%, respectively, in the EXO and in the ENDO models. In the period 1990-1999, actual growth averaged only 1.9% in the EU-14. Given that distortionary policies were ignored, these differences might, *inter alia*, reflect the dramatic increase in the total tax burden, which in the EU-14 area rose from 32.6% of GDP in 1970 to 41.5% in 1995 ([30]).

The simulation results reproduce the stylised fact of a negative correlation between schooling and fertility. In a model with perfect foresight, it could be argued that households should also choose fertility rates to maximise some optimisation function. Therefore, treating fertility as an exogenous variable is not entirely satisfactory, although being a necessary assumption to keep the models used tractable.

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[30] The rise in marginal tax rates, which are the relevant ones for welfare analyses, was even higher.

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