Regulation under Wealth Constraints*

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Abstract

This article studies the problem of regulating a monopolist with unknown marginal cost. The problem described differs from Baron and Myerson [1982] because we suppose that the regulator faces a cash-in-advance constraint. The introduction of such a constraint may lead to the collapse of the incentive system.

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1 Introduction

The problem we concentrate on is the following: a public authority faces a monopolistic producer with unknown cost[1]. The authority tries to provide a public utility (bridge, road or sewer system) to a bunch of consumer. This public utility is produced by the monopolist and financed by government transfers. The contract offered by the government to the firm specifies the quantities that should be produced and a level of transfer. We assume that the government doesn't know the cost function of the firm.

The originality of this paper is to add up to the standard problem macroeconomics constraints. We will suppose that the public authority has only limited funds at disposal. This constraints limits the possibilities for the government to buy the full consumer surplus associated with the public utility. We believe that such constraints may be particularly relevant for the case of developing countries.

We show that the presence of a wealth constraint distorts -in a non linear way- the quantity produced by each type of firm. These 'third best' distortions that come on top of the traditional 'second best' distortions are necessary to fulfill the wealth constraint. But these distortions may lead to the collapse of the incentive system: it may be impossible to separate the different types of firms. And hence, when the government is constrained, bunching is a non trivial issue.

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[1] The framework is similar to Baron and Myerson [1982] except that the firm has no fixed cost.

2 Model

Our model is a simple model of adverse selection: the principal (public authority) contracts with an agent (monopolistic private firm) for the provision of a public utility. The agent is responsible of the production and the principal finances the production with transfers. At the time of contracting, the principal does not know the cost conditions under which the firm can produce. We will assume that the good is produced by the firm with a technology exhibiting constant return to scale. The cost function of the firm is θq, where θ is a constant marginal cost and q is the quantity produced[2]. The marginal cost is private information to the firm. The principal only knows that $\theta \in \Theta \equiv \{\theta_1, \theta_2\}$ with $\theta_1 < \theta_2$ and the probabilities v_1 and $v_2 = 1 - v_1$ of agent being θ_1 and θ_2. We call $\Delta\theta = \theta_2 - \theta_1$.

The utility of the agents is:

$$U^A = T - \theta q$$

Where T is the transfer paid by the principal to the agent. Firm accepts the contract if it gets more than its outside option normalized to zero.

When the agent produces a quantity q, the principal collects a surplus $S(q)$. We assume that $S' \geq 0$, $S'(0) = +\infty$, $S'' < 0$ and $S''' > 0$. Our assumptions on S ensures that it is optimal to have all types producing.

The utility of the principal is:

$$U^P = S(q) - T$$

The regulator offers a contract specifying the transfer T and the quantity q. We call T_1, q_1, the transfer paid to the type θ_1 agent when he produces q_1 and similarly, T_2, q_2, the transfer and production of θ_2 agent.

The cash in advance constraint limits the transfer: they cannot exceed an upper limit denoted $\overline{T}$.

3 Results

3.1 Second best equilibrium

Without cash in advance constraint, the objective of the principal is:
Program $[P1]$

$$\max_{q_1,q_2,T_1,T_2} v_1(S(q_1) - T_1) + v_2(S(q_2) - T_2)$$

s.t. $\forall\, i, j = 1, 2$:

$$T_1 - \theta_1 q_1 \geq T_2 - \theta_1 q_2 \qquad (IC_1)$$

$$T_2 - \theta_2 q_2 \geq T_1 - \theta_2 q_1 \qquad (IC_2)$$

$$T_1 - \theta_1 q_1 \geq 0 \qquad (IR_1)$$

$$T_2 - \theta_2 q_2 \geq 0 \qquad (IR_2)$$

The two relevant constraints of this problem are IC_1 and IR_2.

[2]Equivalently, we could interpret q as the quality of the good produced.

Proposition 1 *The solution to the problem [P1] is given by:*

$$q_1^{SB} = S'^{-1}(\theta_1) \quad (1)$$

$$q_2^{SB} = S'^{-1}(\theta_2 + \frac{v_1}{v_2}\Delta\theta) \quad (2)$$

$$T_1 = \theta_1 q_1^{SB} + \Delta\theta q_2^{SB}$$

$$T_2 = \theta_2 q_2^{SB}$$

This solution is standard.

3.2 Wealth constrained equilibria

We now introduce the wealth constraint. The constraint implies that the principal cannot transfer the agent more than $\overline{T}$. We said that the constraint is *relevant* if the maximal transfer $\overline{T}$ is smaller than the highest transfer paid by the principal in the second best equilibrium[3]. The principal's optimization program becomes:
Program $[P2]$:

$$\max_{q_1,q_2,T_1,T_2} v_1(S(q_1) - T_1) + v_2(S(q_2) - T_2)$$

s.t. (IC_1), (IC_2), (IR_1), (IR_2) and

$$T_1, T_2 \leq \overline{T} \quad (WC)$$

Lemma 1 *When $\overline{T} \leq \theta_1 q_1^{SB} + \Delta\theta q_2^{SB}$, the efficient type agent (θ_1) will be paid $\overline{T}$.*

Proof. If $\overline{T} < T_1^{SB}$, the solution of $P1$ cannot be replicated in $P2$. Then, at least one of the transfers in $P2$ is given by the constraint (WC). A necessary condition for implementation is: $q_1 \geq q_2$ and $T_1 \geq T_2$. Then the constraint (WC) binds (at least) for T_1. ■

Using the relevant constraints IC_1, IR_2 and the result of lemma 1, the program $[P2]$ can be rewritten as:
Program$[P3]$:

$$\max_{q_1,q_2} v_1(S(q_1) - \overline{T}) + v_2(S(q_2) - \theta_2 a_2)$$

s.t.

$$(\mu_1) \qquad \overline{T} = \theta_1 q_1 + \Delta\theta q_2$$

$$(\mu_2) \qquad T_2 = q_2\theta_2 \leq \overline{T}$$

The solution of this optimization program is given in our second proposition.

Proposition 2 *(i) If $\theta_1 \geq v_1\theta_2$ and $\overline{T} \leq \overline{T}^* = \theta_2 S'^{-1}(\frac{\theta_2\theta_1 v_2}{\theta_1 - v_1\theta_2})$, the equilibrium is a pooling equilibrium:*

$$q_1 = q_2 = \frac{\overline{T}}{\theta_2} \quad (3)$$

$$T_1 = T_2 = \overline{T} \quad (4)$$

[3] The constraint is relevant if: $\overline{T} \leq \theta_1 q_1^{SB} + \Delta\theta q_2^{SB}$.

(ii) otherwise, the equilibrium is a separating equilibrium characterized by the following first order conditions:

$$q_1^{WC} = S'^{-1}(\frac{\mu_1}{v_1}\theta_1) \quad (5)$$

$$q_2^{WC} = S'^{-1}(\theta_2 + \frac{\mu_1}{v_2}\Delta\theta) \quad (6)$$

$$\overline{T} - \theta_1 q_1^{WC} - \Delta\theta q_2^{WC} = 0 \quad (7)$$

And the transfers are give by the constraints WC and IR_2.

$$T_1 = \overline{T} \quad (8)$$

$$T_2 = \theta_2 q_2^{WC} \quad (9)$$

The proof of proposition 2 is relegated to an appendix. Before explaining the result of proposition 2, it is useful to make some comparative static and study the effect of a change in $\overline{T}$. This is done in proposition 3:

Proposition 3 *In the separating equilibrium, the value of μ_1 is a decreasing an convex function of $\overline{T}$, with $\lim_{\overline{T}\to 0}\mu_1 = +\infty$ and $\lim_{\overline{T}\to T_1^{SB}}\mu_1 = v_1$*

Proof. (i) From the first order conditions we have: $\overline{T} = \theta_1 S'^{-1}(\frac{\mu_1}{v_1}\theta_1) + \Delta\theta S'^{-1}(\theta_2 + \frac{\mu_1}{v_2}\Delta\theta)$. Call the right hand side $G(\mu_1)$. Then $\mu_1 = G^{-1}(\overline{T})$. Given our assumptions on S, G is increasing and concave, because S'^{-1} is increasing and concave. Then G^{-1} is decreasing and convex. (ii) At the limit when $\overline{T}$ goes to T_1^{SB}, the problem is identical to the problem $P1$ and therefore the solution is identical. i.e. $\mu_1 = v_1$. When $\overline{T}$ goes to zero, the right hand side of the (7) must go to zero. Given that $S'(0) = +\infty$, we have that $G^{-1}(0) = +\infty$. ■

Now we turn back to the equilibria described in proposition 2. On the top of the traditional second best trade off between efficiency and rent extraction that leads to distortions in q_2, there is now a third best distortion necessary to fulfill the wealth constraint. If we call $\mu_1' = \mu_1 - v_1$, we can rewrite (5) and (6) in order to isolates the second and third best distortions:

$$S'(q_1) = \theta_1 + \frac{\mu_1'}{v_1}\theta_1 \quad (10)$$

$$S'(q_2) = \theta_2 + \frac{v_1}{v_2}\Delta\theta + \frac{\mu_1'}{v_2}\Delta\theta \quad (11)$$

If we compare these expressions with the second best equilibrium, it is clear that the last terms on the right hand member measures the distortions imposed to fulfill the wealth constraint. As established by proposition 3, these third best distortions increases when the constraint becomes more severe (μ_1' increases).

The addition of a third best distortion in q_1 and q_2 may lead to the collapse of the incentive system. It will be the case if the distorted actions doesn't satisfy the necessary condition for implementation, namely keeping q_1 greater than q_2[4] If q_1 is more distorted than q_2, there is a level of μ_1' and a corresponding level of $\overline{T}$ (called $\overline{T}^*$) such that the value of q_1 given by (5) is smaller than the value of q_2 given by (6). Therefore, for these values of the $\overline{T}$, the only feasible mechanism is a pooling mechanism.

[4]This question is not an issue in the second best problem, because only the action of the inefficient agent (θ_2) is distorted. When both actions are modified, the question of keeping the action scheme decreasing becomes crucial.

4 Conclusion

In this paper, we have shown that when the government is constrained on the level of transfer he can make to the firms, there is under-provision of public facilities. Moreover pooling contracts where both types of firm produce the same quantities and receive the same transfer may be the optimal contract. In the classical adverse selection problem, and given that the model satisfies some regularity conditions, pooling contracts are ruled out. In our model, pooling contracts may be optimal as soon as the difference between the highest marginal cost and the lowest one is not too big. So pooling is an issue when the regulator faces cash-in-advance constraints. Wealth constraints may lead to low-powered incentive scheme where the regulator pays a constant fee for a fixed quantity.

A Proof of proposition 2

The first order conditions of $P3$ are:

$$S'(q_1) = \frac{\mu_1}{v_1}\theta_1 \tag{12}$$

$$S'(q_2) = \theta_2 + \frac{\mu_1}{v_2}\Delta\theta + \frac{\mu_2}{v_2}\theta_2 \tag{13}$$

$$\overline{T} - \theta_1 q_1 - \Delta\theta q_2 = 0 \tag{14}$$

$$\mu_2(\overline{T} - \theta_2 q_2) = 0 \tag{15}$$

We know by lemma 1 that $\mu_1 > 0$ if the wealth constraint is relevant. There are two possible solutions to this system of equation: a separating solution when $\mu_2 = 0$ and a pooling solution when μ_2 is positive.

If $\mu_2 > 0$, (15) becomes $\overline{T} = \theta_2 q_2$, then $q_2 = \frac{\overline{T}}{\theta_2}$. Replacing this value in (14), we have $q_1 = q_2 = \frac{\overline{T}}{\theta_2}$.

If $\mu_2 = 0$, the separating solution is given by:

$$S'(q_1) = \frac{\mu_1}{v_1}\theta_1 \tag{16}$$

$$S'(q_2) = \theta_2 + \frac{\mu_1}{v_2}\Delta\theta \tag{17}$$

$$\overline{T} - \theta_1 q_1 - \Delta\theta q_2 = 0 \tag{18}$$

To know which solution applies, we check when μ_2 is positive. As long as q_2 is smaller than q_1, the transfer T_2 is smaller than $\overline{T}$. Therefore, the second wealth constraint is slack when $q_2 \leq q_1$. This corresponds to the following condition:

$$\frac{\mu_1}{v_1}\theta_1 \geq \theta_2 + \frac{\mu_1}{v_2} \tag{19}$$

where the value of μ_1 is given by (18). Take the limit case where (19) is satisfied with equality, and solve for μ_1 we have: $\mu_1 = \frac{v_2\theta_2\theta_1}{\theta_1 - v_1\theta_2}$. As long as the actual μ_1 is smaller than this value (call it μ_1^*), q_2 is smaller than q_1.

μ_1^* is negative if $\theta_1 \leq v_1\theta_2$. In this case, whatever $\overline{T}$, q_2 is smaller than q_1, except in the limit case where $\overline{T}$ is null where both quantities are set equal to zero.

If $\theta_1 > v_1\theta_2$, we have to find the value of $\overline{T}$ that generates value of μ_1 equals to μ_1^*. To do this, we solve (16), (17) and (18) for $\overline{T}$ when $\mu = \mu_1^*$. This gives a value $\overline{T}^* = \theta_2 S'^{-1}(\frac{\theta_2\theta_1 v_2}{\theta_1 - v_1\theta_2})$. We anticipate the results of proposition 3 that shows that μ_1 increases when $\overline{T}$ decreases. Hence, when $\overline{T} \leq \overline{T}^*$ and $\theta_1 > v_1\theta_2$, μ_2 is positive and the solution is the pooling equilibrium. When $\overline{T} \geq \overline{T}^*$, μ_2 is null and the solution is the separating equilibrium.

The second order conditions of $P3$ are always satisfied thanks to the concavity of the problem.

References

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