Inventories, Factor Demand and Capacity Utilization: the Long and Short Run Structure

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Abstract

An ECM is derived from first order conditions of a factor demand model. Decisions on inventory stock and capacity utilization are (endogenously) modelled, by which a large systems of equations results. Within this system the exogeneity of real factor prices as well as sales is tested. The role of inventory stock in the long run, i.e. as a precautionary measure (according to Holt, Modigliani, Muth and Simon (1960)) and/or as a production factor (Kydland and Prescott (1982), Christiano (1988)) are further investigated by impulse response functions. For French industrial sectors (1970.1-1992.1V) inventory stocks turn out to be both a decision variable as well as a residual. The precautionary measure is not rejected, but a linear-quadratic specification seems not to hold. Further, no strong evidence is found for the inventory stock as a production factor.

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1 Introduction

Inventories of finished goods would (hardly) exist if product demand were known at any time and adjusting production was cheap\(^2\). After all, perfect foresight about product demand would induce firms to produce exactly the amount of goods needed since holding or not selling goods is costly. In case of perfect foresight also less idle production capacities would exist than in case of demand uncertainty. It makes, in a similar way as holding inventories, no sense to acquire additional production capacity that will not be fully used. Both the level of inventories as well as capacity utilization rate are, however, not only indicators of demand uncertainty but also of technological developments.

Christiano (1988) emphasizes the 'residual' role of inventories.

\[
\begin{array}{c|c|c|c|c|c}
K_i & (capital), & N_i & (labour) & & \\
\hline
\downarrow & & \downarrow & & \downarrow & \\
\downarrow & & \downarrow & & \downarrow & \\
\downarrow & & \downarrow & & \downarrow & \\
\downarrow & & \downarrow & & \downarrow & \\
Y_i (production) & \rightarrow & - & \rightarrow & S_i (sales) & \rightarrow \\
\hline
V_i (inventories) & = & V_{i-1} + Y_i - S_i
\end{array}
\]

He adopts a model where '...elements of precommitment are important in fixed investment and employment decisions, but minor in consumption decisions...' (page 248). Technology and preference shocks can thus not influence capital and labour decisions. They influence, though, sales. In this way '...inventories buffer consumption from unexpected disturbances in production and buffer production from unexpected disturbances in consumption...' (page 248). Inventory investment plays thus a residual role. In addition to this role of inventories, Christiano assumes inventory stock to be a production input that can substitute physical

\(^2\) Adjusting production should be cheap in comparison with inventory costs.
capital stock. Unrealistically, in this model capital capacities are assumed to be always fully utilized.

Instead of inventories as a production factor, the 'precautionary' motive is often mentioned in the literature. Kahn (1992) and Krane (1994), for example, argue that the cost of stock-outs can exceed the holding costs of inventories. Following this line of reasoning, entrepreneurs always want to hold inventory stocks to avoid the possibility of falling short of demand. In these models, as well as in the model of Christiano (1988), the stock of inventories is a decision variable that entrepreneurs choose optimally. In the former models, though, a target value is to be specified. This target value is often assumed to be the level of sales, to which inventory stocks are geared (according to Holt, Modigliani, Muth and Simon (1960)).

To model these 'inventory objectives', mainly linear-quadratic specifications are chosen by which simple linear decision rules result. As a consequence of the assumption that inventory stock is a decision variable, i.e. a production factor and/or precautionary measure, inventory stocks appear in the long run relationships.

The major aim of this study is to discover to what extent inventories are important in the long run. They are considered in relation with the capacity utilization rate, materials, labour and capital. An entrepreneur's ease to decide upon these four latter factors can be thought of to be of a decreasing order. Capital investment decisions are often taken first because of, for example, time-to-build considerations. Precommitments concerning labour also exist, but the incumbent labour force seems more easy to vary (by hiring and firing) than the capital stock but, less easy to change than the stock of materials. Utilization rates thereafter seem much more easy to vary. The place of inventories in this ranking order is a question to be answered.

In contrast to many other empirical inventory studies, the demand for different production factors (instead of production) in French industrial sectors during 1970.1-1992.1IV, is analyzed.

Also in contrast to most other empirical studies, that adopt either a (non-linear) structural model -being a theoretical or economist approach- or a linear unrestricted model -being a statistical approach-, both approaches are discussed in this study. Starting from a linear-quadratic structural model, first order conditions are derived and rewritten as an Error Correction Model. This latter model is estimated since it is a 'proper' framework to test for restrictions implied by the theory, like the number of cointegration relationships, dynamics, exogeneity and causality.

By adopting this approach time series properties are appropriately accounted for, whereas it remains possible to draw conclusions implied by the theory. Unlike many inventory studies, like Christiano (1988) who derives a steady state model, West (1986) or Ramey (1991) who estimate first order conditions by instrumental estimation methods, a simple but suitable ECM framework is thus adopted.

As a first advantage in comparison with these inventory studies, this framework makes it possible to verify whether sales can be assumed to be exogenous for factor demand. Second, whether inventory stock is significant in the long run -hence is an instrument- or not -hence is a residual- can be investigated. In the former case, a (possible) target value can be discovered. As a third advantage of the framework adopted here, both inventories and capacity utilization changes in reaction to sales impulses are easy to investigate. Both short and long run relations are distinguished, by which the assumptions of Christiano (1988) can be discussed. If production factors are precommitted and inventories play mainly a residual role (see Christiano (1988)), inventories do not play a significant role in the long run. In the

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3 Christiano argues further that the high volatility of inventories, corroborated by empirical findings with aggregate United States data, is largely due to the residual role of inventories.

4 See also production smoothing models, where production and inventories are decision variables. Examples are West (1986) and Ramey (1991). For a criticism of these models and a comparison of a factor demand model with inventories and a production smoothing model, see Peeters (1994,1995a).

5 Notice that this holds even if product prices are fully flexible.

6 See also Dolado, Galbraith and Banerjee (1991) that emphasize the importance of accounting for the time series properties when estimating parameters of the structural model.
(extreme) opposite case; inventories play a role in the long run and no residual role. In this case inventories can be a production factor and/or a precautionary measure. In these analyses, distinguishing different sectors is essential since inventory policies differ among sectors that produce to order and sectors that produce to stock.

The outline is as follows. Section two gives two examples of structural models and discusses the role of inventories herein. Section three gives further details on the methodology. Section four derives an ECM with restrictions from a structural model. Section five provides information on the data. Section six presents ECM estimation results. Section seven concludes.

2 The theory on neoclassical (intertemporal) firm behaviour

The neoclassical theory on firm behaviour adopts either a profit maximizing or a cost minimizing framework. Some differences between the two approaches exist. Under standard specifications, a main difference is that production is assumed to be endogenous in the former framework whereas production is assumed to be exogenous in the latter (see for example Epstein (1981)). In a cost minimizing framework the market structure on the product market—hence the way product prices are determined—is thus irrelevant.

Another difference occurs when inventories are considered. This section consecutively pays attention to inventories when profits are maximized, and when costs are minimized. Finally, the differences between both approaches with respect to inventories are discussed.

2.1 Inventories in a profit maximizing framework

If an entrepreneur aims at maximizing profits, the profit function at time t can be formalized as

\[ P(t) = \sum_{i=1}^{2} P_{x_i}x_{i,t} - C(V_t) \left[ - g(\Delta x_{1,t}\Delta x_{2,t}) \right], \]  

where

\( Y_t = f(x_{1,t}, x_{2,t}), \)

\( Y_t = S_t + \Delta Y_t, \)

\( S_t = \text{MIN}(Y_S, S_{t-1}, f(x_{1,t-1}, x_{2,t-1})), \)

\( Y_{it} = \text{MIN}(Y_{S_{it}}, S_{t-1}, f(x_{1,t-1}, x_{2,t-1}, Y_{D})) \).

\( P_t \) is the product price, \( S_t \) are sales, \( P_{x_{it}} \) is the nominal price of production factor \( X_{it} \) \( (i=1,2) \), \( C(.) \) represent the cost of holding‘ inventory stocks \( V_t \), \( g(.) \) represent adjustment costs, being a function of production factor changes \( \Delta X_{it}=X_{it}-X_{it-1} \) \( (i=1,2) \), \( Y_t \) is production, \( Y_{S_{it}} \) is the ‘notional production, \( X_{S_{it}} \) the ‘notional‘ level of production factor \( i \) and \( Y_{D_t} \) the (unknown) demand for the product.

Equation (2) specifies the profit function with only the two production factors \( X_{1,t} \) and \( X_{2,t} \). Both the product price as well as the input prices are assumed to be exogenous here. For the time being, it is assumed that \( g(.)=0 \) so that a static profit function is investigated.

Equation (2) is the production technology where \( f(.) \) specifies the transformation of the factor inputs into production. Under standard conditions it holds that \( \partial f/\partial X_{it}>0 \) and \( \partial^2 f/\partial X_{it}\partial X_{it}<0 \). From identity (3) it follows that production equals sales if no inventories exist, i.e. \( \Delta Y_t=0 \). When production exceeds (falls short of) sales, the inventory stock accumulates (decumulates).

Demand in (4) is assumed to be unknown at the moment where both prices and production capacity are determined. Ex post, excess capacity and excess demand is thus possible. Three possible cases exist. If \( f(x_{1,t}, x_{2,t})+Y_{D_t} \), as well as \( Y_{D_t} \) are not restrictive, \( S_t=Y_{S_{it}}+Y_{D_t} \). If \( X_{1,t} \) or \( X_{2,t} \) is restrictive, \( f(x_{1,t}, x_{2,t})+Y_{D_t} \) will determine sales. If demand is restrictive, demand will determine sales. If no inventories exist, i.e. \( V_t=0 \), (4) equals the often used determination rule of sales in rationing models (see for example Drèze (1991), page 5-6).

Restriction (3), rewritten as \( S_t=V_t-\Delta Y_t \), and (2) can consecutively be substituted in (1). The

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7 As should be noticed, specification (1) does not account for stockout costs. Stockout costs are, though, accounted for in section 4. For example Krane (1994) pays considerable attention to the importance of stockout costs.
optimal demand for the production factors is then implicitly determined by

\[
\frac{\partial f}{\partial X_{ij}} = \frac{P_{x_{ij}}}{P_t}, \quad i=1,2. \tag{5}
\]

The realization of the optimal factors is feasible if sufficient factor supply exists (see (4)). The optimal inventory stock is implicitly determined by

\[
\tau_1 E\{\tau_{t+1}P_{t+1}\Omega_t\} - \tau P_t = \frac{\partial C}{\partial V_t} \tag{6}
\]

Here it is assumed that the entrepreneur is rational. \(E\) represents the rational operator sign, \(\Omega_t\) the information set and \(\tau\) the discount factor.

2.2 A cost minimizing framework

If an entrepreneur aims at minimizing costs, the costs function at \(t\) can be formalized as

\[
\sum_{i=1}^{2} P_{x_{ij}} X_{ij} + C(V_t) \left[ 1 + g(\Delta X_{i1}, \Delta X_{22}) \right], \tag{7}
\]

subject to restrictions (2)-(4).

In contrast with (1) the product price, \(P_t\), does not appear here. In econometric practice, from equation (2) either an explicit function for \(X_{ij}\) from (2) is derived or \(X_{ij}\) is approximated. Substitution in (7) of this expression is then represented by

\[
f^*(X_{22}, P_{x_{22}}) + P_{x_{22}} X_{22} + C(V_t) \left[ 1 + g(\Delta X_{i1}, \Delta X_{22}) \right], \tag{8}
\]

where \(f^*\) denotes the flexible cost function, being a function of the other production factor, output and the own nominal factor price. Necessary conditions for flexible cost functions can be found in Dievert and Wales (1987).

The restriction (3) can be substituted in (8), rendering \(f^*_i = f^*(X_{22}, S_i, \Delta V_t, P_{x_{22}})\). The optimal demand for the production factor \(X_{ij}\) is then implicitly given by the equation following from the Shephard lemma, i.e.

\[
\frac{\partial f}{\partial P_{x_{ij}}} = X_{ij} \tag{9a}
\]

and the first order condition for \(X_{22}\) results from (8), i.e.

\[
\frac{\partial f_{i2}}{\partial X_{22}} + P_{x_{22}} = 0. \tag{9b}
\]

Here again, optimal factor demand is feasible if factor supply suffices. The optimal demand for inventories is implicitly given by

\[
\tau_1 \frac{\partial f}{\partial V_t} - \tau P_t \frac{\partial C}{\partial V_t} + \frac{\partial C}{\partial V_t} = 0. \tag{10}
\]

2.3 A comparison

If there are no inventories \((V_t=0, \forall t)\) and no factor input or demand restrictions, it holds that \(S_t=X_t\), \(S_t\) (see (3) and (4)). Production is endogenous in the profit maximizing framework since it is determined by the optimal factor inputs for \(X_{i1}\) \((i=1,2)\). Production is exogenous in the cost minimizing framework since the variable production factor \(X_{i1}\) is chosen such that (3) holds. In most factor demand studies the assumption \(V_t\) is made and demand equations similar to (5) or (9a)-(9b) are estimated\(^9\).

If inventories exist \((V_t>0)\), demand must have been restrictive—at least once—by which over-production occurred. If neither factor input nor demand restrictions existed, a conflict between identity (3) and restriction (4) existed unless \(V_t=0\). After all, both \(Y_t=S_t, \Delta V_t\) and \(S_t=Y_S, \Delta V_t\) should hold.

The inventory stock, being the accumulated result of differences between sales and production

\(^9\) Notice that (5) as well as (9a)-(9b) do not take the restriction (4) into account. Accounting for this rationing restriction is difficult since demand is not observed. If a demand equation is specified, a likelihood function can be written taking account of the different regimes. Estimation is, though, still not trivial (see Snoeijens, 1981, page 64-75).

\(^{10}\) For an overview of factor demand studies, see Peeters (1995b), chapter 1.
(see (3)), can either be regarded as a variable a producer decides upon—*an instrument*—or as a variable resulting from production falling short of or exceeding sales—*a residual*.

In case of inventories as an instrument, \( V_t \) is determined according to (6) when profit maximizing is assumed, and according to (10) if cost minimizing is assumed. As (3) should hold under both frameworks, (5) and (9) can then only hold coincidentally if *sales is exogenous*.

In case of inventories as residual, (5) and (9) under both frameworks hold and the inventory stock follows from (3), i.e. \( V_t-Y_t-S_{t}+P_{t+1} \), after the realization of sales. Conditions (6) and (10) will thus not be satisfied. Also in this case it is assumed that *sales is exogenous*.

An important difference between both frameworks with regards to inventories is the separability of factor inputs and inventories. From (5) follows that the optimal demand for the production factors is independent of the inventory stocks and sales\(^{11}\). If the specification of (separable) adjustment costs, \( g(\cdot) \) in (1), is taken into account this conclusion is not changed. On the contrary, from (9) follows that the optimal demand for the production factors depends on the inventory stocks; \( f^{*}_t \) is usually not separable in \( Y_t \) and \( P_{t+1} \), and \( Y_t \) and \( X_{2t} \). Production (including inventories) appears thus in the first order conditions here\(^{12}\).

This indicates that, for example, if inventory stock rise during a very long period—*ceteris paribus*—the optimal factor demand derived in the profit maximizing framework does not change. So there is no difference in optimal factor demand whether inventory stocks exist or not. In the cost minimizing approach, however, the demand for factors will change: if inventories increase *ceteris paribus*, production increases and the demand for production factors will fall since \( f^{*}_t \) is convex in \( Y_t \).

To summarize this section, inventory stocks of final products are often neglected but can be incorporated in neoclassical (intertemporal) firm models. Sales are incorporated if inventory investment is modelled as a precautionary measure. The profit maximizing or cost minimizing frameworks differ however with regards to inventory stocks. Even in a very basic cost minimizing framework, like (7), inventories are modelled since they are a part of production. Whether the stock of inventories is chosen optimally by entrepreneurs *and/or* is to be treated as a residual, remains a question to be answered.

### 3 Methodology

The solution of the models specified in the previous section can be estimated in different ways. One possibility is to estimate (5)-(6) or (9)-(10) by an instrumental estimation method, like the Generalized Method of Moments (GMM). This method is consistent, though not efficient. Another possibility is to estimate the system of equations by Full Information Maximum Likelihood (FIML), being an efficient estimation method. To follow this latter avenue, assumptions concerning the marginal processes of prices and sales are to be made.

In order to estimate the solutions (5)-(6) or (9)-(10) any way, though, the production \( (g) \) or cost function \( (f^{*}) \) as well as the specification of the inventory holding cost function \( (C(V)) \) need to be parameterized. The former is not difficult since many functional forms have been investigated (succesfully) in literature. About a specification of inventory costs, however, a consensus has not been reached yet. Many studies use the specification introduced by Holt, Modigliani, Muth and Simon (1960) but much empirical corroboration for this specification has not been provided\(^{13}\).

In order to estimate the solutions (5)-(6) and (9)-(10) any way, with whatever specification, stationarity of the specified relations should hold when standard estimation methods are used. As the structural models specified by (1) or (7) do not directly provide a framework for drawing inferences on time series properties, an Error Correction Model

\(^{11}\) As should be noticed, inventories matter for optimal input demand if they appear interrelatedly in the production function (see Kydland and Prescott (1982) or Christiano (1988)) and/or in the adjustment cost function.

\(^{12}\) This never holds for the profit maximizing framework, even if inventories are included in the production function (see previous footnote), since \( Y_t \) does not appear in first order conditions.

\(^{13}\) See for example Rossana (1993) and Peeters (1995a) who test Holt’s specification in an ECM model and structural models, respectively.
will be derived. Within an ECM time series properties are easily verified and 'correct' test statistics can be obtained. Furthermore, short and long run relations concerning inventories, production factors and sales are easy to investigate.

One disadvantage is that structural parameters are not obtained directly. Another disadvantage is that the estimation of an ECM only allows linear-quadratic specifications in the original objective function (1) or (7). This is a limitation that, though, in almost all other empirical models on inventories is disregarded.

A description of the methodology is as follows. In the following section a linear-quadratic structural model, without separability between production factors and inventories (like (7)), is specified in more detail. Adjustment costs are specified to account for the dynamics in the aggregate time series that are used to estimate the model solution. The expectations of the firm are assumed to be rational. First order conditions are derived and a closed form is obtained by choosing the process of factor prices and sales suitably (according to, for example, Nickell (1985)). This closed form solution is rewritten as an ECM. The number of cointegration relations, the zero parameter restrictions and the exogeneity assumptions implied by the structural model are then tested in an (unrestricted) ECM. The estimation results obtained with this ECM by using French data, are presented in section six.

4 The derivation of an ECM from a structural (intertemporal) model for physical capital stock, labour, materials and inventory demand

An entrepreneur uses capital stock $K^*_t$, for production, but has a potential capital stock $K_t$. Thus $K^*_t=K_tU_t$, where $U_t$ represents the utilization rate. Other production factors are labour $(N_t)$ and materials $(M_t)$. Inventories are represented by $V_t$. For this entrepreneur, the objective function at time $t$ can be formalized as

$$
\text{MIN}_{K_t,N_t,M_t,U_t,V_t} E\left[ \gamma_0 \sum_{k=0}^\infty \gamma^k (K_t^{\gamma_1} N_t^{\gamma_2} V_t^{\gamma_3} U_t^{\gamma_4} M_t^{\gamma_5}) + P_t F_{t+1} + C(V_t,N_t) + 0.5\gamma_1 K_t^{\gamma_6} + 0.5\gamma_2 K_t^{\gamma_7} + \gamma_3 K_t^{\gamma_8} \right] | \Omega_t.
$$

In this function $\gamma_{0-5}$ are the adjustment costs parameters that, like the parameters in the restricted cost function, are to be estimated. By the pre-multiplication of 0.5, the adjustment cost parameters in the Euler equations are not pre-multiplied by constants.

Inventory stock (in levels) is included as an argument in the restricted cost function $F^r_t$. Like capital and labour, it is treated as a production factor (according to Kydland and Prescott (1982) and Christiano (1988)). $I_t$ represents gross investment. The first term, $F^r_t$, represents the restricted cost function. The second term represents the costs of gross investment, the third term the total wage sum. The fourth term represents the costs of holding inventories. The last three term represent adjustments costs. Like in many other studies, it is assumed here that capital stock and labour are quasi-fixed production factors, i.e. adjustment costs are incurred when their (potential) stocks are changed. On the contrary, materials can be acquired without additional costs.

The potential capital stock is accumulated according to

$$
K_t = (1-\kappa)K_{t+1} + I_t.
$$

(12)

The depreciation rate, $\kappa$, is assumed to be constant here. For the inventory holding costs, for the time being, the specification of Holt et al. (1960) is adopted, i.e.

$$
C(V_t,N_t) = 0.5\eta_0(V_t,\eta_1 N_t)^{2}.
$$

(13)

This specification assumes that costs are incurred when inventory stocks deviate from $\eta_1 N_t$. In the 'optimal' or 'preferred' situation, inventories are kept in line with sales: $V_t = \eta_1 N_t$.

The first order conditions for $K_t$, $N_t$, $M_t$, $U_t$ and $V_t$ are given by

$$
\gamma_0 \sum_{k=0}^\infty \gamma^k (K_t^{\gamma_1} N_t^{\gamma_2} V_t^{\gamma_3} U_t^{\gamma_4} M_t^{\gamma_5}) + P_t F_{t+1} + C(V_t,N_t) + 0.5\gamma_1 K_t^{\gamma_6} + 0.5\gamma_2 K_t^{\gamma_7} + \gamma_3 K_t^{\gamma_8} \right] | \Omega_t = 0
$$

(14a)

$$
\frac{\partial F_t}{\partial K_t} = M_t
$$

(14b)

$$
\gamma_0 \sum_{k=0}^\infty \gamma^k (K_t^{\gamma_1} N_t^{\gamma_2} V_t^{\gamma_3} U_t^{\gamma_4} M_t^{\gamma_5}) + P_t F_{t+1} + C(V_t,N_t) + 0.5\gamma_1 K_t^{\gamma_6} + 0.5\gamma_2 K_t^{\gamma_7} + \gamma_3 K_t^{\gamma_8} + \gamma_4 K_t^{\gamma_9} \right] | \Omega_t = 0
$$

(14c)

$$
\frac{\partial F_t}{\partial U_t} = U_t
$$

(14d)
\[
\tau_1 \frac{\partial \gamma_i^*}{\partial \gamma_i} + \eta \delta_i(V_{i-1} - \eta i_{i-1}) = E \left[ \tau_{i+1} \frac{\partial \gamma_{i+1}}{\partial \gamma_i} | \Omega_i \right] = 0. \tag{14e}
\]

As analyzed in section two, the first order condition (14e) does not hold if inventory stocks do not exist (i.e., \( V_i = 0 \), \( \forall i \)). Neither does it hold if inventory stock is only a 'residual', thus not a decision variable.

If inventory stock is a decision variable, though, \( V_i \) appears in the long run relationships. If the specification of Holt et al. (1960) holds, i.e. \( \eta = 0 \), and inventory stock is not a production factor, the relation \( V_i = \eta_i S_i \) exists in the long run. This is investigated by Rossana (1993). However, if inventory stock is both geared towards sales as well as a production factor, the relation \( V_i = \eta_i S_i \) does not necessarily hold.\(^{14}\)

If a linear-quadratic cost function for \( f_i^* \) is adopted, (14a)-(14e) can be rewritten as the system

\[
\bar{A}_1 X_{i+1} + \bar{A}_2 X_i + \bar{B}_1 Z_{i+1} + \bar{B}_2 Z_i + \bar{a} = 0, \tag{15}
\]

where \( X_i = [K_i N_i M_i U_i V_i]^t \) and \( Z_i = [P_{i+1} P_{i} P_{i-1} M_i S_i]^t \)^t. The matrices \( \bar{A}_i \) (\( i = 1, 2, 3 \)), \( \bar{B}_i \) (\( i = 1, 2 \)) and vector \( \bar{a} \) contain reduced form parameters. In the following, it is assumed that the discount factor \( \tau_1 \) is constant, i.e. \( \tau_1 = \tau \), by which the stable solution\(^{13}\) has the representation

\[
X_i = \begin{bmatrix} a \\ w_0 \end{bmatrix} \bar{G}_i + \begin{bmatrix} I_5 - B_0 \end{bmatrix}^{-1} \begin{bmatrix} A & B_1 \\ 0_4 & I_4 \end{bmatrix} \begin{bmatrix} I_5 - B_0 \end{bmatrix}^{-1} \begin{bmatrix} 0_4 & 0_4 \\ 0_4 & B_1 \end{bmatrix} \Gamma_i, \quad i = 2, 3, p, \tag{16}
\]

where

\[
\Gamma_i = - \sum_{k=1}^{p} \bar{G}_k \Gamma_k, \quad i = 2, 3, p-1, \quad \Pi = \sum_{k=1}^{p} \bar{G}_k I_p, \quad \Pi = \alpha \beta_i.
\]

Three important features result from this derivation. By the theoretical model (11) with restrictions (12)-(13) it holds that

(i) dynamics in the system are determined by the marginal process assumption (17), and by the first order autoregressive terms of \( X_i \) as a consequence of the adjustment cost specification in (11);

(ii) four exogenous variables are assumed to exist, being \( Z_i \), by which thus no causality from \( X_i \) to \( Z_i \) is to be found;

(iii) at least five cointegration relations are to be found if inventory stock is a decision variable. In this case the endogenous variables possibly are \( X_i = [K_i N_i M_i U_i V_i]^t \); or

(iiiib) at least four cointegration relations are to be found if inventory stock is not a decision variable. In this case the endogenous variables possibly are \( K_i, N_i, M_i, U_i \).

\(^{16}\) The process (17) can also be in first differences, by which among others \( W_i - I_4 \).
In section six the model (19) is estimated unrestricted and the features are tested. Attention is paid to the significance of inventory stocks in the long run, which hinges upon the fact whether (iiia) or (iiib) is true. If (iiia) is true, (14a)-(14c) must be satisfied simultaneously. Sales should then be important in the long run. The inventory cost specification of Holt et al. (1960), i.e. \( \eta_d(V_t - \eta_1S_t) \), and/or inventory stock as a production factor can then uphold (14c). If (iiib) is true, however, (14e) is not necessarily satisfied. Inventory stock might still be important in the short run because of the appearance in (14a)-(14d), but do not matter in the long run; neither \( V_t = \eta_1S_t \) is an objective nor inventory stock is a production factor.

5 Data and descriptive statistics

The data used in the empirical analyses are quarterly, seasonally adjusted, and range from 1970.1 to 1992.1V. Five French sectors are distinguished: the intermediate goods, the professional equipment, the durable consumption goods (consumer equipment), the transport equipment and the consumption goods sector. The series of gross investment (\( I_t \)), the average weekly working hours (\( N_t \)), the volume of materials (\( M_t \)), the volume of inventory investment (\( \Delta V_t \)), the nominal investment price (\( P_{IC} \)), the nominal wage (\( P_{Wt} \)), the nominal materials price (\( P_{Mt} \)) and sales (\( S_t \)) are from the National Accounts\(^{12}\). Physical capital stock (\( K_t \)) is constructed by means of the Perpetual Inventory Method, using a benchmark\(^{13}\), gross investment and a constant depreciation rate of 0.025 per quarter. To calculate the stock of inventory stock a (sectoral) benchmark is used. This benchmark as well as data on the utilization rate per sector were kindly provided by the French national bureau of statistics (INSEE, Paris). Unfortunately, the utilization rate only exists per sector from 1976 onwards. For this reason, only the period 1976.1-1992.1V is considered in the empirical analyses.

Two measurements of the utilization rate exist, being a rate with and without workers that can be hired in the very short run\(^{19}\). As differences between these rates certainly exist on levels, it turns out that fluctuations between the two rates are not very divergent. Only the utilization rate excluding the additional workers is used since it measures the ‘existing’ undercapacity of sectors. As should be noticed, this utilization rate concerns physical capital stock only (\( U_t = K_t \), see (11)).

In table 1 simple correlations between sales and inventory stocks, and between sales and the utilization rate are presented. Inventory stocks and utilization rates are varied from three lags up to and until three leads. All variables are in growth rates, where this rate is calculated as \( \hat{w} = (W_{t+1} - W_t) / W_t \). \( W_t \), \( S_t \), \( V_t \), \( U_t \), to eliminate possible (left) seasonal effects.

Reminding that the sample concerns 1976.1-1992.1V, the period under investigation can be considered to be a period of abundant production, more than a period of supply shortages. For this reason, changes in sales will more be determined by changes in product demand than by changes in product supply.

If sales (or let us say demand) increases occur, inventory stocks could be expected to decrease instantaneously and/or in the near future. On the contrary, utilization rates can be expected to increase, instantaneously and/or in the near future. The turning points of the inventories’ and utilization rates’ cycles presented in table 1 can thus be expected to have opposite signs.

In table 1 the bold figures are the lowest and highest correlation of the cycles. For the professional equipment, durable consumption goods and consumption goods sector it follows that the inventory and utilization cycle are opposite since signs differ. When only leads \( (i = 1, 2, 3) \) are considered for these three sectors, it follows that sales correlates more strongly with utilization rates than with inventory growth rates, but return points occur later. This may indicate that inventories are depleted first, and capacity adjustments occur later.

The correlations for the intermediate and transport equipment sector show opposite results. In both sectors, inventory stocks are not depleted when demand shocks occur (and sales


\( ^{13} \) The benchmark is taken from 'Comptes et indicateurs économiques: Rapport sur les comptes de la Nation' and concerns net capital stock in constant prices of 1980.

\( ^{19} \) These rates are obtained by using the margin of disposable capacity measured with and without additional hiring (‘avec embauche’ et ‘sans embauche’). As the first is always higher than the second, the utilization rate of the latter is always higher.
### Table 1 Simple correlations of growth rates

<table>
<thead>
<tr>
<th>Sector</th>
<th>(\text{Corr}(\delta, \mu))</th>
<th>(\text{Corr}(\delta, \nu))</th>
<th>(\text{Corr}(\delta, \lambda))</th>
<th>(\text{Corr}(\delta, \mu, \lambda))</th>
<th>(\text{Corr}(\delta, \mu, \lambda, \nu))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate goods</td>
<td>0.09</td>
<td>0.11</td>
<td>0.21</td>
<td>0.34</td>
<td>0.48</td>
</tr>
<tr>
<td>Professional equipment</td>
<td>0.23</td>
<td>0.28</td>
<td>0.42</td>
<td>0.52</td>
<td>0.59*</td>
</tr>
<tr>
<td>Durable consumption</td>
<td>0.16*</td>
<td>0.10</td>
<td>0.01</td>
<td>-0.09</td>
<td>-0.13*</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>0.30</td>
<td>0.25</td>
<td>0.18</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>Consumption goods</td>
<td>0.27</td>
<td>0.38</td>
<td>0.34*</td>
<td>0.34</td>
<td>0.34*</td>
</tr>
</tbody>
</table>

6.1 Rank and exogeneity tests

For each sector the ECM (19) is estimated with the RATS (3.11) program of Johansen, that is modified by Hansen. In order to estimate this model, each of the nine variables should be integrated of order one or zero. Unit root test statistics, that are not given here, show that all variables have at most a unit root.\(^{20}\) The price variables \((P_i, P, P_M)\), though, had to be deflated by the product price since some nominal prices are (nearly) 1(2). These real prices are used in the further analyses.\(^{21}\)

For each sector rank tests are carried out. In the first panel in table 2 the hypothesis that four or less cointegration relationships exist is tested against the hypothesis that five or less cointegration relationships exist. The statistic presented in the first panel is the p-value for this test. For the sector of durable consumption goods and transport equipment the p-values are rather high (12% and 20%). For these sectors the hypothesis of four cointegration relationships is thus not rejected (at the 5%-level). For the other three sectors the hypothesis of four or less cointegration relationships is rejected. This implies that five cointegration relationships (or more) may exist.

The cointegration relations can be associated with the steady state of equations (14a)-(14c), according to the theory in section four. In the following the assumption of five relationships is maintained for all sectors, i.e. the matrices \(\alpha\) and \(\beta\) (see (19)) have dimensions 9 by 5.

In applying these rank tests a constant term in each equation was accounted for, included in (19) as a drift. Testing the hypothesis that there is no drift (see panel two in table 2) shows that the hypothesis is rejected, except for the transport equipment sector where a p-value of 11% is found. Results presented in the following, are obtained by accounting for a drift. For the transport equipment sector, analyses were also carried out without drift. Significant differences in final conclusions for this sector, between the results with and

---

\(^{20}\) For the utilization rates the assumption of a unit root seems a bit odd since they are restricted to the range \([0,1]\). Over the sample 1976.1-1992.1IV, though, a unit root can not be rejected.

\(^{21}\) This is not in contradiction with the structural model (11), iff the discount factor (assumed to be a constant) is also in real terms.
Table 2  Rank and exogeneity tests

<table>
<thead>
<tr>
<th>Sector</th>
<th>Intermediate goods</th>
<th>Professional equipment</th>
<th>Durable consumption goods</th>
<th>Transport equipment</th>
<th>Consumption goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>(O-(II)) ≤ 4</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.12)</td>
<td>(0.20)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>rt-drift</td>
<td>10.8</td>
<td>11.6</td>
<td>14.4&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3.6</td>
<td>15.7&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>[4]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z&lt;sub&gt;W&lt;/sub&gt;</td>
<td>44.8&lt;sup&gt;a&lt;/sup&gt;</td>
<td>65.0</td>
<td>90.7&lt;sup&gt;a&lt;/sup&gt;</td>
<td>92.5&lt;sup&gt;a&lt;/sup&gt;</td>
<td>70.9&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>weak exo.</td>
<td>[20]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P&lt;sub&gt;W&lt;/sub&gt;</td>
<td>6.7</td>
<td>4.5</td>
<td>23.9&lt;sup&gt;a&lt;/sup&gt;</td>
<td>16.9&lt;sup&gt;a&lt;/sup&gt;</td>
<td>10.5&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>weak exo.</td>
<td>[5]</td>
<td>(0.25)</td>
<td>(0.49)</td>
<td></td>
<td>(0.06)</td>
</tr>
<tr>
<td>P&lt;sub&gt;W&lt;/sub&gt;</td>
<td>5.3</td>
<td>11.1</td>
<td>12.4&lt;sup&gt;a&lt;/sup&gt;</td>
<td>19.3&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.9&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>weak exo.</td>
<td>[5]</td>
<td>(0.38)</td>
<td>(0.05)</td>
<td>(0.03)</td>
<td>(0.41)</td>
</tr>
<tr>
<td>P&lt;sub&gt;W&lt;/sub&gt;</td>
<td>7.0</td>
<td>11.8</td>
<td>3.8&lt;sup&gt;a&lt;/sup&gt;</td>
<td>17.9&lt;sup&gt;a&lt;/sup&gt;</td>
<td>16.8&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>weak exo.</td>
<td>[5]</td>
<td>(0.22)</td>
<td>(0.04)</td>
<td>(0.59)</td>
<td></td>
</tr>
<tr>
<td>S&lt;sub&gt;W&lt;/sub&gt;</td>
<td>4.7</td>
<td>12.5</td>
<td>25.7&lt;sup&gt;a&lt;/sup&gt;</td>
<td>14.6&lt;sup&gt;a&lt;/sup&gt;</td>
<td>20.6&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>weak exo.</td>
<td>[5]</td>
<td>(0.43)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>V&lt;sub&gt;W&lt;/sub&gt;</td>
<td>21.3&lt;sup&gt;a&lt;/sup&gt;</td>
<td>34.8&lt;sup&gt;a&lt;/sup&gt;</td>
<td>17.7&lt;sup&gt;a&lt;/sup&gt;</td>
<td>9.5</td>
<td>11.7&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>weak exo.</td>
<td>[5]</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>

<table>
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<tr>
<th>X&lt;sub&gt;i&lt;/sub&gt; + Z&lt;sub&gt;i&lt;/sub&gt;&lt;sup&gt;**&lt;/sup&gt;</th>
<th>Z&lt;sub&gt;i&lt;/sub&gt;</th>
<th>P&lt;sub&gt;W&lt;/sub&gt;P&lt;sub&gt;M&lt;/sub&gt;</th>
<th>S&lt;sub&gt;V&lt;/sub&gt;</th>
<th>P&lt;sub&gt;W&lt;/sub&gt;P&lt;sub&gt;M&lt;/sub&gt;</th>
<th>V&lt;sub&gt;i&lt;/sub&gt;</th>
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<td>54.8&lt;sup&gt;a&lt;/sup&gt;</td>
<td>32.8&lt;sup&gt;a&lt;/sup&gt;</td>
<td>46.5&lt;sup&gt;a&lt;/sup&gt;</td>
<td>91.3&lt;sup&gt;a&lt;/sup&gt;</td>
<td>[37] (0.00)&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>[37] (0.00)&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
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<td></td>
<td>[37] (0.00)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>18.2&lt;sup&gt;a&lt;/sup&gt;</td>
<td>11.2&lt;sup&gt;a&lt;/sup&gt;</td>
<td>25.4&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>[18.2] (0.15)&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>16.7&lt;sup&gt;a&lt;/sup&gt;</td>
<td>25.0&lt;sup&gt;a&lt;/sup&gt;</td>
<td>24.9&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>[16.7] (0.21)&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>10.3&lt;sup&gt;a&lt;/sup&gt;</td>
<td>21.3&lt;sup&gt;a&lt;/sup&gt;</td>
<td>26.6&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>[10.3] (0.67)&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>15.3&lt;sup&gt;a&lt;/sup&gt;</td>
<td>22.2&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>[15.3] (0.29)&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>26.6&lt;sup&gt;a&lt;/sup&gt;</td>
<td>26.6&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>[26.6] (0.29)&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
<td>[24] (0.11)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>53.6&lt;sup&gt;a&lt;/sup&gt;</td>
<td>40.6&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
<td>[32] (0.01)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>[53.6] (0.01)&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
<td>[32] (0.01)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>46.0&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[24] (0.01)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

The figures in brackets are p-values. The figures in square brackets are degrees of freedom of the $\chi^2$ distribution. $\bar{X}_i$ contains all variables except for the variables that are causally influenced.

Z$^2$ = [P<sub>W</sub>P<sub>M</sub>][P<sub>M</sub>][V<sub>i</sub>]

** Causality test statistics with the lowest Akaike value (see Mosconi and Giannini (1992)).

* P-value of about zero.

without a drift, were not found.

The third panel in table 2 contains exogeneity and causality test statistics. These tests are carried out in order to verify whether or not exogeneity assumptions in the theoretical model, concerning prices and sales by (17), hold.

For each price and for sales a test statistic of weak exogeneity for the parameters in the long run ($\beta$) is calculated. This is a standard test, that in the full ECM only imposes restrictions on the factor loadings $\alpha$ (see for example Urbain (1993), p. 67). The null-hypothesis is $H_0: \alpha_i = 0$, $k = 6, 7, 8, 9$, $i = 1, 2, 5$ if the weak exogeneity of $Z_i$ is tested. This hypothesis is $H_0: a_{i,j} = 0$, $i = 1, 2, 5$ with $k = 6$ if the weak exogeneity of $P_{ij}$ is tested. Similarly, $k = 7, k = 8, k = 9$ if the weak exogeneity of $P_{ij}P_{ij}S_i$ is tested, respectively.

The results for the exogeneity of the three prices and sales simultaneously, i.e. $Z_i$, are presented in the first row of the third block of table 2. These tests show the rejection at the 1%-level of weak exogeneity of $Z_i$, for all sectors. The univariate exogeneity test statistics (see also the third block) show that the weak exogeneity of the (real) investment price is rejected in the durable consumption good and transport equipment sector. These results imply that in these sectors it is important to specify the (marginal) processes of investment prices in order to estimate the parameters of $\beta$ consistently. Weak exogeneity of wages for the parameters of $\beta$, as well as weak exogeneity of material prices are also rejected in some sectors. Weak exogeneity of sales is rejected in the durable consumption goods and consumption goods sector.

These results are important in several respects. In many studies sales is assumed to be (weakly or even strongly) exogenous, an assumption that is rejected here for the durable goods and consumption goods sectors. For some prices in several sectors the same holds. The rejection of weak exogeneity of factor prices implies the rejection of strong exogeneity of the (real) factor prices. An explanation for this result can be monopolistic competition on the output markets or monopolism on factor input markets.

As should be noticed, real wages are only rejected (at the 1%-level) in the transport equipment sector. In this sector none of the three prices are weakly exogenous for the long term parameters. This can indicate that monopolistic competition at the output market occurs; production factors, and consequently production, cause the product price (thus real factor
prices).

Furthermore, in the third block also weak exogeneity test statistics of inventories for the long run parameters are presented. If inventories were only a residual factor, the hypothesis of weak exogeneity should hold. As follows, the weak exogeneity hypothesis is not rejected for the transport equipment and consumption goods sector. So, in these sectors inventories could be a pure residual.

In panel four of table 2 causality tests statistics are presented. They are carried out along the lines of Mosconi and Giannini (1992) with their RATS (version 3.11) program. The statistics proposed by them account for the non-stationarity of the variables. A problem with the test procedure is that testing of causality of (a) variable(s) is only possible with respect to all other variables in the system. As another disadvantage of this procedure, several possibilities exist if the number of variables whose causality is tested, exceeds the number of cointegration relationships. For a detailed discussion on the method, see Mosconi and Giannini (1992).

For each sector the system of those variables for which (individually) weak exogeneity was accepted, is considered. Only these variables can be strongly exogenous. Per sector, this selected system of variables is indicated by \( Z^*_i \) in the fourth panel in table 2. As interpreting the results as well as the avenue to proceed is not straightforward, the results will be discussed per sector.

For the intermediate goods sector, the weak exogeneity of each of the three prices and sales was accepted overwhelmingly (even at 22%-level). Therefore, the hypothesis that the five other variables \( (X_i) \) do not cause these four variables \( (Z_i) \) is tested. The test statistic is 54.4 and accepted at the 3%-level. Univariate causality tests (see also panel four in table 2) show that the hypothesis that the vector of eight (other) variables do not cause the investment price is accepted (even at the 15%-level). This conclusion also holds for the wage, the price of materials and sales. The non-causality of factors to the three prices is accepted (at 1%-level). As a parsimonious system of variables is desirable in the further analyses, it is assumed in the following, as a consequence of the analyses here, that prices and sales are strongly exogenous in the intermediate goods sector.

For the professional equipment sector the proceeding is similar. The hypothesis that factors do not cause \( Z_i \), though, is not accepted. The non-causality of factors and sales to the system of three prices is accepted at 14%. Therefore, it is assumed in the following that the three prices are not caused by the production factors, the utilization rate, inventories and sales. Sales are thus not assumed to be exogenous.

For the durable consumption goods sector, only wages and material prices are not caused by the other variables (at 11%-level). Therefore, the investment price and sales are not assumed to be exogenous.

For the transport equipment sector, the hypothesis that \( \{X_i, P_{L,i}, P_{M,i}, P_{S,i}\} \) does not cause sales and inventories is rejected. Individual tests with respect to sales and inventories, though, accept non-causality at the 1%-level. As a consequence of this (less decisive) result, for this sector the full model with nine variables is considered in the following. Thus none of the variables is assumed to be exogenous.

For the consumption goods sector the hypothesis that the investment price, the wage and inventories are not caused by the other variables, is rejected. An individual test shows that the inventory stock is caused by the other factors. Therefore, only the investment price and the wage can be assumed to be exogenous in this sector.

To summarize, the test statistics indicate that for four sectors a partial model can be considered\(^2\). For the intermediate goods and professional equipment sector the three factor prices, for the durable consumption goods sector wages and material prices, and for the consumption goods sector the investment price and wages will be assumed (strongly) exogenous. For the transport equipment sector we are left with the rather unfortunate result that not the partial system of equations (like (18)) is to be used in the further investigations, but the full system of nine equations.

\(^2\) It could be argued that the selection procedures of the exogenous variables in table 2 are somewhat biased, since the exogeneity of prices is more preferred than the exogeneity of sales and/or inventories. From an economic point of view, however, the acceptance of the former is less wrong than the rejection of the latter.
6.2 The long and short run structure

A common practice in all kinds of cointegration analyses is to impose restrictions on the parameters of the long run structure, before presenting the parameters of the short run structure (see for example Juselius (1994) and almost all her colleagues).

Proceeding in this way is (always) rather arbitrary and, in particular, makes no sense here. From a theoretical point of view, no restrictions on the five cointegration relations in (19) exist. This follows from system (14), due to the restricted cost function $f_t^*$ that incorporates the inventories, the production factors and the utilization rate interrelatedly. It is furthermore unfortunate that standard errors are not directly available for the cointegration relations. In addition to this, combinations of the five relations are also long run relationships.

The parameter estimates of the short and long run structure, with information concerning weak and strong exogeneity from table 2 but without additional restrictions on the long run parameters, are presented in appendix 1. Significant long run parameters (at the 5%-level) are indicated by a superscript * in table A.1. Significant short run parameters (at the 5%-level) are presented in table A.2. Standard errors for these long and short run parameters where obtained by estimating the ECM with non-linear restrictions22.

In these ECM analyses the order of variables is chosen such that the 'most exogenous' variables appear first, whereas the endogenous variables appear thereafter in the order of decision making. The order that is chosen is $S_t, P_{t-1}, P_{t-2}, P_{t-3}, K_t, N_t, M_t, U_t, V_t$. Capital precedes labour since capital is assumed to be more predetermined than labour. Labour precedes materials and materials precede the utilization rate. Inventories are assumed to be most flexible and consequently occur last. This order is chosen with regards to the impulse responses, presented in the following subsection. The results presented in the subsequent subsections are thus based on these ECM results.

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22 Because of the large system of equations, it was impossible to take all non-linear parameter restrictions on $\alpha$ and $\beta$ into account simultaneously. First, parameter estimates of the short run parameters, $\alpha, \Gamma$ were obtained by using the estimates of $\beta$. In a second stage, standard errors of $\beta$ were obtained conditional on the estimates of the short run parameters.

6.3 Inventories as decision variable

Three questions about inventories are tried to be answered in this subsection. The first question is whether inventory stocks are significant in the long run. If the answer to this question is affirmative, the second question is to what extent inventories play a role as a decision variable in comparison with their role as a residual. The role of inventories can be mixed, i.e. a decision variable (precautionary and/or production factor) as well as a residual.

A third question, also conditional on the affirmative answer to the first question, concerns the nature of the role of inventories in the long run. Are inventories geared towards sales, according to the specification of Holt et al. (1960) and/or can inventories be considered to be a production factor?

The significance of inventory stocks in the long run can be tested by LR tests. The test statistics of the hypothesis $H_0: \beta_{j-0}, j=1,2,3,4,5$, for all sectors, are presented in the first row of table 3. The results show that the hypothesis is rejected at the 5%-level for all sectors, except for the professional equipment sector. According to these findings, inventory stock is a decision variable in the intermediate goods, the durable consumption goods, the transport equipment and consumption goods sector. For the professional equipment sector the role of inventories in the long run is less clear. As follows from the parameter estimates in table A.1, indicated by a * if they have significant t-values, also in the professional equipment sector inventory stock is (individually) often significant in a long run relation.

Inventory stock is thus not only a residual, but also a decision variable. In order to discover to what extent inventory stock is important in the long run, in each sector, the residuals of the long run relations -taking account of inventory stocks- are compared with these residuals for the case where $\beta_{j-0}, j=1,2,3,4,5$. As should be stressed emphatically here, these calculations are no formal tests. As far as I know, formal tests on the explanatory power of (a) variable(s) in the long run only do not exist. The following procedure is followed, though, because insights on the significance are important for this study.
As five long term equations exist in both cases, five residuals are obtained. Let $\Sigma_1$ represent the covariance matrix of the residuals for the case where inventories are included. Let $\Sigma_2$ represent this matrix for the case where inventories are not included. Then it follows that both $\Sigma_1$ and $\Sigma_2$ have dimension 5 and (theoretically) it should hold that the determinant of $\Sigma_1$, $|\Sigma_1|$, is less than $|\Sigma_2|$.

These ratio of these determinants is given in the lower line of table 3. So this is the percentage of the (co-)variances explained by $Y$. If the ratio equals 0, inventories are very (and possibly only) important in the long run in comparison with the short run. If the ratio equals 1, inventories are not important in the long run in comparison with the short run.

The ratio’s calculated for the five French sectors are presented in the second row of table 3. The results show that the explanatory power of inventories in the long run is approximately 0% in the durable consumption goods sector. For this sector, a closer look at this sector shows that for the volatility of inventories is extremely high. So despite the fact that inventories are significant in the long run (see the first row in table 3), inventory stock is not very important as a decision variable in comparison with its role as a residual.

The results for the intermediate sector show that the major role of inventory stock is a residual role. On the contrary, for the other three sectors it turns out that inventory stock is more a decision variable than a residual.

These results suggest thus that, for three of the five sectors, inventory stock as a decision variable explains the major part of the variance of the residuals of the long run relations.
Christiano (1988) found with aggregate American data that the major part of the volatility of inventory investment is due to its residual role. Comparing the results for French sectors here and Christiano’s results with aggregate data is difficult, since different models, estimation methods and data are used. One important difference is, though, that Christiano only accounts for inventories as a production factor. So he disregards the possibility of inventory investment as a precautionary motive, like Holt’s inventory objective function. This can be an explication for the fact that three out of five sectors here, find more evidence for the role of inventories in the long run than as a residual.

Once again I want to stress that the results in the lower part of table 3 are obtained in a very heuristic way. No formal tests are carried out since, to the best of my knowledge, they do not exist.

The role of inventories in the long run is further investigated by impulse response functions. The ECM (19) is rewritten as a VAR(2), and thereafter inverted into a vector-MA. For each sector the structure and order of variables is chosen according to the analyses in appendix 1. With the vector-MA impulse responses are calculated, using the program MULTI of Lütkepohl (1990). This program has the advantage that also confidence intervals are calculated.

With the model of nine variables, 81 impulse response graphs can be drawn. The impulses of sales are, though, most interesting for the analyses here since sales shocks are assumed to be exogenous and to affect the production factors, the utilization rate and inventories in models like (11) in the short and long run. To measure the pure responses, disturbances are orthogonalized. For each sector then three figures are given, see graph 1-15. The first graph shows the response of sales and the second the response of inventories, both in reaction to impulses in sales shocks. The third graph shows the response of utilization rates to inventory impulses. Impulse responses are calculated for a horizon of 25 quarters.

The upper graphs show that in response to the positive sales shock, the response of sales is positive in the short run but returns to a certain level close to zero after 25 periods. As should be noticed, a return to zero is not necessary since a constant term is included in the ECM (and thus the vector-MA).

The second graphs (in the middle) show that in response to the positive sales shock, inventories are depleted in the short run for all sectors, except for the intermediate sector. In the middle run (say 8-12 quarters), on the contrary, the inventory shock grows. In the long run (25 quarters) the response of inventory stocks to the shock fades away.

If the objective of Holt et al. (1960), i.e. \( V_t/n_t S_t \), were true, a positive sales shock should result in increases in inventory stocks in the end. From the graphs 2, 5 and 14 it follows that inventory stocks decrease in the end. Furthermore, comparing the upper graphs with the middle graphs shows rather different responses (and thus speed of responses) over the horizon of 25 quarters. Whether the ratio of inventories and sales is constant over time, i.e. \( V_t/S_t \), is thus doubtful.

Christiano (1988) models the inventory stock as a production input that influences the production capacity positively. Following this line of reasoning, utilization rates should decrease (increase) instantaneously if inventory stocks increase (decrease). To investigate whether this holds for the French sectors under investigation here, the reaction of utilization rate to inventory stock impulses are graphed. The lower graphs show the results.

The graphs show that only for the intermediate goods sector utilization rates decrease in the short run indeed. For all other sectors utilization rates hardly change in the very short run. In addition to this, the confidence intervals indicate that the hypothesis of no changes in utilization rates cannot be rejected (at the 5%-level).

<table>
<thead>
<tr>
<th>Variable</th>
<th>( S_t )</th>
<th>( P_{ij} )</th>
<th>( P_{ij} )</th>
<th>( P_{ij} )</th>
<th>( K_t )</th>
<th>( N_t )</th>
<th>( M_t )</th>
<th>( U_t )</th>
<th>( V_t )</th>
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<tbody>
<tr>
<td>Intermediate good</td>
<td>0.31*</td>
<td>0.09</td>
<td>0.13*</td>
<td>0.09</td>
<td>0.03</td>
<td>0.01</td>
<td>0.20*</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>Professional equipent</td>
<td>0.18*</td>
<td>0.28*</td>
<td>0.11*</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.30*</td>
<td>0.03</td>
<td>0.10*</td>
</tr>
<tr>
<td>Durable consumption goods</td>
<td>0.21*</td>
<td>0.11</td>
<td>0.08*</td>
<td>0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.10*</td>
<td></td>
</tr>
<tr>
<td>Transport equipent</td>
<td>0.10</td>
<td>0.02</td>
<td>0.02</td>
<td>0.11</td>
<td>0.002</td>
<td>0.14</td>
<td>0.04*</td>
<td>0.17</td>
<td>0.06*</td>
</tr>
<tr>
<td>Consumption goods</td>
<td>0.17</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.02</td>
<td>0.11</td>
<td>0.23*</td>
<td>0.03</td>
<td>0.18*</td>
</tr>
</tbody>
</table>

*Significant at the 5%-level.

The forecast horizon is 10 quarters.
Finally, to investigate by which factors the inventory stock can be forecast, forecast error variance decompositions are given in table 4. The forecast horizon is chosen to be only 10 quarters since the calculation of standard errors is very time intensive. Also these calculations are carried out with the program MULTI of Lütkepohl (1990).

As seen before, according to the results in table 3 inventory stock is mainly a residual in the intermediate goods sector and durable consumption goods sector. For these sectors it could thus be expected that sales (or demand in a demand restricted regime) explains the variance in inventories for the largest part. The results in table 4 show that for the intermediate goods sector a (significant) 31% holds and for the durable consumption goods sector a (significant) 21% indeed.

But also for the other sectors it holds that sales should be an important explanator if inventory stocks are held according to the specification of Holt et al. (1960). For all sectors, except for the intermediate goods sector, it holds however that sales is not the main explanator of the forecast error variance of inventory stock. From this follows that inventories as a precautionary measure, specified as a quadratic specification in inventories and sales, is not the most appropriate specification.

7 Conclusions

In this study structural factor demand models with inventories are specified. Linear decision rules for the production factors, utilization rate and inventory stock are derived and further investigated in an ECM. This ECM is applied to five industrial sectors of France. The main focus in the analyses is on the role of inventories in the long run.

The theoretical and empirical analyses lead to the following conclusions:

(i) For most sectors, both sales as well as factor prices do not turn out to be strongly, let alone, weakly exogenous (see table 2). The often adopted assumption in partial models, like factor demand models under perfect competition, may thus lead to biased estimation results. The same holds for production smoothing models, in which the exogeneity of sales is almost always assumed;

(ii) Inventory stocks and utilization rates are determined by prices and production factors, as well as by sales. This (re-)emphasizes the importance to investigate full systems of variables instead of only production, sales and inventories (see Granger and Lee (1989) or West (1986)) or only labour equations with wages (see for example Engsted and Haldrup (1994)). Probably, the decision process of firms should even be casted in a much more general model including financial structures of the firm (see for example Hay and Louri (1994));

(iii) If the inventory stock is not important in the long run, it plays only the role of a (pure) 'residual'. If inventory stock appears in the long run relationships of factor demand, it is a decision variable. In the literature two explanations of inventories as a decision variable exist. The first explanation is that inventory stocks are held as a precautionary motive. This is specified by a 'desired' value of the inventory stock that is in line with sales. Examples are production smoothing models (West (1986), Ramey (1991)) or Kahn (1992) and Krane (1994)). A second explanation is that inventory stocks are a production factor (or buffer). This is specified by Kydland and Prescott (1982) and Christiano (1988);

(iv) For the French sectors distinguished here, inventories play the role of both a decision variable and as residual. In the durable consumption goods sector inventory stocks are mainly a residual. This might be due to the fact that sales (thus demand in a demand restricted regime) is very volatile in this sector. This makes it hardly possible to target inventory stocks. In the professional equipment, transport equipment and consumption goods sector the role of inventories as a decision variable dominates;

(v) It is difficult to discover whether inventories are held as a precaution or as a buffer (production factor). Impulse response functions show that in response to sales shocks, in the short run inventory stocks are depleted and production capacities more intensively utilized. In the long run the effect of inventory stocks is not always positive, but the role of inventory stocks as a precautionary motive could still hold. The specification of a constant rate of inventory stocks with respect to sales, specified by Holt et al. (1960), might however be too simple. The role of inventory stocks as a production factor is not significant. This also followed from the analyses by Christiano
(1988), who finds very small parameter estimates for the inventory coefficient in the production function;

(vi) Inventory stocks in the intermediate sector are mainly determined by sales. This may indicate the production to order in this sector. In (aggregate) inventory studies, consequently, this sector should be investigated separately. This is already done by Granger and Lee (1989), but not by Christiano (1988) for instance.

Improvements of the study here could easily be made. Modelling inventory costs, though, will remain difficult. The more realistic the inventory cost specification, the more difficult it will be to estimate the model solutions. Furthermore, reliable measurements on inventory stocks are difficult to obtain. Nevertheless, from an economic point of view studying inventory behaviour seems important to understand factor demand decisions for instance.

Appendix I  The long and short run structure

This appendix presents the parameter estimates of the ECM (19). Table A.1 presents the long run parameters, table A.2 presents the short run parameters. Zero restrictions on the matrices $\alpha$ and $\beta$, resulting from the exogeneity tests in table 2, are taken into account. Conclusions from these results are discussed in subsection 6.3.

<table>
<thead>
<tr>
<th>Table A.1: Long run structure, $\hat{\beta}$</th>
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<tr>
<td>Order of variables:</td>
</tr>
<tr>
<td>$S_1, S_2, X_1, X_2, \ldots, X_k, Y_1, Y_k$</td>
</tr>
<tr>
<td>Intermediate goods</td>
</tr>
<tr>
<td>$-0.3, -0.1, 0.3, -0.2, 0.1, 0.2, -0.3, 0.2$</td>
</tr>
<tr>
<td>$-0.5, -0.3, -0.1, 0.1, 0.2, 0.3, -0.4, 0.4$</td>
</tr>
<tr>
<td>$-0.6, -0.4, -0.2, 0.2, 0.3, 0.4, -0.5, 0.5$</td>
</tr>
<tr>
<td>$-0.7, -0.5, -0.3, 0.3, 0.4, 0.5, -0.6, 0.6$</td>
</tr>
<tr>
<td>Professional equipment</td>
</tr>
<tr>
<td>$-0.8, -0.6, -0.4, 0.4, -0.6, 0.6, -0.8, 0.8$</td>
</tr>
<tr>
<td>$-0.9, -0.7, -0.5, 0.5, -0.7, 0.7, -0.9, 0.9$</td>
</tr>
<tr>
<td>$-1.0, -0.8, -0.6, 0.6, -0.8, 0.8, -1.0, 1.0$</td>
</tr>
<tr>
<td>Consumer goods</td>
</tr>
<tr>
<td>$-1.1, -0.9, -0.7, 0.7, -0.9, 0.9, -1.1, 1.1$</td>
</tr>
<tr>
<td>$-1.2, -1.0, -0.8, 0.8, -1.0, 1.0, -1.2, 1.2$</td>
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Figures with a superscript * are significant at the 5%-level.
### Table A.2 Short run structure, $a$ and $\hat{a}$

| Intermediate goods | $\frac{\Delta z_t^a}{\Delta z_t}$ | $\frac{\Delta z_t}{\Delta z_t}$ | $\frac{\Delta z_t^a}{\Delta z_t}$ | $\frac{\Delta z_t}{\Delta z_t}$ | $\frac{\Delta z_t^a}{\Delta z_t}$ | $\frac{\Delta z_t}{\Delta z_t}$ | $\frac{\Delta z_t^a}{\Delta z_t}$ | $\frac{\Delta z_t}{\Delta z_t}$ | $\frac{\Delta z_t^a}{\Delta z_t}$ | $\frac{\Delta z_t}{\Delta z_t}$ | $\frac{\Delta z_t^a}{\Delta z_t}$ | $\frac{\Delta z_t}{\Delta z_t}$ |
|--------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| Professional equipment |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |
|                       |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |
| Durable consumption goods |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |
| Transport equipment |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |
| Consumption goods |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |                                  |

**References**


Kranc, S.D. (1994), 'The distinction between inventory holding and stockout costs: implications for target inventories, asymmetric adjustment, and the effect of aggregation on


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