Workpackage 3

IUAP Study Day

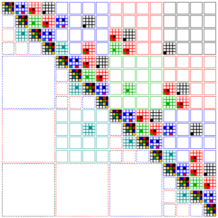
- Teams
  - UCL/INMA
  - KUL/KUL1
  - KUL/KUL2
  - ULg/SYST
  - UG/SYSTeMS
- Themes
  - Optimization
  - Stability and Model Reduction
  - Linear algebra, Multiresolution and Simulation

Structured matrices

Structured matrices depend on $\mathcal{O}(n)$ data rather than $\mathcal{O}(n^2)$ data (like a dense matrix). Examples are

- Sparse matrices
- Patterned matrices (Hankel, Toeplitz, FFT, Wavelet)
- Hierarchical matrices
- Rank structured matrices

Matrix problem complexity is then $\mathcal{O}(n^2)$ rather than $\mathcal{O}(n^3)$.



Empty submatrices are low rank and colored ones are full rank

Structured matrices

Continued collaborations and joint publications between

- KUL1
- KUL2
- UCL

Developed techniques are useful for other workpackages

- WP1 : Identification
- WP5 : Biomedical data analysis
- WP3 : Simulation, graph analysis, structured optimization, multiresolution analysis

Example: companion matrix

- Computing the roots of a monic polynomial $p(z) = p_0 + p_1 z + p_2 z^2 + \ldots + p_{n-1} z^{n-1} + z^n$ amounts to computing the eigenvalues of a 'companion' matrix H:

$$H = \begin{bmatrix} 0 & & & -p_0 \\ 1 & \ddots & & -p_1 \\ & \ddots & & \vdots \\ & & 1 & -p_{n-1} \end{bmatrix}.$$

- LAPACK solves this problem by the QR-algorithm but destroys the structure and requires $\mathcal{O}(n^3)$ flops

Reformulation of the problem

Rewriting the companion matrix as $H = U + R$:

$$H = \begin{bmatrix} 0 & & & 1 \\ 1 & \ddots & & \\ & \ddots & & \\ & & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & & & -p_0 - 1 \\ 0 & \ddots & & -p_1 \\ & \ddots & 0 & \vdots \\ & & 0 & -p_{n-1} \end{bmatrix}$$

where H is Hessenberg, U is unitary and R is rank one, allows to preserve this structure along the QR algorithm iterations.

This results in an $\mathcal{O}\left(n^2\right)$ algorithm which was succesfully applied to polynomials of degree up to a few thousands.

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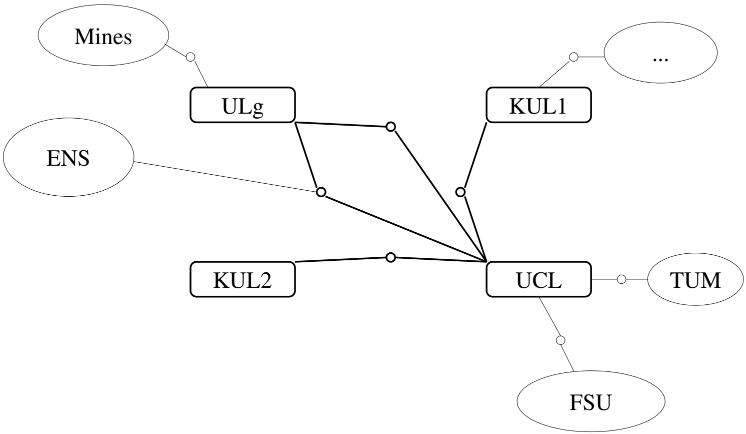
Low-rank approximations: joint publications



Low-rank approximations: geometry of $\mathrm{S}_+(p, n)$

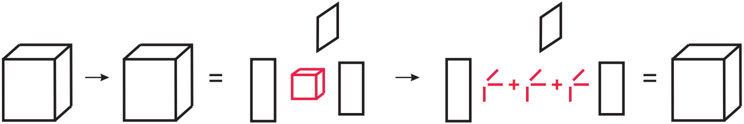
$\mathrm{S}_+(p, n)$ is the set of all symmetric positive-semidefinite $n \times n$ matrices of rank p:

$$\mathrm{S}_+(p, n) = \{YY^T : Y \in \mathbf{R}^{n \times p}, \mathrm{rk}(Y) = p\}.$$

$S_+(p, n)$ leads to

- various geometries: quotient or embedded;
- various computational problems: geodesics, means, statistics...;
- various applications:
  - covariance matrices (e.g., in finance)
  - semidefinite programming (e.g., for maximal cut and sparse PCA)
  - kernels (e.g., in machine learning)
  - diffusion tensors (e.g., in medical imaging).

Low-rank approximations: tensors



$\mathcal{A}$ Rank-(R_1, R_2, R_3) approx. Rank-R approx.

Applications of the best rank-(R1, R2, R3) approximation:

- Dimensionality reduction for independent component analysis (ICA)
- Dimensionality reduction for CANDECOMP
- Image analysis and recognition
- Multi-way data analysis