Workpackage 2 : Network, Dynamics and Control

A coordinated study of coordination : Clustering and synchronization phenomena

Some collaborative investigations within the IAP

IAP Dynamical Systems, Control and Optimization

Study Day 28 / 05 / 2009, Mons

Workpackage 2 : Network, Dynamics and Control

Teams involved:

- UCL/INMA
- KUL/ KUL1 & KUL2
- UGent/SYSTeMS
- ULB
- ULg/SYST

Main themes:

- Multi-agent systems
- Control and optimization
- Infinite dimensional systems
- Nonlinear control applications

Workpackage 2 : Network, Dynamics and Control

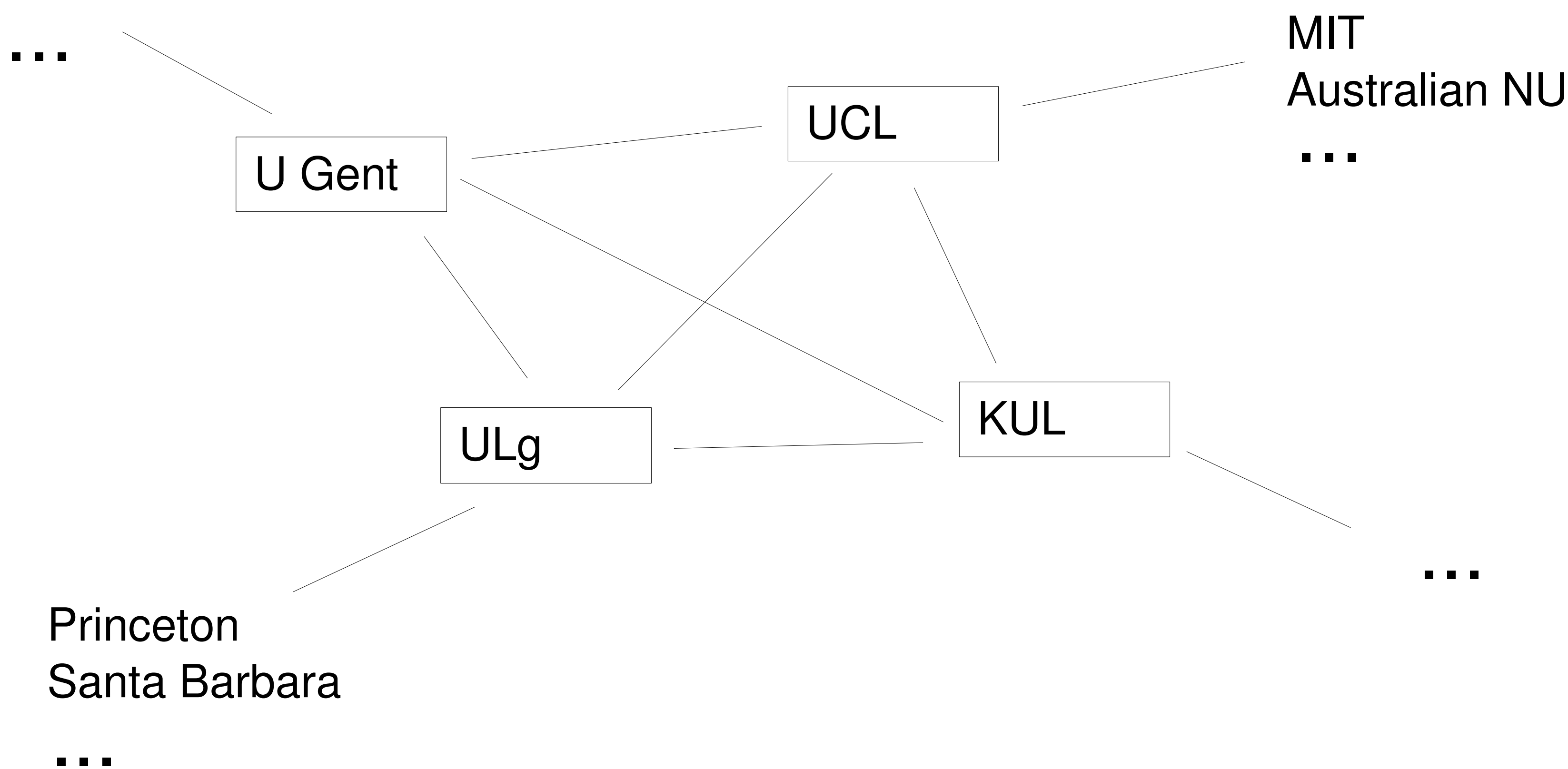
Teams involved:

- UCL/INMA
- KUL/ KUL1 & KUL2
- UGent/SYSTeMS
- ULB
- ULg/SYST

Main themes:

- Multi-agent systems
- Control and optimization
- Infinite dimensional systems
- Nonlinear control applications

Workpackage 2 > Multi-agent systems



Complementary expertise

U Gent		
	Distributed state estimation	Clustering phenomena
	Platoons and formations	Heterogeneous agents
ULG		
	Distributed power systems	Synchronization / clustering
	Coordinated motion design	Nonlinear spaces, integrate & fire oscillators
KUL		
	Communication networks	Organizing clusters
		Chaotic oscillators, Hodgkin-Huxley
UCL		
	Distributed controller synthesis	Information flow
		State-dependent networks

Diverse levels of interaction

Workshop

[Synchronization in Complex Networks](), KUL (J. Suykens), 17/10/2008
with international & IAP partners.

Seminars exchanging speakers among UCL, ULg, KUL

Coordinated research on synchronization & clustering

UGent: Aeyels
Heterogeneous agents
De Smet PhD < 2008

UCL: Blondel
State-dependent graph
Hendrickx PhD = 2008

ULg: Sepulchre
Nonlinear spaces, int.&fire
Sarlette PhD > 2008

Success story I > Global consensus behavior on the circle

U Gent	Kuramoto model $\frac{d}{dt}\theta_k = \omega_k + \alpha \sum \sin(\theta_j - \theta_k)$	Consensus in $\mathbb{R}$ $\frac{d}{dt}x_k = \sum a_{jk}(x_j - x_k)$
UCL	Influence of networks, interaction graphs, information flow	
ULg	Coordinated motion: agree on steering angle θ_k, trajectory curvature ω_k	

<u>Consensus on the circle</u> $$\frac{d}{dt}\theta_k = \omega_k + \sum a_{jk} \sin(\theta_j - \theta_k)$$

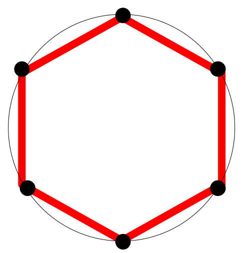
<u>Q: Global synchronization behavior ?</u>

Success story I > Path A : simple dynamics, full geometry

ULg

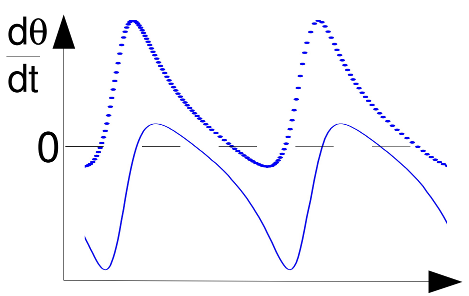
$$\frac{d}{dt}\theta_k = \sum a_{jk} \sin(\theta_j - \theta_k)$$

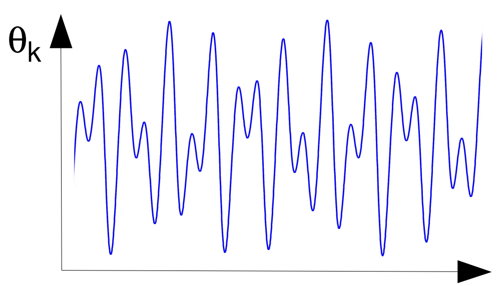
-local equilibria with undirected graphs
-alternative control laws for global synchronization
-extension to compact homogeneous manifolds



UCL Q: **directed** information flow

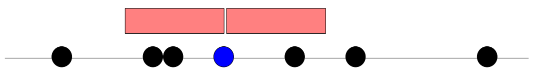
ULg A: stable nonstationary behavior

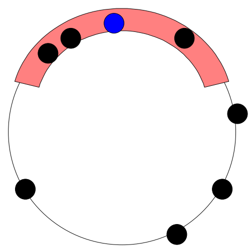




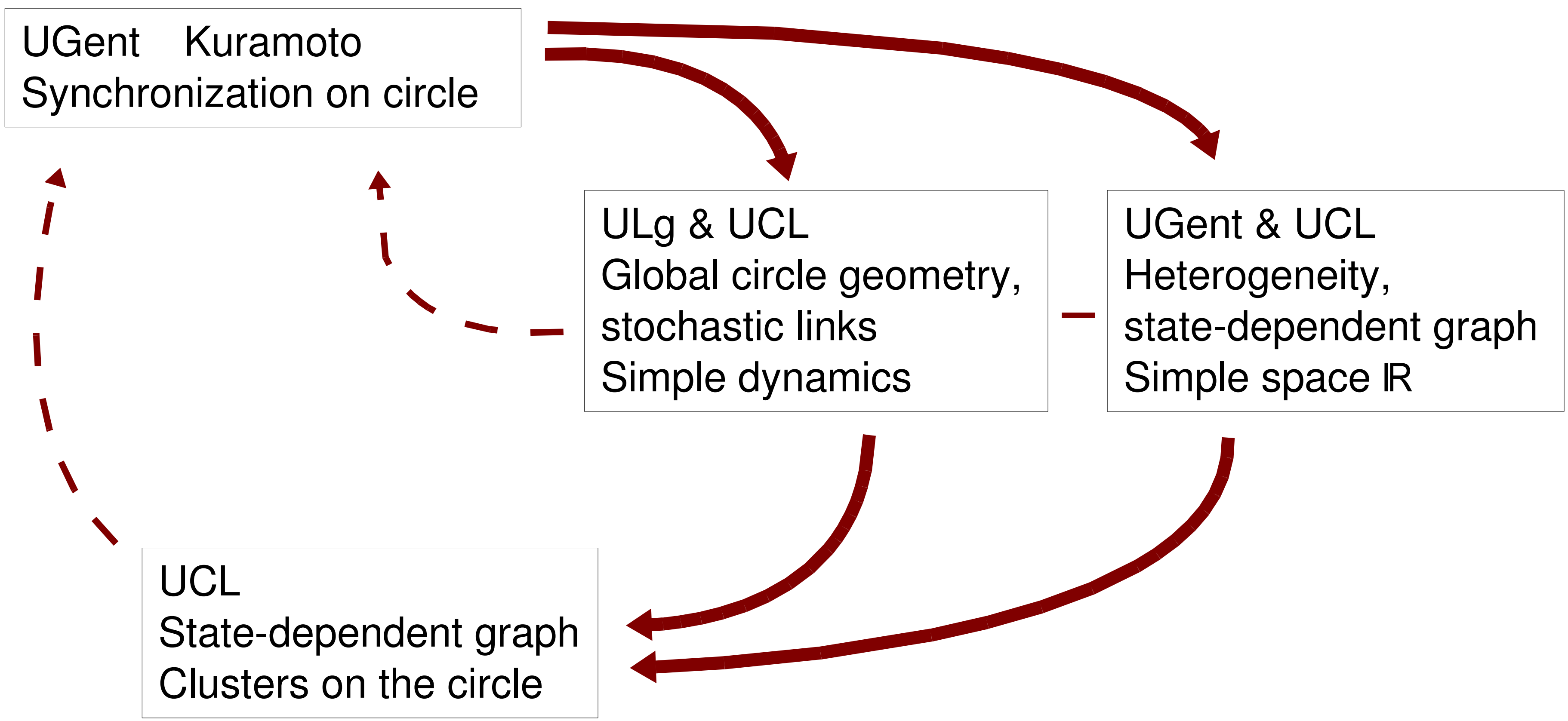
ULg+UCL Global synchronization by stochastic “gossip” communication selection

Success story I > Path B : complex interaction, linear space

UGent	Heterogeneous model: $\omega_k \neq \omega_j$	
	Motion synchronization phenomena for Nonlinear interactions on $\mathbb{R}$	
UCL	State-dependent interaction graph	
	Cluster formation “Krause” model on $\mathbb{R}$	

UCL (ULg)	State-dependent interaction graph Cluster formation on the circle	

Success story I > A closed loop of research



Success story II > Clustering in integrate & fire oscillators

Kuramoto oscillators: continuous, sinusoidal coupling

ULg — Quadratic integrate & fire oscillators with impulsive coupling

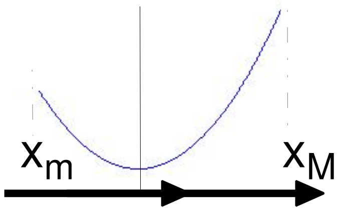
neuron

firing

WP5

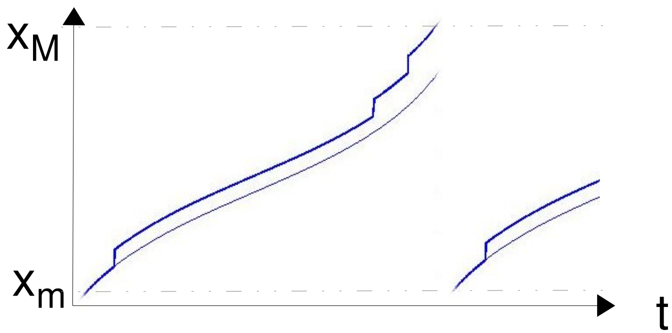
flow for $x_k \in [x_m, x_M)$

$$\frac{d}{dt}x_k = S + x_k^2$$



reset & fire for $\exists k : x_k = x_M$

$$x_k(t_+) = x_m$$
$$x_{i \neq k}(t_+) = x_i(t) + \epsilon$$



Success story II > Clustering in integrate & fire oscillators

ULg
A. Mauroy

Conjecture: If $S > 0$, $\epsilon > 0$ and $x_m+x_M > 0$
then there is a **stable** solution with separate traveling clusters

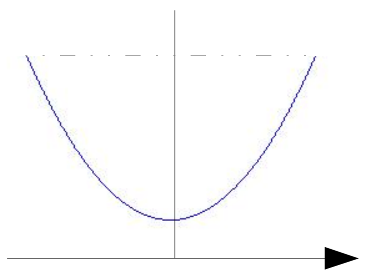
Proof: OK for $x_m > 0$

ULg&UCL

For $x_m+x_M = 0$: charact. eq. of the Jacobian
is a palindromic polynomial

$$P(z) = \sum e_k \left(z^k + z^{N-k}\right)$$

$$0 < e_{k+1} < e_k \leq e_0 = 1$$



Hope: prove marginal stability of cluster solution for $x_m+x_M = 0$

Success story II > Clustering in integrate & fire oscillators

UCL&MIT Prof. A.Megretski (MIT) gets the problem through postdoctoral stay from J.Hendrickx, UCL.
Proof: palindromic polynomial has all zeros on the unit circle

ULg Quadratic integrate & fire oscillators feature cluster solutions with

marginal stability for $x_m+x_M = 0$

stability for $x_m+x_M > 0$ if $x^2+\epsilon x - S < 0$ on $[x_m, x_M]$

stability for $\frac{d}{dt}x_k = \exp(x^2)$

Ongoing research: reduce conditions in the proof

Success story II > An open network of research