The universal 2-category containing n-composable monad morphisms and its associated nerve.

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We start by reviewing two explicit descriptions of the universal 2-category containing an adjunction, by Schanuel and Street [1] and by Riehl and Verity [2]. We describe the isomorphism between both models explicitly and in a topological way.

Building on both models, we introduce an explicit model for the universal 2-category containing n-composable (lax) monad morphisms. These 2-categories can be assembled together to form a cosimplical object in simplicial categories.

The associated nerve $N_{\mathbf{Mnd}} : \mathbf{sCat} \longrightarrow \mathbf{sSet}$ associates to a 2-category $\mathcal{C}$ the category whose objects are monads in $\mathcal{C}$ and whose morphisms are (lax) monad morphisms, following Street in [3]. We conjecture that if $\mathcal{C}$ is now a simplicial category whose homspaces are quasicategories, its nerve $N_{\mathbf{Mnd}}(\mathcal{C})$ is a quasicategory.

References

[1] Stephen Schanuel and Ross Street, *The free adjunction*, Cahiers Topologie Géom. Différentielle Catég. 27(1):81–83, 1986.

[2] Emily Riehl and Dominic Verity, *Homotopy coherent adjunctions and the formal theory of monads*, arXiv:1310.8279, 2013.

[3] Ross Street, *The formal theory of monads*, J. Pure Appl. Algebra, 2(2):149–168, 1972.