Reconstruction of Hopfish algebras

Joost Vercruysse *

Bialgebras are algebras that also have a coalgebra structure satisfying an appropriate compatibility condition; Hopf algebras are bialgebras possessing an *antipode*. It is well-known that bialgebras (resp. Hopf algebras) over a commutative base ring k can be characterised as those k-algebras whose category of modules is monoidal (resp. closed monoidal) such that the forgetful functor to k-modules is strict monoidal (and resp. preserves internal Homs). Motivated by examples arising from non-commutative geometry (especially related to the so-called "quantum torus"), Tang, Weinstein and Zhu introduced a generalization of Hopf algebras which they termed "Hopfish algebras"; roughly in their work the coalgebra and antipode structure morphisms are replaced by bimodules.

Although the equivalent notion of a bialgebra in this new setting is quite obvious, giving a suitable definition of the antipode (that is, defining a Hopf-like object) is more tricky. We advocate another definition than the one which was proposed initially by Tang, Weinstein and Zhu, and show that our notion is invariant under arbitrary Morita equivalence and an appropriate category of representations becomes rigid, which allows an internal characterization and a reconstruction theorem for Hopfish algebras.

References

[1] X. Tang, A. Weinstein, C. Zhu, *Hopfish algebras*, Pacific J. Math. 231 (2007), 193–216.

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