

A looping-deloooping adjunction for topological spaces

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In [1], Farjoun and Hess introduced *twisted homotopical categories*, a framework for monoidal categories that come with a looping-deloooping adjunction, in which a formal theory of bundles is available. This recovers twisted cartesian products for simplicial sets, and twisted tensor products for chain complexes. Although much of this kind of structure was inspired by classical constructions and results holding for topological spaces, it does not seem possible to construct a full twisted homotopical structure that recovers principal bundles in spaces. However, we provide a *weak twisted homotopical structure*, by showing that the relation between the loop space functor that Milnor introduced in [2] and the classifying space functor is a sort of adjunction between pointed spaces and topological groups. The argument leads to a classification of principal bundles over a fixed space, as a dual version of the well-known classification of bundles with a fixed group. Such a result clarifies the deep relation that exists between the theory of bundles, the classifying space construction and the loop space, which are very important in topological K -theory, group cohomology and homotopy theory.

REFERENCES

- [1] Farjoun, and Hess, *Normal and conormal maps in homotopy theory*, Homology Homotopy Appl. Vol. 14, Number 1 (2012), 79–112.
- [2] Milnor, *Construction of universal bundles, I*, Annals of Mathematics, Second Series, Vol. 63, Number 2 (1956), 272–284.