Equivariant Euler characteristics

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Let $\mathcal{C}$ be a finite category and A a group acting on it. Under appropriate assumptions the equivariant Euler characteristics $\chi_r(\mathcal{C}, A)$ are defined for all $r \geq 0$. The significance of these invariants are unclear in general. However, in the very special case described below they seem to carry some rather interesting information.

Let G be a finite group, p a prime number. Then $\mathcal{C} = \mathcal{S}_G^{p+*}$, the Brown poset of nontrivial p-subgroups of G, is a G-poset under the conjugation action. The Quillen conjecture, $|G|\widetilde{\chi}_0(\mathcal{S}_G^{p+*}, G) = 1 \iff O_p(G) \neq 1$, Webb's theorem, $\widetilde{\chi}_1(\mathcal{S}_G^{p+*}, G) = 0$, and the Knörr-Robinson formulation of Alperin's Weight Conjecture, $\widetilde{\chi}_2(\mathcal{S}_G^{p+*}, G) = z_p(G)$, are all statements about equivariant Euler characteristics. There is at present no interpretation of the higher equivariant Euler characteristic $\chi_r(\mathcal{S}_G^{p+*}, G)$ for $r \geq 3$.

Equivariant Euler characteristics are defined in terms of Euler characteristics of centralizer subcategories. In my talk I'd like to present results from my preprint [1] containing an analysis of centralizer subcategories mainly of subgroup categories associated to finite groups.

References

[1] J.M. Møller, *Euler characteristics of centralizer subcategories*, arXiv:1502.01317.