

# Torsion Theories for Crossed Modules

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Following [3], in a category  $\mathbf{C}$  with pullbacks and pushouts, we consider a full replete subcategory  $\mathbf{C}_0$ , both reflective and coreflective in  $\mathbf{C}$ , as the class of null objects in  $\mathbf{C}$ . Accordingly, we define constant morphisms, kernels and cokernels (with respect to  $\mathbf{C}_0$ ). We can then extend the well known notion of torsion theory  $(\mathcal{T}, \mathcal{F})$  given in a pointed category in [4] (previously introduced in [2] for the abelian case) by using exact sequences (with respect to  $\mathbf{C}_0$ ). This general context includes factorization systems  $(\mathcal{E}, \mathcal{M})$  in  $\mathbf{C}$  as torsion theories in the category of arrows  $\text{Arr}(\mathbf{C})$  with respect to its subcategory  $\mathbf{C}$ . In particular, in the category  $\text{XMod}(\mathbf{C})$  of crossed modules in a semi-abelian category  $\mathbf{C}$ , if we consider as null objects the trivial crossed modules  $H \rightarrow H$  given by the identities with the conjugation action, we find that central extensions form a torsion class, whose corresponding torsion free class is given by normal subobjects. We will see how this torsion theory is related to the (pointed) torsion theory in  $\text{XMod}(\mathbf{C})$  studied in [1] given by the same torsion free class of normal subobjects, but with abelian objects as corresponding torsion class.

## REFERENCES

- [1] Bourn, D. and Gran, M., *Torsion theories in homological categories*, J. Algebra 305 (2006) 18–47.
- [2] Dickson, S. E., *A torsion theory for Abelian categories*, Trans. Amer. Math. Soc. 121 (1966) 223–235.
- [3] Grandis, M., Janelidze, G. and Márki, L., *Non-pointed exactness, radicals, closure operators*, J. Aust. Math. Soc. 94 (2013) 348–361.
- [4] Janelidze, G. and Tholen, W., *Characterization of torsion theories in general categories*, Categories in algebra, geometry and mathematical physics, 249256, Contemp. Math., 431, Amer. Math. Soc., Providence, RI, 2007