The categorical origins of entropy

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Entropy is fundamental to many parts of mathematics (such as dynamical systems, information theory and probability theory) as well as many branches of applied science, but it is not often considered by the kind of people who attend workshops on category theory and algebraic topology. However, I will show that the concept of Shannon entropy is present in the heartlands of pure mathematics, whether we like it or not.

Specifically, I will describe a categorical machine which, when fed as input the concepts of topological simplex and real number, produces as output the concept of Shannon entropy. The most important component of this machine is the notion of "internal algebra" in an algebra for an operad (generalizing the notion of monoid in a monoidal category). The resulting characterization of Shannon entropy can be stripped completely of its categorical garb, to obtain a simple and entirely elementary characterization. This last theorem is joint work with John Baez and Tobias Fritz (arXiv:1106.1791).