Hochschild cohomology, lax descent, and paracyclic structure

Steve Lack *

Thanks to the universal property of the simplex category Δ, every comonad gives rise to a(n augmented) simplicial set, and one can then linearize and normalize to obtain a chain complex. Many homology theories arise in this way.

On the other hand cyclic homology involves a simplicial object together with an action of the cyclic group in each degree, subject to various conditions. The resulting structure is called a cyclic object, although it turns out that more general things called paracyclic objects can also be used. The fundamental example of a cyclic object is the Hochschild complex of a ring, where in each degree n we have the n-fold tensor power of the ring, and the cyclic group acts on this by cyclically permuting the factors.

Generalizing the classical construction of a simplicial object from a comonad, Böhm and Stefan have shown how to construct paracyclic structure from a pair of comonads with a distributive law between them, along with certain further ingredients.

I shall describe the universal nature of this construction of paracyclic structure, using a 2-categorical version of Hochschild cohomology in which the construction becomes the cap product.

*Joint work with Richard Garner and Paul Slevin