

Relative homotopy theory

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Characterization of presheaf categories is a classical problem in category theory. Simple criteria determining if a category is equivalent to a category of presheaves were found by M. Bunge in her Ph.D. thesis.

A similar question in homotopy theory was treated by W.G. Dwyer and D. Kan. They introduced the concept of a *small* subcategory \mathcal{O} of *orbits* in a simplicial category \mathcal{M} , such that \mathcal{M} equipped with a *set* of orbits carries a model structure Quillen equivalent to the simplicial presheaves $\mathcal{P}(\mathcal{O})$ with the projective model structure.

In this talk we will present a generalization of Dwyer-Kan theorem. We will show that a category \mathcal{M} equipped with a locally small *class* of orbits is Quillen equivalent to the category of *relative* simplicial presheaves $\mathcal{P}(\mathcal{M}, \mathcal{O})$.

As an application we show that the equivariant model structure on the category of diagrams of spaces, introduced by E. Dror Farjoun, is Quillen equivalent to the category of *relative* simplicial presheaves, even though the orbit category is no longer small, extending similar result about Bredon homotopy theories. It also extends our previous result with Dwyer that the category of maps of spaces with the equivariant model structure and the contravariant functors from spaces to spaces are Quillen equivalent.