

# Categorical groups: a bit of theory and some applications to homological and homotopical algebra

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The aim of this mini-course is to introduce categorical groups as a topic of research, interesting in and of itself, and to show that they are a higher dimensional gadget useful for studying familiar constructions and for understanding better certain classical results in (homological) algebra.

In the first lesson, I will introduce the notion of a categorical group with several examples from homological algebra, ring theory, algebraic topology and algebraic K-theory. I will also discuss the links among categorical groups, internal groupoids in groups, and group extensions.

The second lesson will be devoted to the theory of symmetric categorical groups, starting from strong homotopy kernels and cokernels and ending at long exact sequences of homology categorical groups, which leads to the axiomatic notion of an abelian 2-category.

In the third lesson, two applications of the higher dimensional point of view introduced in Lessons 1 and 2 will be discussed. First, I will show that strong homotopy kernels reveal the Snail Lemma, a generalization of the Snake Lemma that remains completely hidden if we look at the Snake Lemma from the classical “1-dimensional” point of view. Second, we will see that Sinh’s homotopical classification of categorical groups subsumes the Mac Lane - Schreier theory of group extensions.

If you wish to enjoy this mini-course, please come armed with basic knowledge of monoidal categories, 2-categories, and internal categories. Sections 7.1, 7.2 and 8.1 of [1] and Sections 6.1 and 6.4 of [2] cover all the needed material. Monoidal categories and 2-categories are also covered by [3], which will be made available to the participants.

## REFERENCES

- [1] Francis Borceux, *Handbook of Categorical Algebra 1*, Cambridge University Press (1994).
- [2] Francis Borceux, *Handbook of Categorical Algebra 2*, Cambridge University Press (1994).
- [3] Pierre-Alain Jacqmin, *Classification of Cat-Groups*, Master Thesis at UCL (2013).

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\*Joint work with a lot of good friends.