Semi-abelian categories, semi-localizations and torsion theories

Marino Gran

Semi-abelian categories [10] provide a suitable axiomatic context to study the (co)homology of non-abelian algebraic structures (such as groups, compact groups, crossed modules and Lie algebras), torsion and radical theories, and commutators. In this mini-course a brief introduction to some elementary properties of these categories will be given, before focusing on recent developments concerning torsion theories [1, 4, 6, 9]. In particular the relationship between torsion theories and semi-left-exact reflections in the sense of [3] will be explained, as well as the connections with radicals, closure operators, categorical Galois structures, and factorizations systems [2]. Some recent work concerning semi-localizations of semi-abelian categories [9], and the relationship with some investigations on reflective subcategories of the category of groups [5] will also be discussed, leading to some interactions with the mini-course given by Jérôme Scherer.

Concerning the preliminaries to follow these lectures, some basic knowledge of the properties of regular categories and of abelian categories will be helpful. Most of the needed categorical background can be found in the notes [8], and in the second chapter of [7].

REFERENCES

[1] D. Bourn and M. Gran, *Torsion theories in homological categories*, Journal of Algebra 305 (2006) 1–10.

[2] A. Carboni, G. Janelidze, G.M. Kelly, and R. Paré, *On localization and stabilization of factorization systems*, Appl. Categ. Structures 5 (1997) 1–58.

[3] C. Cassidy, M. Hébert, and G.M. Kelly, *Reflective subcategories, localizations and factorizations systems*, Journal Australian Math. Society, 38 (1985) 287–329.

[4] M. M. Clementino, D. Dikranjan, and W. Tholen, *Torsion theories and radicals in normal categories*, Journal of Algebra 305 (2006) 98–129.

[5] E. Dror Farjoun and J. Scherer, *Conditionally flat functors in homotopy theory and in group theory*, Preprint (2013).

[6] T. Everaert and M. Gran, *Monotone-light factorisation systems and torsion theories*, Bull. Sciences Mathématiques (2013) 996–1006.

[7] P.J. Freyd, *Abelian categories. An introduction the the theory of functors,* Harper's Series in Modern Mathematics (1964), available online as a Reprint in Theory and Applications of Categories, at the address http://www.tac.mta.ca/tac/reprints/articles/3/tr3abs.html

[8] M. Gran, *Notes on regular, exact and additive categories*, available for the participants in the Summer School on Category Theory and Algebraic Topology, EPFL, 11-13 September 2014.

[9] M. Gran and S. Lack, *Semi-localizations of semi-abelian categories*, Preprint (2014) arXiv:1405.1092

[10] G. Janelidze, L. Márki and W. Tholen, *Semi-abelian categories*, Journal of Pure and Applied Algebra, 168 (2002) 367–386,