Categorification in Representation Theory

Vanessa Miemietz

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Explicitly: $\bullet \mapsto V, \qquad \mathrm{End}_{\mathcal{A}}(\bullet) \ni a \mapsto \rho(a) \in \mathrm{End}_{\mathcal{V}ect_{\Bbbk}}(V).$

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vector space	dimension
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Categorification: the opposite - not constructive!

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$\rightsquigarrow$ Categorification in representation theory.

Why?

More information in the higher structure
$\rightsquigarrow$ new information about the decategorified object;
now have additional information about natural transformations of these functors.

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Examples in representation theory

- categorification of Kac–Moody algebras *[Khovanov–Lauda, Rouquier]* ($\rightsquigarrow 4$-dimensional topological quantum field theories (TQFT)?)
- categorification of Heisenberg algebras *[Khovanov]*
- categorification of Lie superalgebras *[Brundan–Stroppel]*
- categorification of Hall algebras (for cyclic quivers) *[Stroppel–Webster]*
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$\rightsquigarrow$ proof of Broué's abelian defect group conjecture for symmetric groups, proof of Kazhdan–Lusztig conjectures for all Coxeter systems, counterexample to James' conjecture for Hecke algebras, counterexamples to (and refinements of) Lusztig's conjectures

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- Algebras often appear as convolution algebras of functions on certain spaces.
- Example: Hecke algebra $\mathcal{H} := \mathrm{Fun}_{B \times B}(G, \mathbb{C})$

  G conn. red. alg. group (e.g. GL_n), B Borel (e.g. $\left\{\begin{pmatrix} * & * & * \\ & \ddots & * \\ & & * \end{pmatrix}\right\}$)

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Issue: Difficult to work with, so find more algebraic descriptions.

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- the 2-category $\mathfrak{A}_{\Bbbk}^{f}$ whose
  - objects are small idempotent complete $\Bbbk$-linear additive categories with finitely many indecomposable objects up to isomorphism and finite-dimensional morphism spaces
    (that is, equivalent to the category of finitely generated projective modules over a finite-dimensional $\Bbbk$-algebra);
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- it is finitary;
- there is a weak involutive equivalence $(-)^*\colon \mathscr{C} \to \mathscr{C}^{\mathrm{op},\mathrm{op}}$ such that there exist adjunction morphisms $F \circ F^* \to \mathbb{1}_{\mathtt{i}}$ and $\mathbb{1}_{\mathtt{j}} \to F^* \circ F$.

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Fact: $\mathscr{S}$ is fiat (for W finite) and categorifies the Hecke algebra.

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Goal. Classify simple 2-representations for interesting 2-categories.

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H-cells: intersections of left and right cells

Example

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Example. $W = \langle s, t | s^2 = 1 = t^2, stst = tsts \rangle$ of type $B_2 = C_2$. Cells are

1

s, sts	st
ts	t, tst

$stst$

$\mathcal{H}$-cell reduction

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In the example, take $\mathcal{H} = \{\theta_s, \theta_{sts}\}$, then $\mathscr{S}_{\mathcal{H}}$ has cell structure

$$\boxed{\mathbb{1} = \theta_1}$$

$$\boxed{\theta_s, \theta_{sts}}$$

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Theorem 1. [Mackaay–Mazorchuk–M–Zhang] There is a bijection

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Upshot: concentrate on $\mathscr{C}_{\mathcal{H}} \rightsquigarrow$ smaller! We call this **$\mathcal{H}$-cell reduction**.

Representations of Hecke algebras

[Lusztig]: (W, S) Coxeter group

$\mathcal{H}$ a two-sided cell or diagonal H-cell $\rightsquigarrow$ **asymptotic algebra** $\mathrm{A}_{\mathcal{H}}$ (via $q \to 0$)

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Idea: Asymptotic algebras are easier to understand.

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Proposition. $\mathscr{E}nd_{\mathscr{S}_{\mathcal{H}}}(\mathbf{C}) \cong \mathscr{A}_{\mathcal{H}}$.

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Combining Theorems 1 and 2, this yields

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- completes classification in all finite Coxeter types apart form H_3, H_4
- for few H-cells in types H_3, H_4, $\mathscr{A}_{\mathcal{H}}$ is not well-understood

Thank you!

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