Special Session in memory of F.W. Lawvere
CT 2023 Louvain-la-Neuve

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Thank you to the
organizers;

and to

M.M. Clementino and J. Picado
for making the interview with Lawvere in 2007 (Braga, Portugal), including the picture:

and to

Fatima and Danilo Lawvere
for friendship and help

CT 1987 in Louvain-la-Neuve and Leuven

Lawvere:

"I dedicate this talk to the fundamental unity of the Indoeuropean languages"

Unification

The unification of mathematics is an important strategy for learning, developing and using mathematics

(Lawvere and Schanuel, in “Conceptual Mathematics” 2009)

From topology to algebra; steps in the unification

Algebraic topology; homology functors

$$\text{Top.Spaces} \rightarrow \text{Abelian groups}$$

Later

$$\text{Simplicial sets} \rightarrow \text{Abelian groups};$$

A simplicial set itself as a functor $\Delta^* \rightarrow$ Sets;
adjoint functors.

From Categories to Category Theory

Lawvere:
The Category of Categories as a **Foundation** for Mathematics
(La Jolla 1965)

The past 100 years' tradition of "foundations" as justification has not helped mathematics very much. In my own education I was fortunate to have two teachers who used the term "foundations" in a common-sense way . . . :

Foundations of Algebraic Topology, published in 1952 by Eilenberg (with Steenrod), and the Mechanical Foundations of Elasticity and Fluid Mechanics, published in the same year by Truesdell.

Category of Sets

…I reasoned that Grothendieck's theory of Abelian categories should have a non-linear analogue whose examples would include categories of sheaves of sets; …and noted that, on the basis of my work on the category of sets, such a theory would have a greater autonomy than the Abelian one could have.

Mac Lane books:

Homology 1963

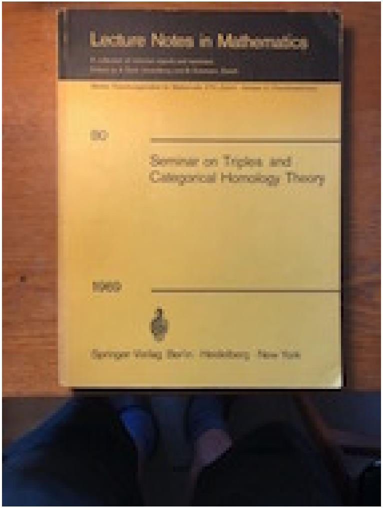
Categories for the Working Mathematician 1971.

The title of the latter indicates that in 1971, category theory is seen not just as a tool for Algebraic Topology/Homological Algebra, but for <u>every</u> working mathematician.

The slogan is "Adjoint functors arise everywhere".
(Mac Lane 1971)

Zürich 1966

Eckmann's seminar 1966-67



(Triple = Monad = (T, η, μ))

Cafe Odeon 1966

1967-1973

1967-1969: Chicago
1969-1971: Halifax. Not renewed due to political activities
1971-1972: Aarhus
1972-1973: Perugia
1973-2023: Buffalo

Categorical Dynamics 1967

Tangent bundle of M:

$$T(M) = M^D$$

M a object (space) in an unspecified Cartesian closed category, D a postulated infinitesimal object.

Since the late 1970s "Synthetic Differential Geometry": postulate further a commutative ring R, ("the line"), and require $R \times R \to R^D$ to be an iso; then R is called "a ring of line type".
Now develop axiomatically;
in a topos, with the insight brought about from Elementary Toposes: "just argue as if the topos were SETS"

Hanne, Anders, Bill, Fatima

Marie, Danilo, Silvana, Joachim, Hans

Aarhus Open House, Summer of 1973, 1978, 1983

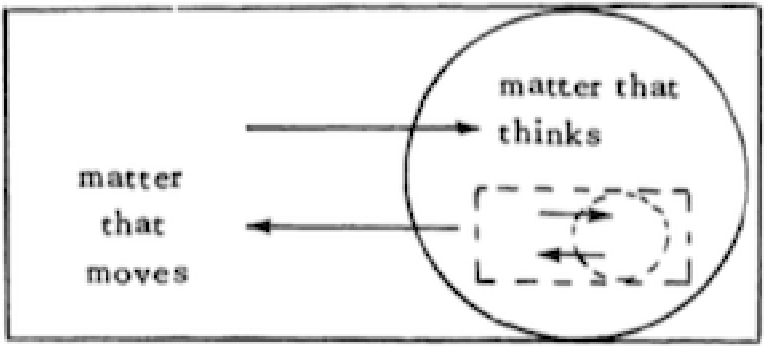
"TOPOS THEORETIC METHODS IN GEOMETRY", Aarhus Math. Institute, Various Publication Series No. 30 (1979).

Among the scheduled events were "Discussion on philosophy", with Joyal and Lawvere (780516 and 780517), as well as an elaboration (780524).

Some of Lawvere's contributions in this discussion are contained in Cahiers 1980, with title

TOWARD THE DESCRIPTION IN A SMOOTH TOPOS OF THE DYNAMICALLY POSSIBLE MOTIONS AND DEFORMATIONS OF A CONTINUOUS BODY

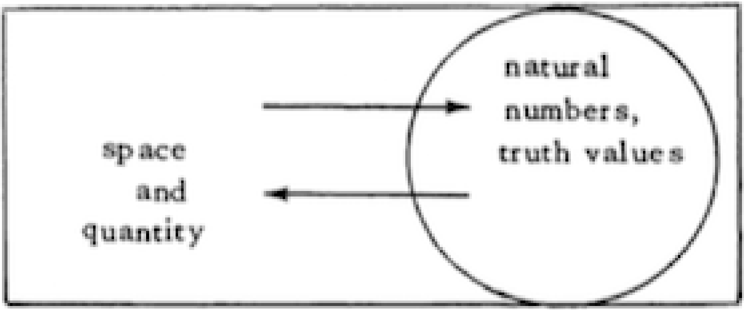
Scientific world picture



(Cahiers 1980)

Mathematical world picture

Mathematics: studying space and quantity and their relations
Logic of mathematics: constructing concepts and reasoning for this



I agree with Brouwer that $\mathbb{N}$ is subjective, in contrast to the continuum which is objective (quote from 780517)
In the circle may add: continuum.

The continuum

the continuum .. [is] primarily a **concept** derived **directly** from our historical-scientific experience with the world of matter-in-motion

not only a mental from construction $\mathbb{N}$ and Ω: (the subjective idealizations of iteration and truth respectively)

A model for the continuum *may* be constructed (Dedekind, Cauchy, ...) from $\mathbb{N}$ and Ω and the denoted $\mathbb{R}$. Thus: subjective, and leads to to space filling curves, which do not exists.
[But] space is here now (quote from 780517)

Space vs. Quantity

(From 780524)

The axioms of Euclid are not so bad.
Problems come when making them rigorous. Nobody would use Hilbert's book for a high school text book.

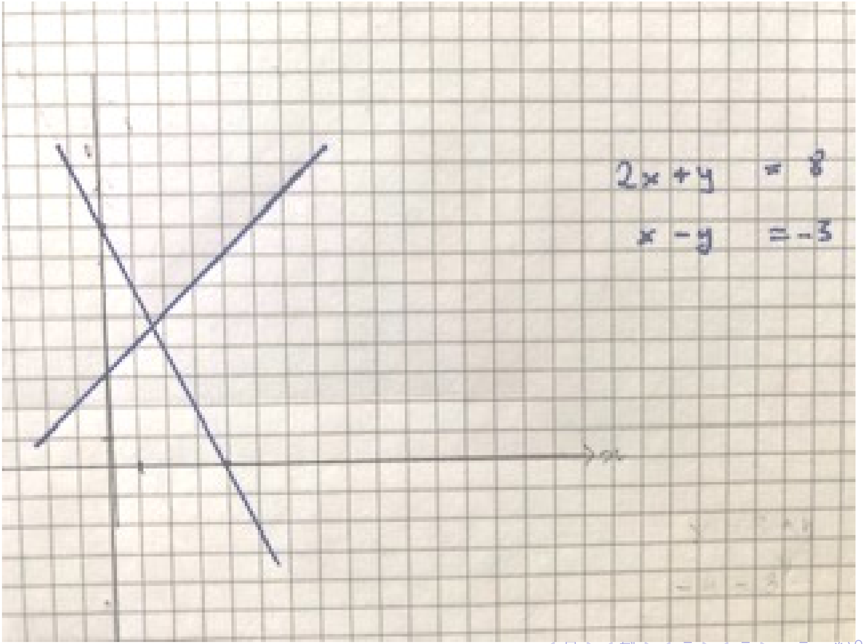
Math: study of space forms and quantitative relations, and their relationship. Cannot we take this idea directly and arrive at axioms for mathematics. Slightly schematically

(Basic variable quantity: distance).

This picture is not good enough to be an end. Because there are, e.g., distinct space forms.

Roughly:
S a topos of Spaces
Q a Picard closed additive category of physical dimensions (with objects types of quantities: length, area, volume, mass, time-lapse, pure quantity (ratios of quantities of same physical dimension, like π)
linear maps

Space vs. Quantity



Conceptual Construction vs Formal Deduction

Inside the logic of mathematics, have a non-antagonistic contradiction,
LCC: Logic of Conceptual Construction
LFD: Logic of Formal Deduction
Nearly every piece of mathematics involves usually an advance in both.
LFD is not the whole thing: this idea "*waves the flag of LFD to oppose LCC", especially the purpose of math., (clarifying, guiding,)* (from 780517).

LFD is overestimated, LCC underestimated

Concepts

The existence of Euclidean Geometry is more important than any specific Theorem in it.

The fact that there must be a Continuum is more important than the way you "construct" it.

Euclidean geometry was from the outset based on the **concepts** plane, line, point, - derived from experience from the real world, say, architecture. "Line" a concept derived from taut strings, typically made of linen, whence the name.

Grothendieck

Grothendieck was in Buffalo in the spring of 1973, and gave a series of colloquium talks.

Lawvere coming back from Perugia only in time for the last of these.
According to Michael Wright:
Alas, none of Gs talks to the Buffalo Math Colloquium (as distinct from the Lecture Courses) were recorded and Bill attended and made notes only of the last talk. . . . Bill gave an account of this Talk in the course of the discussions held in Fougeres in 2005, which are now (finally) in the course of being transcribed.
The talk contained
. . . an overarching view of how G saw Narrow-sense logic as fitting within (broad-sense) Geometry what Bill referred to in passing as "Grothendiecks Vision of how to by-pass Logic".

Good cocepts imply fewer proofs.
LCC over LFD.

Schanuel

Photo: R.Walters, Sydney 1988

Schanuel and Lawvere worked together on a daily and private basis for decades. In the Buffalo winters, they (with Fatima) went together to Oxaca in Mexico. - Lawvere writes in an obituary for Schanuel 2014:

We studied deep into the nights; sometimes Fatima heard us giggle, because we had discovered how simply some results could be proved. In the morning she typed the notes that I had left on her table.

Danilo Lawvere:

She [Fatima] made sure that dad went to conferences with notes or drafts, and she ensured that deadlines were met, letters were answered as well as all the extensive class notes we have. *Conceptual Mathematics* [with Schanuel, 2009] was in part conceived by her, and she was the driving force in getting it done, and then published.

Archives; Danilo Lawvere

Danilo:

There is a lot to do with respect to the archive, but my mom [Fatima Lawvere] sure did prepare the way.

First, I want to say that the plans for archiving and making available all of my dads work is already underway, and it is something I will be working on for several years.

Some of the first steps will be ensuring that the materials on the website have links, then move toward unpublished works, and then comments to TAC, selected correspondence etc. I may be able to get some assistance from the math department here and am looking at the possible places to put the papers (possibly University of Buffalo).

Dads website we are planning to update
`https://www.acsu.buffalo.edu/~wlawvere/`

Archives; links (url)

Michael Wright has recorded and scanned a lot of material from Lawvere, including scannings of notes I took from Lawveres lectures over the years. These scannings will be made public on his
https://archmathsci.org/category/mathematics/
I have made some of these scannings of my notes available, e.g.
https://users-math.au.dk/kock/780524.pdf
where "780524" is the date of the lecture. Also 780516, 780517, 780518, 780519
Braga Interview (M.M. Clementino and J. Picado interviewing Lawvere in 2007):
http://www.mat.uc.pt/ picado/lawvere/interview.pdf
I have an obituary article forthcoming in the Magazine of the EMS, partly based on the Braga Interview, "Lawvere: A lifelong struggle for the unity of mathematics.
Best source: https://www.acsu.buffalo.edu/~wlawvere/

Thank you !

“. . . out of fashion. . . ”

“Nevertheless there are areas which are out of fashion, for instance category theory and set theoretic topology, also mathematical logic, partially, judging from the number of posts”

Vasco Alexander Schmidt, inteviewing Matthias Kreck, in the chapter “The Unity of Mathematics”,
in Springer’s “Mathematics Unlimited - 2001 and Beyond”