ABSTRACT

Freezing is an important step in the manufacturing process of ice cream and sorbet, since the operating conditions have a strong influence on the micro-structure, and consequently on the sensorial attributes of the final product. This paper presents a model for a continuous sorbet crystallization process approach based on the moments model approach and its validation with experimental data. The moments model was used to characterize the influence of the operating conditions during sorbet freezing such as: the evaporation temperature of the refrigerant fluid, the mix flow rate and the dasher rotation speed, on the final product characteristics such as: ice crystal size and draw temperature.

The moments model includes heterogeneous nucleation and growth phenomena, has low computational requirements and is quite adapted for optimization or process control tasks.

1. INTRODUCTION

In conventional continuous ice cream freezing process, scraped-surface heat exchangers (SSHE) are used, with a refrigerant fluid vaporizing on the wall of the SSHE and ice crystals are formed and scraped off by rotating blades. To attain a smooth texture, a good creaminess, a high scoopability and a cooling sensation, an objective of this freezing process is to deliver a product containing ice crystals as small as possible.

A number of works in literature have been performed to study the role of size of ice crystals on microstructure and sensorial properties of ice cream and sorbets. For example Muse and Hartel, (2004) showed that ice crystal size affects the hardness, and an ice cream with large ice crystals is harder than one with small ice crystals. Russel et al (1999) have reported a clear correlation between the “delectability” of the ice-cream and the mean crystal size. The creaminess attribute is also partly related to the microstructure of ice cream, a high density of evenly sized particles resulting in a realistic sense of creaminess (Richardson et al. 1993).

To optimize this ice cream freezing process Inoue et al. (2007) established an empirical model of the ice cream process specific to the device and the mix recipe used based on empirical results and response surface approach. But this previous approach is not possible to predict the behaviour of any other formula or any other ice cream freezing process. In order to predict ice crystallisation kinetics, ice crystal size, concentration, draw temperature, it is necessary to have a better knowledge of the crystallization process coupled to the heat transfer inside the SSH.

In this paper a model of the crystallization of a sorbet in a scraped surface heat-exchanger including heat transfer, nucleation and growth phenomena is presented. It uses the moments approach to solve the
population balance equation. One of the main advantages of this model is its numerical efficiency which allows us to use it in optimization and process control.

2. MODEL FORMULATION

2.1. Crystallization model and moments method

Population balance modelling has been applied to a number of industrial processes and form an appropriate framework for the dynamic modelling of ice crystallisation process. The population balance equation (PBE) allows to describe the state of the system by means of a density function, and to take into account the different processes that control the density function such as breakage, coalescence, growth and convective transport of the particles. The resulting equation is a nonlinear partial derivative equation which requires a numerical method to be solved:

\[
\frac{\partial \psi}{\partial t} + \vec{v} \cdot \nabla \psi + \frac{\partial}{\partial L} \left( G \psi \right) = \frac{B}{\text{Birth}} - \frac{D}{\text{Death}}
\]

where \( t \) is time, \( \psi(L,x,y,z,t) \) is the density function (crystal distribution function), \( L \) is a characteristic dimension, \( \vec{v} \) the velocity vector, \( G \) the time rate change of the characteristic dimension, \( B \) and \( D \) the terms of formation of new crystals and removal of existing crystals respectively.

The method of moments is an alternative to the direct discretization approach for the numerical solution of the PBE. It transforms the PBE into a set of ordinary differential equations (Randolph and Larson, 1971). This method has modest computational requirements permitting to use it efficiently in control design. The final physical model in this case is a “reduced” and not a simplified one, since no assumptions on the physicochemical parameters are made and the result is very close to the complete one. The only restriction is the limited amount of the extracted information.

The moment \( j \) is defined as:

\[
M_j = \int_0^\infty L^j \psi(L,t) dL
\]

Multiplying equation (1) by \( L^j \) and integrating the both sides with respect to \( L \) from \( L=0 \) to \( L=\infty \) leads to the following equations for the evolution of the moments:

\[
\int_0^\infty L^j \left( \frac{\partial \psi}{\partial t} + \vec{v} \cdot \nabla \psi + \frac{\partial}{\partial L} \left( G \psi \right) - B + D \right) dL = 0
\]

The number of the moments finally used defines the number of state variables of the problem (plus the internal energy which is also a state variable).

2.2. Nucleation and growth terms

Cooling crystallization is a process in which the mechanisms of nucleation, growth and fragmentation occur in a competitive way and a numerical method suitable for solving the PBE should deal with these three mechanisms.

Homogeneous and heterogeneous nucleations on the evaporator wall have been considered to initiate the crystallization according to the following kinetics:

\[
B = \alpha D \left( T - T_s \right)^\varphi + \alpha' D \left( T - T_c \right)^\varphi
\]

(4)
where $D$ is the Dirac function, $\alpha$ is a coefficient independent on crystal length for the homogeneous primary nucleation law and $\alpha'$ is a coefficient also independent on crystal length for the heterogeneous primary nucleation on the evaporator wall and $S$ is the wall surface.

Growth term is expressed as a function of the difference between product temperature and saturation temperature:

$$G = \beta(T - T_s)$$

where $\beta$ is a growth rate constant, $T_s$ the saturation temperature and $\gamma$ a power law parameter in equation (5).

2.3. Final model

Based on assumptions and equations (4) and (5) for the right term, the final form of the equation (3) is given as:

$$\frac{dM_j}{dt} - jG \mu_{j-1} - L_e \dot{\gamma} B = 0$$

(6)

Considering the heat flux through the evaporator wall and the viscous dissipation as a heat source, a volumetric energy balance is finally added to the system of equations:

$$\frac{dM_0}{dt} - B = 0$$

$$\frac{dM_1}{dt} - GM_0 - L_e B = 0$$

$$\frac{dM_2}{dt} - 2GM_1 - L_e^2 B = 0$$

$$\frac{dM_3}{dt} - 3GM_2 - L_e^3 B = 0$$

$$\frac{du}{dt} = h_e S(T_e - T) + \mu \dot{\gamma}^2$$

(7)

where $h_e$ is the convective heat transfer coefficient between product and the evaporator wall (figure 2), $T_e$ is the evaporation temperature, $\mu$ is the product viscosity and $\dot{\gamma}$ is the shear rate. Model parameters are shown in table 1. Product viscosity was modeled by Thomas law (equation 8), and the solution viscosity $\mu_s$ was based on an empirical law based on experiments.

$$\mu = \mu_s \left(1 + 2.5\phi_g + 10.5\phi_g^2 + 0.00273 \exp(16.6\phi_g) \right)$$

(8)

Axial and radial dispersion are neglected and a plug flow is considered. To simulate the model, the five differential equations (7) are integrated for a period of time corresponding to the residence time distribution.

Table 1: Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$h_e$ [W/m²·C]</th>
<th>$\alpha$ [s⁻¹·m⁻³·K⁻¹]</th>
<th>$\alpha'$ [s⁻¹·m⁻³·K⁻¹]</th>
<th>$\beta$ [m·s⁻¹·K⁻¹]</th>
<th>$\delta$</th>
<th>$\delta'$</th>
<th>$\gamma$</th>
<th>$L_e$ [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>2400</td>
<td>0</td>
<td>7.6x10⁸</td>
<td>5x10⁻⁷</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

The model has been developed in MATLAB software. A typical simulation needs 0.05 second to be performed on a computer with a Core Duo 2GHz Intel processor.

3. MATERIAL AND METHOD
Freezing of a lemon sorbet mix was carried out in a laboratory scale scraped surface freezer, a WCB Technohoy® MF50. The ingredients of the lemon sorbet mix are: water, lemon juice, sucrose, fructose, xanthane gum and guar gum. With a 25.2°C Brix, the lemon sorbet mix density is 1110 kg/cm³ and its initial freezing point is -2.6°C.

The equipment used had a variable capacity from 25 to 75 kg/h. The inner diameter of the heat exchange tube of the freezer was 5cm and the length is 0.40cm (total volume of cylinder 0.785 cm³). Inside the cylinder (figure 1), there is a 'solid' dasher provided with two scraper blades mounted in a floating manner on the rotor (dasher volume = 0.351cm³). The factory settings allowed the variation of the dasher speed from 250 to 1000 rpm.

Figure 1: Dasher shape and blades disposition in the SSHE

Measurement of the ice crystal size distribution was performed immediately at the outlet of the freezer by a Mettler-Toledo Lasentec® Focused Beam Reflectance Method probe (FBRM Model S400A-8). The FBRM probe provides a laser back-scattering measurement. For this we insert the FBRM directly at the outlet pipe of the crystallizer, focusing a laser beam forward through a sapphire window at the probe’s tip, and collecting the laser light scattered back to the probe (figure 2). The FBRM measurement have been already applied to numerous pharmaceutical industrial processes and can be considered as an innovative tool which has almost never been used in food process for studying ice crystallization. It is the only method online that allows measurements at high concentration ice suspensions. We can follow ice crystal size distribution evolution during the freezing in a continuous freezer (A. Haddad and al., 2010).

Figure 2: FBRM probe

The draw temperature of the sorbet was measured by a calibrated Pt100 probe. The probe was inserted into the flow product at the outlet pipe of the freezer.
4. RESULTS

4.1. Influence of dasher speed

Dasher speed has a significant impact on the draw temperature by the frictional energy transmitted to the product. This influence is taken into account in the model by a variation of the viscosity and the viscous dissipation energy. The predicted draw temperatures and the measurement data, showing an increase of the temperature of 1°C when dasher speed is varied from 600 to 900 RPM, are plotted in figure 3. Indeed the increase of dasher speed alters the energy balance.

![Figure 3: Experimental data and model predictions of draw temperature vs dasher rotation speed (mix flow rate 35 kg/h, evaporation temperature -15°C)](image)

An expected effect due to dasher rotation speed on crystal size distribution is related to the reduction of time between two consecutive scrapes on the wall. Another expected effect is due to the increase of product temperature which tends to reduce the crystal growth and leads to smaller crystals, as it was observed by Sodawala and Garside (1997). In our case, no significant influence on the mean chord length as a function of the dasher speed has been observed experimentally. The average value for these experiments was around 6.4 µm.

4.2. Influence of evaporation temperature

Evaporation temperature is known to have a significant effect on ice crystal size and draw temperature. A reduction of the evaporation temperature directly increases the cooling rate which intensifies the nucleation rate and the growth rate. Usually the effect on nucleation is more significative than the growth effect. Consequently, the average diameter size tends to decrease, due to the higher number of small crystals. The effect on draw temperature is more intuitive: a reduction of evaporation temperature intensifies the heat transfer, the ice content increases, and consequently the draw temperature decreases significantly (figure 4).

Those two effects are well reproduced by the model. The experimental measurements show an important reduction of the chord length as a function of a decreasing evaporation temperature. The model also shows a tendency very similar to the experimental one (figure 5).
Figure 4: Predicted draw temperature and measured temperature as a function of evaporation temperature 
(mix flow rate = 35 kg/h, dasher speed = 750 rpm)

Figure 5: Predicted chord length and measured chord length as a function of evaporation temperature 
(mix flow rate= 35 kg/h, dasher speed = 750 rpm)

4.3. Influence of mix flow rate
The residence time for the product is directly changed by altering the mix flow rate. Consequently, the exit temperature should mechanically increase with an increasing flow rate with the same evaporation temperature. This has been confirmed experimentally and results of measurement are compared with simulation results in figure 6.

![Figure 6: Comparison of predicted draw temperature and measured draw temperature as a function of mix flow rate (evaporation temperature=-15°C, dasher speed=750 rpm)](image)

We observe a maximum difference of 9% between simulated and experimental draw temperature results. These differences are lower when mix flow rate is increased. Concerning the simulated average chord length, the effect of mix flow rate is not significant experimentally for the range of operating conditions.

5. CONCLUSION

The moments method is an alternative way to solve the population balance equation, but is not based on physical simplifications. The only difference concerning the assumptions between a class method and the moments method for the resolution of the PBE is the radial mixing of the scraper, not taken into account in the moments method. The mathematical model developed in this work predicts both draw temperature and chord length of a sorbet freezing in a SSHE and highlights the pertinence of this approach. The comparison between measurements and simulations show that the influences of the main operating variables of the process are well predicted.

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6. REFERENCES


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