97th Peripatetic Seminar on Sheaves and Logic
at the Université catholique de Louvain, on the occasion of Rudger Kieboom's 65th birthday

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Saturday 31st January 2015

9:00 *Registration*

Chair: **Borceux**
9:30 **Janelidze**: "Historical remarks on internal object actions and action representability"
10:00 **Van der Linden**: "When is a double central extension *universal*?"
10:30 **Adámek**: "Colimits of Monads"

11:00 *Coffee*

Chair: **Clementino**
11:30 **Guitart**: "Algebraic Structures over Autograph"
12:00 **Van Opdenbosch**: "Regularity for relational algebras and some examples"
12:30 **Tholen**: "Topological categories, quantaloids, and total cocompleteness"

13:00 *Lunch*

Chair: **Colebunders**
15:00 **Caenepeel**: "Hopf categories"
15:30 **Trentinaglia**: "Reduced smooth stacks?"
16:00 **Everaert**: "A Galois theorem for normal extensions"

16:30 *Coffee*

Chair: **Stubbe**
17:00 **Gurski**: "Invertible objects in braided monoidal categories"
17:30 **Jacqmin**: "Exactness properties and cofiltered limit completion"
18:00 **Leinster**: "The Euler characteristic of an associative algebra"

18:30 *End*

19:30 *Conference dinner*

Sunday 1st February 2015

Chair: **Bourn**
9:00 **Vitale**: "Gabriel-Zisman and Brown exact sequences for internal groupoids"
9:30 **Metere**: "Extension theory and the calculus of butterflies"
10:00 **Heredia**: "Bicategorical homotopy pullbacks"

10:30 *Coffee*

Chair: **Johnstone**
11:00 **Lucyshyn-Wright**: "Completion, closure, and density relative to a monad, with examples in functional analysis and sheaf theory"
11:30 **Avery**: "Codensity and the Giry monad"
12:00 **Schaeppi**: "Universal geometric tensor categories"
12:30 **Kieboom**: "KT-fields and sharply 3-transitive groups"

13:00 *End*

97th Peripatetic Seminar on Sheaves and Logic
abstracts

Jiří Adámek (Braunschweig) *Colimits of Monads*
The category of monads on Set has all coequalizers. (This does not hold e.g. for monads over graphs.) A necessary and sufficient condition for a collection of monads to have a coproduct is the existence of arbitrarily large common pre-fixed points. Moreover, a simple formula for computation of coproducts of monads on Set will be presented.

Tom Avery (Edinburgh) *Codensity and the Giry monad*
An ultrafilter on a set can be viewed as a finitely additive probability measure taking only the values 0 and 1. The functor sending a set to the set of ultrafilters on it is part of a monad, and there is an analogous monad on the category of measurable spaces in which ultrafilters are replaced by probability measures, due to Giry. So, the ultrafilter monad is a primitive version of the Giry monad. A theorem of Kennison and Gildenhuys characterises the ultrafilter monad as the codensity monad of the inclusion of finite sets into sets. I will give an analogous characterisation of the Giry monad, namely as the codensity monad of a forgetful functor from a certain category of convex sets to measurable spaces.

Stefaan Caenepeel (Brussel) *Hopf categories*
We introduce Hopf categories enriched over braided monoidal categories. The notion is linked to several recently developed notions in Hopf algebra theory, such as Hopf group (co)algebras, weak Hopf algebras and duoidal categories. Several classical properties from Hopf algebra theory generalize to Hopf categories, such as for example the fundamental theorem for Hopf modules.

Tomas Everaert (Brussel) *A Galois theorem for normal extensions*
For a particular class of (not necessarily admissible) Galois structures, the normal extensions are precisely those extensions that are 'locally' split epic and trivial, and this fact can be used to prove a 'Galois theorem' for normal extensions.

Joint work with Mathieu Duckerts-Antoine.

René Guitart (Paris) *Algebraic Structures over Autograph*
The category of autographs is the topos of actions of the free monoid on 2 generators. We will show how graphic algebras in the sense of Burroni are autographic algebras. We will examine how algebraic structures over graphs (in the sense of realization of a projective sketch in graph, or in the sense of algebra of a monad on graphs) are also algebraic structures over autographs. Furthermore many other structures are algebraic over autographs. Especially this is a natural basis for the description of knots or links, as well as for the description of n-categories.

Nick Gurski (Sheffield) *Invertible objects in braided monoidal categories*
An object of a monoidal category is invertible if it is invertible when viewed as an element of the monoid of isomorphism classes of objects. In algebraic contexts, these objects are akin to 1-dimensional vector spaces, and in geometric contexts, line bundles. Invertible objects in symmetric monoidal categories carry a natural action of the cyclic group of order 2. In this talk, I will discuss the braided case, and the natural group action that arises. This is joint work with Ed Prior.

Benjamín Alarcón Heredia (Granada) *Bicategorical homotopy pullbacks*
The homotopy theory of higher categorical structures has become a relevant part of the machinery of algebraic topology and algebraic K-theory, and this paper contains contributions to the study of the relationship between Bénabou's bicategories and the homotopy types of their classifying spaces. Mainly, we state and prove an extension of Quillen's Theorem B by showing, under reasonable necessary conditions, a bicategory-theoretical interpretation of the homotopy-fibre product of the continuous maps induced on classifying spaces by a diagram of bicategories $\mathcal{A} \to \mathcal{B} \leftarrow \mathcal{A}'$. Applications are given for the study of homotopy pullbacks of monoidal categories and of crossed modules.

This is a joint work with A. M. Cegarra and J. Remedios

Pierre-Alain Jacqmin (Louvain-la-Neuve) *Exactness properties and cofiltered limit completion*
Cofiltered limit completion of a small finitely complete category $\mathcal{C}$ can be described by the Yoneda embedding into the category $Lex(\mathcal{C}, Set)^{op}$ (Gabriel - Ulmer). Several exactness properties have been already proved to be preserved by this completion. For example: regularity (Barr), coregularity, additivity, abelianness (Day - Street), co-Mal'cev coregularity (Gran - Pedicchio), Mal'cev Barr-exactness with pushouts (Borceux - Pedicchio). We present here a general description of a family of such properties which recover the above examples.

George Janelidze (Cape Town) *Historical remarks on internal object actions and action representability*
This talk is an attempt to clarify the origins of the concepts of its title — in category theory and in homological algebra.

Rudger Kieboom (Brussel) *KT-fields and sharply 3-transitive groups*
There exists a categorical equivalence between the category of sharply 3-transitive permutation groups on a set X with at least 3 elements, labeled by 0, 1 and a third chosen point ∞ and the category of so-called "KT-fields" (named after Helmut Karzel and Jacques Tits) which are couples (F, ε), where $F = (F, +, .)$ is a neardomain (see [2]) and $\varepsilon\colon (F^*, .) \to (F^*, .)$ is a multiplicative automorphism of $F^* = F \setminus \{0\}$ satisfying the identity

$$\varepsilon(1 - \varepsilon(x)) = 1 - \varepsilon(1 - x) \text{ for all } x \in F^*, x \neq 1.$$

This is joint work with Ph. Cara.

[1] Ph. Cara, R. Kieboom and T. Vervloet, A categorical approach to loops, neardomains and nearfields, Bull. Belg. Math. Soc. Simon Stevin 19 (2012), 845–857.
[2] W. Kerby and H. Wefelscheid, Über eine scharf 3-fach transitiven Gruppen zugeordenete algebraische Struktur, Abh. Math. Sem. Univ. Hamburg 37 (1972), 225–235.

Tom Leinster (Edinburgh) *The Euler characteristic of an associative algebra*
It was Steve Schanuel who established that the right way to "count" a topological space is to take its Euler characteristic – that Euler characteristic is to spaces what cardinality is to sets. Pursuing his insight leads to a general definition of the Euler characteristic, or magnitude, of a (suitably finite) enriched category. Different choices of base category produce cardinality-like invariants in different branches of mathematics. I will describe what happens over Vect. In particular, every associative algebra A gives rise canonically to a certain Vect-enriched category, which in turn has an Euler characteristic; and as I will show, this is a recognizable homological invariant of A. This invariant perhaps deserves to be known as the Euler characteristic of an associative algebra.

Rory Lucyshyn-Wright (Cambridge) *Completion, closure, and density relative to a monad, with examples in functional analysis and sheaf theory*
Given a monad T on a suitable enriched category B equipped with a proper factorization system (E,M), we define notions of T-completion, T-closure, and T-density. We show that not only the familiar notions of completion, closure, and density in normed vector spaces, but also the notions of sheafification, closure, and density with respect to a Lawvere-Tierney topology, are instances of the given abstract notions. The process of T-completion is equally the enriched idempotent monad associated to T (which we call the idempotent core of T), and we show that it exists as soon as every morphism in B factors as a T-dense morphism followed by a T-closed M-embedding. The latter hypothesis is satisfied as soon as B has certain pullbacks as well as wide intersections of M-embeddings. Hence the resulting theorem on the existence of the idempotent core of an enriched monad entails Fakir's existence result in the non-enriched case, as well as adjoint functor factorization results of Applegate-Tierney and Day.

Giuseppe Metere (Palermo) *Extension theory and the calculus of butterflies*
In [2], Bourn presents a cohomological classification of extensions by considering suitable groupoids of pretorsors. This talks is inspired by the observation that Bourn's result, and consequently the classical Schreier-Mac Lane Theorem, can be conveniently understood via the language of internal butterflies (i.e. a normalization of a specific class of internal distributors see, [4, 1]).

Actually, our approach gives a recipe to deal with the theories of extensions in those varieties of semi-abelian algebras, where a simple notion of crossed module is defined, such as, for instance, groups, Lie-algebras over a field, rings and all categories of interest in the sense of Orzech [6]. In fact, we extend Bourn's result to the wider class of extensions of a crossed module by a (discrete) object. This situation has been studied, in the case of group extensions by Dedecker ([3], see also [5]).

This is joint work with Alan S. Cigoli.

[1] O. Abbad, S. Mantovani, G. Metere and E. M. Vitale, Butterflies in a semi-abelian context, *Adv. Math.* **238** (2013) 140–183.
[2] D. Bourn, Internal profunctors and commutator theory; applications to extensions classification and categorical Galois theory, *Theory Appl. Categ.* **24** (2010) 451–488.
[3] P. Dedecker, Cohomologie de dimension 2 à coefficients non abéliens, *C. R. Acad. Sci. Paris* **247** (1958) 1160–1163.
[4] S. Mantovani, G. Metere and E. M. Vitale, Profunctors in Mal'tsev categories and fractions of functors, *J. Pure Appl. Algebra* **217** (7) (2013) 1173–1186.
[5] B. Noohi, On weak maps between 2-groups (2008) arXiv:math/0506313v3.
[6] G. Orzech, Obstruction theory in algebraic categories I, II, *J. Pure Appl. Algebra* **2** (1972) 287–314, 315–340.

Daniel Schäppi (Sheffield) *Universal geometric tensor categories*

The notion of geometric tensor categories gives a precise characterization of those tensor categories associated to a particular class of objects in algebraic geometry (called Adams stacks; these include classical objects such as projective varieties). In fact, the 2-category of Adams stacks is contravariantly equivalent to the 2-category of geometric tensor categories.

I will explain how enriched Grothendieck topologies can be used to associate a geometric tensor category to any tensor category in a universal way. This shows that geometric tensor categories are coreflective among all tensor categories. It follows in particular that the 2-category of Adams stacks is complete and cocomplete.

Walter Tholen (Toronto) *Topological categories, quantaloids, and total cocompleteness*

In this talk we present, and expand upon, a recent result by Richard Garner ("Topological functors as total categories", TAC 29 (15) 406-421, 2014, by which the notion of topologicity of a concrete functor is subsumed under the concept of total cocompleteness of enriched category theory. Motivated by some key results of the 1970s, we develop all needed ingredients from the theory of quantaloids in order to place essential results of categorical topology into the context of quantaloid-enriched category theory, a field that previously drew its motivation and applications from other domains, such as quantum logic. The talk is based on joint work with Lili Shen ("Topological Categories, quantaloids and Isbell adjunctions", York University, 2015).

Giorgio Trentinaglia (Lisboa) *Reduced smooth stacks?*

An arbitrary Lie groupoid gives rise to a groupoid of germs of local diffeomorphisms over its base manifold, known as its *effect*. The effect of any bundle of Lie groups is trivial. All quotients of a given Lie groupoid determine the same effect. It is natural to regard the effects of any two Morita equivalent Lie groupoids as being "equivalent". In this talk we will describe a systematic way of comparing the effects of different Lie groupoids. In particular, we will rigorously define what it means for two arbitrary Lie groupoids to give rise to "equivalent" effects. For effective orbifold groupoids, the new notion of equivalence turns out to coincide with the traditional notion of Morita equivalence. Our analysis is relevant to the presentation theory of proper smooth stacks.

Tim Van der Linden (Louvain-la-Neuve) *When is a double central extension* universal*?*

It is easy to prove that a double central extension over an object X which is initial amongst such is necessarily the zero double extension over $X = 0$. This explains why it does not make sense to define universality of higher central extensions in the way done classically for one-fold central extensions.

The aim of my talk is to introduce an appropriate notion of universality for higher central extensions which does extend the theory of one-fold central extensions and prefect objects to higher degrees in a non-trivial way. This work is done in semi-abelian categories, but the results are new even for groups.

We shall see that a universal double central extension of X by $\mathrm{H}_3(X)$ exists as soon as $\mathrm{H}_2(X) = \mathrm{H}_1(X) = 0$. I will also describe a simple construction for such universal extensions.

Karen Van Opdenbosch (Brussel) *Regularity for relational algebras and some examples*

Topological and approach spaces are known to be relational $\mathbb{T}$-algebras for suitable monads $\mathbb{T}$ on Set, laxly extended to Rel.

Top is known to be isomorphically described as $(\beta, 2)-\mathsf{Cat}$ for the ultrafiltermonad with the Barr extension [1] or as $(\mathbb{F}, 2)-\mathsf{Cat}$ for the filtermonad [7]. App can be isomorphically described as $(\mathbb{B}, 2)-\mathsf{Cat}$ for the prime functional ideal monad $\mathbb{B}$ [6] or as $(\mathbb{I}, 2)-\mathsf{Cat}$ for the functional ideal monad $\mathbb{I}$ [2, 3]. Both $\mathbb{F}$ and $\mathbb{I}$ are power-enriched monads and their extension to Rel is the Kleisli extension. For the monad $\mathbb{B}$, we use the initial extension to Rel.

In this talk we look at objects in $(\mathbb{T}, 2)-\mathsf{Cat}$ as spaces and we explore the topological property of $\mathbb{T}$-regularity. When applied to $(\beta, 2)-\mathsf{Cat}$, β-regularity is known to be equivalent to the usual regularity of the topological space [5].

Consider now a monad $\mathbb{T}$, power-enriched by the monad morphism $\tau : \mathbb{P} \to \mathbb{T}$, with the Kleisli extension. We prove that under mild conditions, even when abandoning improper elements, $\mathbb{T}$-regularity is too strong since it implies the space to be indiscrete. Both $\mathbb{I}$ and $\mathbb{F}$ satisfy these mild conditions, hence $\mathbb{I}$- and $\mathbb{F}$-regularity are way too strong to give interesting results. In the case of the functional ideal monad $\mathbb{I}$, we present weaker conditions in order to describe the usual notion of regularity in App.

For the prime functional ideal monad $\mathbb{B}$, a submonad of $\mathbb{I}$, the situation is different. Again abandoning improper elements to avoid uninteresting results, we explain that $\mathbb{B}$-regularity is equivalent to the approach space being topological and regular. We also present weaker conditions based on the monad $\mathbb{B}$ describing the usual regularity in App.

This is joint work with E. Colebunders and R. Lowen [4].

[1] M. Barr, Relational algebras. Lect. Notes Math., 137:39-55, 1970.
[2] E. Colebunders, R. Lowen and W. Rosiers, Lax algebras via initial monad morphisms: APP, TOP, MET and ORD. Topology Appl., 158:882-903, 2011.
[3] E. Colebunders, R. Lowen and K. Van Opdenbosch, Approach spaces, functional ideal convergence and Kleisli monoids, preprint.
[4] E. Colebunders, R. Lowen and K. Van Opdenbosch, Regularity for relational algebras and the case of approach spaces, submitted.
[5] D. Hofmann, G. J. Seal and W. Tholen, Monoidal Topology, A Categorical Approach to Order, Metric and Topology. Encyclopedia of Mathematics and its Applications, Cambridge University Press, 2014.
[6] R. Lowen and T. Vroegrijk, A new lax algebraic characterization of approach spaces. Quad. Mat., 22: 199-232, 2008.
[7] G. J. Seal, Canonical and op-canonical lax algebras. Theory Appl. Categ., 14:221-243, 2005.

Enrico Vitale (Louvain-la-Neuve) *Gabriel-Zisman and Brown exact sequences for internal groupoids*
In order to give a unified treatment of various exact sequences arising in algebraic topology and in non abelian homological algebra, Gabriel and Zisman [1] and Brown [2] associated a six terms exact sequence of groups and pointed sets respectively to a morphism of pointed groupoids and to a fibration of groupoids. In collaboration with Duskin, Kieboom, Kasangian and Metere, we have generalized Gabriel-Zisman sequence to pointed n-groupoids [3, 4]. In this talk, I will report on a different generalization of Gabriel-Zisman and Brown sequences : we pass from pointed groupoids in Set to groupoids internal to any pointed regular category. In particular, exploiting results by Everaert, Kieboom and Van der Linden on the homotopy theory of internal categories [5], we show that the generalized Brown sequence is a special case of the generalized Gabriel-Zisman sequence.

Joint work with S. Mantovani and G. Metere

[1] P. Gabriel and M. Zisman : Calculus of fractions and homotopy theory, Springer 1967.
[2] R. Brown : Fibrations of groupoids, Journal of Algebra 1970.
[3] J. Duskin, R. Kieboom and E. M. Vitale : Morphisms of 2-groupoids and low-dimensional cohomology of crossed modules, Fields Institute Communication Series 2004.
[4] S. Kasangian, G. Metere and E. M. Vitale : The ziqqurath of exact sequences of n-groupoids, Cahiers de Topologie et Géométrie Différentielle Catégorique 2011.
[5] T. Everaert, R. Kieboom and T. Van der Linden : Model structures for homotopy of internal categories, Theory and Applications of Categories 2005.

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