lmat1235

2022

Some notions of mathematical logic

5.00 credits	30.0 h + 15.0 h	Q2

Teacher(s)	Van der Linden Tim ;Vitale Enrico ;
Language :	French > English-friendly
Place of the course	Louvain-la-Neuve
Prerequisites	LMAT1121, LMAT1131
Main themes	We start with a naive viewpoint on the concept of a set. Within this framework we introduce ordinals and cardinals. While developing this theory we see clearly that our naive approach is not tenable. We then start studying Zermelo and Fraenkel's axiomatic set theory. We give particular attention to problems of independence and (in)coherence, taking as particular examples the axiom of choice and the continuum hypothesis. In parallel we sketch the basics of propositional calculus and predicate calculus, that is to say, the structure of first-order languages, which we need to fully understand and solve the problems that appear within set theory.
Learning outcomes	**At the end of this learning unit, the student is able to :** 1 **Contribution of the course to learning outcomes in the Bachelor in Mathematics programme.** **By the end of this activity, students will have made progress in :** - Recognising and understanding a basic foundation of mathematics. He will in particular have developed his capacity to : - Choose and use the basic tools of calculation to solve mathematical problems. - Recognise the fundamental concepts of important current mathematical theories. - Establish the main connections between these theories, analyse them and explain them through the use of examples. - Identify, by use of the abstract and experimental approach specific to the exact sciences, the unifying features of different situations and experiments in mathematics or in closely related fields. - Show evidence of abstract thinking and of a critical spirit. He will in particular have developed his capacity to : --Argue within the context of the axiomatic method --Recognise the key arguments and the structure of a proof. --Construct and draw up a proof independently. --Evaluate the rigour of a mathematical or logical argument and identify any possible flaws in it. **Learning outcomes specific to the course.** **By the end of this activity, students will be able to :** - Reason within the framework of propositional and predicate calculus, make a natural deduction. - Recognise whether a given collection of objects forms a set. - Use the theory of ordinals and cardinals to determine the size of a set, and to compare the sizes of two given sets. - Use transfinite induction and Zorn's lemma. - Understand the status of the axiom of choice and the continuum hypothesis within the axiomatic settings of Zermelo--Fraenkel and von Neumann--Bernays--Gödel.
Evaluation methods	The assessment aims to test knowledge and understanding of concepts, examples and fundamental results, the ability to build a coherent reasoning, mastery of demonstration techniques introduced during the course. The assessment consists of a final oral exam. To establish the final grade, we will take into account the oral exam and active participation in the practical work.
Teaching methods	The learning activities consist of lectures and exercise sessions. The lectures aim to introduce fundamental concepts, to motivate them by giving examples and proving results, to show their reciprocal connections and their connections with other courses in the programme for the Bachelor in Mathematics. The exercise sessions aim to teach the basic calculation techniques of propositions and predicates, that is to say, of first-order structures and languages.
Content	The aim of this activity is to make the laws that govern mathematical reasoning when it is presented as a formal theory explicit. We examine which particular languages to use, which properties to take as a starting point, which deduction rules are commonly admitted.

	The spirit and the presentation are the same as in any other course in Mathematics: we give definitions, build chains of propositions, prove theorems.
	The following themes are touched upon in the framework of this course:
	- Propositional logic, Heyting's and Boole's algebras, consistency and coherence, Dedekind's representation theorem, Stone's completeness theorem, axiom of choice.
	- Predicative logic, conditions of Frobenius and Beck.
Inline resources	Website moodle. The site contains the lecture notes, the problems for the exercise sessions and a detailed overview of the course.
Bibliography	• K.J. Devlin, Fundamentals of Contemporary Set Theory, Springer, 1979 • H. Herrlich, G. E. Strecker, Category Theory, 3 ed., Sigma Ser. Pure Math., vol. 1, Heldermann Verlag, 2007 • K. Hrbacek, K.T. Jech, Introduction to Set Theory, 3rd Edition, Marcel Dekker, 1999 • F. W. Lawvere, R. Rosebrugh, Sets for Mathematics, Cambridge University Press, 2003
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Learning outcomes
Additionnal module in Mathematics	APPMATH	5		🔍