UCLouvain

lepl1103

2022

EDPs and complex analysis

5.00 credits	30.0 h + 30.0 h	Q1

Teacher(s)	. SOMEBODY ;Chatelain Philippe ;Hendrickx Julien ;Winckelmans Grégoire (coordinator) ;
Language :	French
Place of the course	Louvain-la-Neuve
Prerequisites	This course assumes that you have acquired the basic notions of analysis (ordinary differential equations (ED0) and methods of solving 1st order and 2nd order ODE, functions of several variables and partial derivatives) and those of gradient, divergence and Laplacian as taught in courses LEPL1102 and LEPL1105. Finally, it supposes to follow in parallel the course of Physics LEPL1203 for the concept of wave equation which is approached there.
Main themes	Partial differential equations (PDEs): classification (hyperbolic, parabolic, elliptical), links with physical phenomena, method of characteristics for hyperbolic PDEs, solutions in infinite domain (by Green's functions), solutions in finite domain (by separation of variables) with self-adjoint operators, eigenvalues and eigenfunctions, orthogonality and development of the series solution of eigenfunctions, solutions in semi-infinite 1-D domain (by similarity variable). Functions of a complex variable: elementary functions, branching point(s) and cut(s), limit and continuity, differentiability and Cauchy-Riemann equations, integration, Cauchy's theorem and Cauchy's integral formulas, series, residue theorem and maps (definite integrals), conformal transformations.
Learning outcomes	**At the end of this learning unit, the student is able to :** 1 **Contribution of the course to the program framework** AA1.1, AA1.2 AA6.1 **Course specific learning outcomes** At the end of this course, the student will be able to: Define and explain the fundamental properties of the different types of first-order and second-order, linear and quasi-linear PDEs. Understand and differentiate the fundamental physical phenomena governed by PDEs: hyperbolic case (also transport equation and wave equation), parabolic case (diffusion equation), elliptical case (Laplace and Poisson equations). For each type, define and apply appropriate initial and/or boundary conditions (Dirichlet, Neumann, Robin). Apply the method of characteristics to solve PDEs of order 1, and to solve the 1-D wave equation. Understand the general theory on self-adjoint operators: eigenvalues and eigenfunctions, orthogonality of eigenfunctions, development of a function as a series of eigenfunctions. Apply the method of separation of variables to solve Laplace's equation in a rectangle, a (sector of) circle, a (sector of) ring. Apply the method of separation of variables to solve the wave equation in a segment, a rectangle, a circle (Bessel equation). Apply the method of separation of variables to solve the diffusion equation in a segment, a rectangle, a circle. Obtain the similarity solution of the diffusion equation in a semi-infinite segment: case with fixed jump at the origin, case with temporal oscillation at the origin. Understand the definition of the elementary functions of a complex variable, obtain the possible branching point(s) of a function and choose an appropriate cut(s), evaluate a function in one or more branches. Understand the concepts of limit, continuity and differentiability of a function, and make the connections with Laplace's equation. Obtain the series expansion of a function. Calculate the pole(s) of a function, and use the residue theorem to evaluate definite integrals. Understand the concept of conformal transformation, and be able to apply it in simple cases. The contribution of this teaching unit to the development and mastery of the skills and achievements of the program(s) can be accessed at the end of this sheet, in the section "Programmes/training offering this teaching unit (TU)".
Evaluation methods	The students are evaluated individually, with a written exam. The APE (APE1 to APE12) are not graded, but solutions are put on the Moodle site. This too allows the students to continiously evaluate their level of comprehension and acquisition of competences

Teaching methods	The course is organized in 12 courses (CM1 to CM12), given in a large auditorium, and into 12 sessions of « learning through exercises » (APE1 to APE12) that are realized, in part, in tutored groups (with one assistant-tutor per group) and, for the rest, out of the tutored groups.
Content	**Partial differential equations (PDE) :** 1st and 2^{nd} order PDE: presentation, classification (hyperbolic, parabolic, elliptic) and links with physical phenomena (transport equation, wave equation, diffusion equation, Laplace's equation, Poisson's equation), Cauchy problem and method of characteristics for hyperbolic PDE, initial and/or boundary conditions (Dirichlet, Neumann, Robin), solutions in infinite domain (by Green's functions) for the diffusion equation, and for Poisson's equation. self-adjoint operators, eignevalues and eigenfunctions, orthogonality of eigenfunctions. Developpement of functions in series of eigenfunctions. Helmholtz problem. Bessel functions of the 1st and 2^{nd} kind. Method of separation of variables for problems in infinite domain: Laplace's equation in 2-D (rectangle, circle, annulus, sector of circle or annulus) ; wave equation in 1-D and in 2-D, diffison equation in 1-D and in 2-D. Similarity solutions for the diffusion equation in 1-D semi-infinite domain. **Functions of a complex variable, f(z) :** Recall the complex plane and the complex numbers. Definition of elementary functions: z^a, exp(z), log(z), a^z, sin(z), sinh(z), arcsin(z), etc. Branch point(s) and branch cut(s), Riemann surface(s). Limits and continuity, derivability, holomorphic (analytic) functions, entire functions, Cauchy-Riemann equations and links with Laplace's equation. Integration, Cauchy theorem and consequences: Cauchy integral formula, Taylor and Laurent series, poles, residue(s) theorem. Evaluation of definite integrals (also using Jordan's lemma). Introduction to conformal transformations and examples of applications.
Inline resources	https://www.moodleucl.uclouvain.be
Bibliography	**Partie EDP :** J.-F. Remacle et G. Winckelmans, syllabus « LEPL1103: Support partiel pour la partie ésuations aux dérivées partielles (EDP)", notes complémentaires : « Modèle LWR du traffic routier », « Fonctions de Bessel de 1ère et de 2ème espèces », « Méthodes de résolution de l'équation de diffusion ». Ouvrage de références: Richard Haberman , « Elementary Applied Partial Differential Equations: with Fourier Series and Boundary Value Problems », Prentice Hall. **Partie Analyse complexe :** G. Winckelmans et J.-F. Remacle : notes complémentaires : « Lemmes de Jordan ». Ouvrages de références : Stephen D. Fisher , « Complex Variables » , Dover (fortement recommandé) ; Georges F. Carrier, M. Krook, Carl E. Pearson, « Functions of a Complex Variable : Theory and Practice » , Hod Books. Les documents du cours (syllabus, notes complémentaires, énoncés et solutions des APE, énoncé et solution de l'évaluation intermédiaire (le cas échéant) sont mis à disposition sur le site Moodle du cours.
Faculty or entity in charge	BTCI

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Learning outcomes
Bachelor in Engineering	FSA1BA	5		