UCLouvain

lphys2211

2021

Group theory

5.00 credits	22.5 h + 22.5 h	Q2

Teacher(s)	Ruelle Philippe ;
Language :	English
Place of the course	Louvain-la-Neuve
Prerequisites	Having followed LMAT1121 and LMAT1131 ou equivalent teaching units in other programmes is an asset.
Main themes	This course is a general introduction to group theory, with a strong orientation towards its use in Physics. Symmetries are fundamental in Physics and are mathematically formalized by the concept of group. Therefore physicists need to know how to formulate a symmetry, how to use it and understand all the consequences of it. In this regard, the notion of representation is central, as it encapsulates the way physical quantities behave under the symmetry transformations. After a brief review of some general aspects of groups, the course will focus on the study and use of their representations. A few concrete applications will show the efficiency of group methods in Physics.
Learning outcomes	**At the end of this learning unit, the student is able to :** 1 **a. Contribution of the teaching unit to the learning outcomes of the programme (PHYS2M and PHS2M1)** 1.1, 1.5, 2.1, 2.3, 3.1, 3.2, 3.3, 3.4. **b. Specific learning outcomes of the teaching unit** At the end of the teaching unit, the student will be able to : 1. formulate a symmetry in terms of a group; 2. analyze the consequences of a symmetry by the use of representations of the associated group ; 3. understand the importance of group representations in Physics ; 4. calculate characters of representations ; 5. identify different types of representations ; 6. calculate the reduction of a representation of a finite group in irreducible representations, and identify the associated invariant subspaces ; 7. calculate the algebra of a matrix group and determine its dimension ; 8. characterize a representation of su(2) ; 9. calculate su(2) Clebsch-Gordan coefficients.
Evaluation methods	The evaluation is based on a written exam, with the objective to test the knowledge, the understanding and the use in simple and concrete problems, of the mathematical notions and techniques developed in the course. Emphasis will be put on the capacity to analyze a new but simple situation, rather than on the proof of mathematical and abstract results. In case compulsory take-home projects are submitted during the semester, these will contribute to the final mark. This contribution will be used in every exam session.
Teaching methods	The teaching consists of lectures and tutorials. The lectures aim at introducing the fundamental concepts of group theory which are central to understand its role in physics. In particular certain concepts developed in other courses of the undergraduate physics program are revisited from a purely group theoretic point of view, thereby showing the full relevance of group methods. The most useful results of group theory are presented and the associated methods are made completely explicit. The tutorials are meant to get familiar with the theoretical material and the methods presented during the lectures. Attendance to both the lectures and the tutorials is required.
Content	The course consists of two main parts, whose content is detailed hereafter. Depending on the time availability, the material marked with a star, somewhat more advanced, might not be discussed. 1. **Finite groups :** ' basic notions, properties and examples (invariant subgroup, direct and semi-direct product, conjugacy classes, left and right cosets, quotient group, illustrations in symmetric groups) ; ' concept of representation (motivations, definitions, examples, equivalence classes of representations, direct sums, distinction between reducible and irreducible, classification problem) ; ' general results for finite groups (characters, orthogonality relations, irreducible character tables, reduction methods, applications) ; ' tensor product of representations (definition, reduction of tensor products, practical efficiency of tensorial notation, examples) ;

	' mathematical characterization and consequences of symmetries in a concrete physical system (calculation of the normal modes of a mechanical system from the identification of irreducible representations in the symmetry group action) ; ' (*) symmetric groups (irreducible representations associated to Young diagrams, dimensions, characters). 2. **Lie groups and Lie algebras :** ' the group SO(2) (defining representation, infinitesimal generators) ; ' generalization to matrix groups (Lie algebra, exponential map, structure constants, representation of the algebra, group composition law) ; ' the groups SU(2) and SO(3) (group varieties, parametrizations, relation between the two) ; ' the su(2) algebra (irreducible representations, reduction of tensor products, Clebsch-Gordan coefficients) ; ' (*) representations of the su(3) algebra (examples, general structure and classification, reduction of tensor products, applications to the quantum harmonic oscillator) ; ' (*) representations of classical matrix groups (tensorial methods, role of permutation groups, Young diagrams and dimension formulas, peculiarities of the orthogonal groups, application to the Riemann tensor).
Inline resources	The lecture notes are available on the ModdleUCL website of the teaching unit.
Bibliography	Syllabus disponible sur MoodleUCL.
Faculty or entity in charge	PHYS

Programmes containing this learning unit (UE)				
Program title	Acronym	Credits	Prerequisite	Learning outcomes
Minor in Mathematics	MINMATH	5		
Minor in Physics	MINPHYS	5		
Master [120] in Physics	PHYS2M	5		
Additionnal module in Mathematics	APPMATH	5		
Additionnal module in Physics	APPHYS	5		
Master [60] in Physics	PHYS2M1	5		