UCLouvain

# lmat2150

*2021*

# Category theory

| 5.00 credits | 30.0 h + 15.0 h | Q1 |
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| Teacher(s) | Gran Marino ; |
| Language : | English |
| Place of the course | Louvain-la-Neuve |
| Prerequisites | LMAT1131 - linear algebra (first year of the bachelor program in mathematics) or any equivalent cours.<br>LMAT 1231 - multilinear algebra and group theory (second year of the bachelor program in mathematics) or any equivalent cours. |
| Main themes | Categories, functors, natural transformations.<br>Adjoint functors and equivalences of categories.<br>Limits and colimits.<br>Regular, exact and abelian categories.<br>Exact sequences and homological lemmas. |
| Learning outcomes | **At the end of this learning unit, the student is able to :**<br>1<br>**Contribution of the course to learning outcomes in the Master in Mathematics programme.**<br>**By the end of this activity, students will have made progress in :**<br>- Recognise and understand a basic foundation of mathematics.He will have made progress in:<br>-- Recognise the fundamental concepts of some important current mathematical theories.<br>-- Establish the main connections between these theories.<br>- Show evidence of abstract thinking and of a critical spirit. He will have made progress in:<br>-- Identify the unifying aspects of different situations and experiences.<br>-- Argue within the context of the axiomatic method.<br>-- Construct and draw up a proof independently, clearly and rigorously.<br>**Learning outcomes specific to the course.**<br>**By the end of this activity, students will be able to :**<br>- Identify, in his mathematical knowledge, several meaningful examples of categories, functors and natural transformations.<br>- Establish the adjointness of some pairs of functors and the equivalence of some categories.<br>- Construct limits and colimits, eventually using adjoint functors and equivalences of categories.<br>- Recognise and prove some important exactness properties of regular, exact and abelian categories.<br>- Concretely explain different notions and results in the categories of sets, groups, abelian groups and topological groups. |
| Evaluation methods | Assessment is by oral examination. This tests the knowledge and the understanding of the main concepts, examples and fundamental results, the ability to give a coherent proof of a result studied in the course.<br>Students may choose the examination language (English or French). |
| Teaching methods | The course is taught through lectures, and specific exercice sessions. During these sessions, students are asked to give suggestions and formulate ideas on the basis of their previous knowledge in order to solve the problems and analyse specific examples.<br>Special attention is paid to the connections between the new concepts introduced in the course and the ones considered in other courses in the Bachelor and Master in Mathematics. |
| Content | In this course we introduce the basic language and some fundamental results in category theory, in order to explain some mathematical situations encountered in other courses during the bachelor program and the first year of the master program in mathematics.<br>The following subjects are studied :<br>- Definition and examples of categories, functors, natural transformations.<br>- Isomorphisms, monomorphisms and epimorphisms in a category.<br>- Adjoint functors (unit, counit, triangular identities) and their fundamental properties.<br>- Reflective subcategories and equivalences of categories.<br>- Examples of limits and colimits.<br>- Limits and adjoint functors. |

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| | - Definition of regular and of exact category, main properties and examples. Barr-Kock theorem. Mal'tsev categories.<br>- Abelian categories, exact sequences, five lemma, nine lemma, snake lemma. |
| Inline resources | Website MoodleUCLouvain |
| Bibliography | Syllabus pour la partie sur les catégories exactes et abéliennes (disponible sur MoodleUCLouvain).<br>F. Borceux : Handbook of categorical algebra, Vol. 1-2 (Cambridge University Press).<br>P. Freyd : Abelian categories (disponible sur MoodleUCLouvain).<br>S. Mac Lane : Categories for the Working Mathematician (Springer).<br>T. Leinster : Basic Category Theory (Cambridge studies in advanced mathematics)<br>-------<br>Syllabus for the part on exact and abelian categories (available on MoodleUCLouvain).<br>F. Borceux : Handbook of categorical algebra, Vol. 1-2 (Cambridge University Press).<br>P. Freyd : Abelian categories (available on MoodleUCLouvain).<br>S. Mac Lane : Categories for the Working Mathematician (Springer).<br>T. Leinster : Basic Category Theory (Cambridge studies in advanced mathematics). |
| Faculty or entity in charge | MATH |

| Programmes containing this learning unit (UE) | | | | |
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| Program title | Acronym | Credits | Prerequisite | Learning outcomes |
| Master [120] in Mathematics | MATH2M | 5 | | 🔍 |
| Master [60] in Mathematics | MATH2M1 | 5 | | 🔍 |