UCLouvain

lmat1331

2021

Commutative algebra

5.00 credits	30.0 h + 15.0 h	Q2

Teacher(s)	Gran Marino ;Jacqmin Pierre-Alain (compensates Gran Marino) ;
Language :	French
Place of the course	Louvain-la-Neuve
Prerequisites	Cours LMAT1131. Topics discussed in the course : solution of systems of algebraic equations. Arithmetic of polynomial rings and elimination theory. Structure of modules over a principal ideal domain, and application to the classification of linear operators on finite-dimensional vector spaces. *The prerequisite(s) for this Teaching Unit (Unité d'enseignement – UE) for the programmes/courses that offer this Teaching Unit are specified at the end of this sheet.*
Main themes	Solution of systems of algebraic equations. Arithmetic of polynomial rings and elimination theory. Structure of modules over a principal ideal domain, and application to the classification of linear operators on finite-dimensional vector spaces.
Learning outcomes	**At the end of this learning unit, the student is able to :** 1 **Contribution of the course to learning outcomes in the Bachelor in Mathematics programme.** **By the end of this activity, students will have made progress in :** - Recognising and understanding a basic foundation of mathematics. -- Choosing and using the basic tools of calculation to solve mathematical problems. -- Recognising the fundamental concepts of important current mathematical theories. -- Establishing the main connections between these theories, analysing them and explaining them through the use of examples. - Identifying, by use of the abstract and experimental approach specific to the exact sciences, the unifying features of different situations and experiments in mathematics or in closely related fields (probability and statistics, physics, computing). - Showing evidence of abstract thinking and of a critical spirit : -- Arguing within the context of the axiomatic method, recognising the key arguments and the structure of a proof, constructing and drawing up a proof independently. -- Evaluating the rigour of a mathematical or logical argument and identifying any possible flaws in it. -- Distinguishing between the intuition and the validity of a result and the different levels of rigorous understanding of this same result. **Learning outcomes specific to the course. By the end of this activity, students will be able to :** - Factor multivariate polynomials into irreducible factors. - Analyse systems of algebraic equations to determine whether they admit solutions and to represent them geometrically. - Find equations with a given parametrised set of solutions. - Analyse the structure of modules over a principal ideal domain. - Reduce linear operators over a finite-dimensional vector space to canonical forms.
Evaluation methods	Assessment is based on a written examination that focuses on theory and on exercises. The examination tests knowledge and understanding of fundamental concepts and results, ability to construct and write a coherent argument, and competence in calculation techniques.
Teaching methods	Learning activities consist of lectures including supervised exercise sessions The lectures aim to introduce fundamental concepts, to explain them by showing examples and by delineating their use, to show their reciprocal connections and their connections with other courses in the programme for the Bachelor in Mathematics. The supervised exercise sessions aim to teach how to select the appropriate method in the resolution of exercices. The two activities are given in presential sessions.
Content	This course introduces abstract algebraic notions playing an important role throughout the cursus of Bachelor and Master in Mathematics : commutative rings and modules.

	The following topics are discussed : - Commutative rings and ideals, quotient rings, isomorphism theorems, chinese remainder theorem. - Integral domains, localisations, fields of fractions. - Maximal ideals and Krull's theorem. - Polynomial rings. Euclidean rings, unique factorization domains (UFD). The ring of Gaussian integers. - Gauss' theorem : if A is a UFD, then the polynomial ring A[X] is UFD. - Radical, nilradical, nilpotent elements. - Local rings. - Modules, direct sums and direct products, free modules and projectives, modules of finite type. - Exact sequences, tensor products. - Noetherian rings, Hilbert's basis theorem.
Inline resources	Some notes of the course will be available on the website of the course on MoodleUCLouvain. Some documents with the exercices to be solved during the practical classes will also be available.
Bibliography	M. Atiyah & I. MacDonald, Introduction to Commutative Algebra, Add. Wesley 1969 (13-01/ATI/ex. 2).
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)				
Program title	Acronym	Credits	Prerequisite	Learning outcomes
Minor in Mathematics	MINMATH	4		🔍
Bachelor in Mathematics	MATH1BA	5	LMAT1131	🔍
Additionnal module in Mathematics	APPMATH	5		🔍