lmat1241 — Geometry II

2021

6.00 credits	45.0 h + 30.0 h	Q2

Teacher(s)	Bieliavsky Pierre ;
Language :	French
Place of the course	Louvain-la-Neuve
Prerequisites	Prerequisites: LMAT1141 ' Geometry 1, LMAT1122 ' Mathematical Analysis 2, LMAT1131 ' Linear Algebra (or equivalent courses). The prerequisite(s) for this Teaching Unit (TU) are specified at the end of this sheet, in relation to the programs/ training courses that offer this TU. *The prerequisite(s) for this Teaching Unit (Unité d'enseignement – UE) for the programmes/courses that offer this Teaching Unit are specified at the end of this sheet.*
Main themes	Theory of plunged surfaces in the euclidean space of dimension three. Gauss-Bonnet formula. Elements of plane hyperbolic geometry.
Learning outcomes	**At the end of this learning unit, the student is able to :** **Contribution of the course to the learning outcomes of the bachelor's degree program in mathematics.** **At the end of this activity, the student will have progressed in his/her ability to know and understand a fundamental base of mathematics.** **In particular, he/she will have developed the ability to :** I. Select and use fundamental computational methods and tools to solve mathematical problems. II. Recognize the fundamental concepts of some current mathematical theories. III. Establish the major connections between these theories, explain and motivate them with examples. **Course Specific Learning Outcomes.** 1 **By the end of this activity, the student will be able to become familiar with the basic concepts of differential geometry, specifically :** (a) Conceive the notion of a plunging surface in a global context, equipped with an atlas. (b) Use the notion of map change to conceive globally of the notions of fundamental shapes and curvature. (c) Use the techniques of solving differential equations in a concrete geometrical framework: calculation of vector field flows and calculation of geodesics. (d) Conceive the notion of Euler-Poincaré characteristic as a topological invariant. The contribution of this course to the development and mastery of the skills and knowledge of the program(s) is available at the end of this document, in the section "Programs/training courses offering this course".
Evaluation methods	The evaluation is based on a written exam on the one hand (exercises of the type done in the TD's), and on the other hand on the theory (definitions, statements of lemmas, propositions, theorems and their proofs). Each part counts for 50% of the overall grade. We test the knowledge and understanding of fundamental notions and results, the ability to construct and write a coherent reasoning, the mastery of calculation techniques.
Content	The following content is covered in the course : 0. Preliminary : theorem of inverse functions and smooth functions between subsets of R^n (definition).1. Surfaces immersed in R^3 1.1. Definition 1.2 Tangent planes 1.3 Vector fields 1.4 Statement of Cauchy's theorem in the case of the flow of a vector field on a surface. 1.5. Hook of two vector fields 1.6 First fundamental form. 2. Introduction to Stokes' theorem (and relation with the statement seen in the physics course) 2.1 a differential form on a plunging surface (as a dual object of a vector field by the first fundamental form) 2.2 area form and its functional multiples 2.3 Statement of Green's formula (Stokes in the plane) 3. Study of the first fundamental form (PFF)

	3.1 Definition of the covariant derivative associated to the PFF (tangent projection of the usual usual derivative) 3.2 Local form (Christoffel symbol) 3.3 Definition of the Riemann tensor 3.4 Parallel transport of a vector along a path. 3.5 Notion of holonomy angle along a loop 3.6 Algebraic curvature 3.7 Gauss curvature 4. Gauss-Bonnet formula 4.1 First formulation at the local level 4.2 Region, triangulation and Euler-Poincaré characteristic 4.3 Global formulation of the G-B formula 5. Introduction to the geometry of the hyperbolic plane 5.1 The hyperbolic plane as a sphere in R^{1,2} 5.2 Study of the group SL(2,R) 5.3 The hyperbolic plane as an orbit of the group SL(2,R) 5.4 Metric aspects 5.5 Geodesics as critical length curves 5.6 Calculation of geodesics 5.6 Model of the Poincaré half-plane
Inline resources	The Moodle site contains the course syllabus, the statements and solutions of the exercises for the practical sessions, the answers to recent exams and the detailed course outline. Bibliography Syllabus available on Moodle.
Bibliography	• M. do Carmo, Differential geometry of curves and surfaces. • P. Malliavin, Géométrie différentielle intrinsèque. • M. Berger, B. Gostiaux, Géométrie différentielle : variétés, courbes et surfaces. • J. Milnor, Topology from a differentiable viewpoint.
Faculty or entity in charge	SC

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Learning outcomes
Minor in Mathematics	MINMATH	6		
Bachelor in Mathematics	MATH1BA	6	LMAT1121 AND LMAT1141 AND LMAT1131	
Additionnal module in Physics	APPHYS	6		