UCLouvain

lmat2240

2020

Low-dimensional topology

Due to the COVID-19 crisis, the information below is subject to change, in particular that concerning the teaching mode (presential, distance or in a comodal or hybrid format).

5 credits	30.0 h + 15.0 h	Q2

This learning unit is not being organized during this academic year.

Teacher(s)	Dos Santos Santana Forte Vaz Pedro ;Lambrechts Pascal ;
Language :	English
Place of the course	Louvain-la-Neuve
Main themes	Knots and diagrams, the Jones and Alexander polynomial invariants, knots and the topology of surfaces, the fundamental group, special topics (knots and 3-lanifolds, homological invariants, applications of knot theoru to biology and chemistry), quantum invariants of knots and 3-manifolds, 2 and 4-manifolds, link homology and categorification. Knots and diagrams, the Jones and Alexander polynomial invariants, knots and the topology of surfaces, the fundamental group, special topics (knots and 3-lanifolds, homological invariants, applications of knot theoru to biology and chemistry), quantum invariants of knots and 3-manifolds, 2 and 4-manifolds, link homology and categorification.
Aims	1 Contribution of the course to learning outcomes in the Master in Mathematics programme. By the end of this activity, students will have made progress in: - Knowing and understanding a fundamental socle in mathematics. More precisely, he/she will have developed his/her skills in : -- Identifying the fundamental concepts of contemporary important mathematical theories. -- Explaining mathematical theories by motivating the statements and definitions by examples and counter-examples end in emphasizing the main ideas. - Make proof of abstraction, critical reasoning and spirit. He/She will have developed his/her skills to : -- Emphasizing the unifying aspects of different situations and experiences. -- Reasoning in terms of the axiomatic method. -- Autonomously construct and write a mathematical proof in a clear and rigourous way. -- Be able to distinguish between the intuition of the validity of a result and the different levels of rigorous understanding of the same result. - Make scientific communications. He/She will have developed his/her skills in : -- Structuring an oral presentation, emphasize the key elements, concepts and techniques and adapt it to the listener's level of understanding. - Analysing, in depth and under different points of view, a problem or a complex sistem to extract the essential features and relate them with the proper tools. He/She will have developed his/her skills in : -- Choosing, having in mind its limitations, a method and the tools to solve a mathematical problem. - Learning autonomy in terms of : ' Look for sources in the mathematical literature and evaluate their relevance. -- Read and understand an advanced mathematical text and place it correctly with respect to the aquired knowledge. Learning outcomes specific to the course. The aim of the course is to present some advanced chapters in low-dimensional topology to prepare the student to a research carrer in a related domain. By the end of the course the student will be able to : - Identify a knot as part of the topology of the ambient space. - Show properties of invariants and compute them in order to distinguish knots. - Use the topology of surfaces to obtain the genus of a knot and use it to caracterize knots in terms of prime knots. - Use the fundamental group to study the topology of the knot complement. - Study autonomusly a modern chapter in knot theory. - - - - *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled “Programmes/courses offering this Teaching Unit”.*

Evaluation methods	**Due to the COVID-19 crisis, the information in this section is particularly likely to change.** Assessment is by a written test and by an oral presentation of one of the special topics, chosen by the student. These test knowledge and understanding of concepts, examples and fundamental results, ability to construct a coherent argument, and mastery of the techniques of proof introduced in the course. The examination has around five questions. Students may also choose the examination language (English or French).
Teaching methods	**Due to the COVID-19 crisis, the information in this section is particularly likely to change.** The main part of the course is given through lectures. During the sessions, students are asked to give suggestions and formulate ideas on the basis of their previous knowledge in order to further the course. Several examples of exercises will be given in order to illustrate and check understanding of subjects. The second part of the course consists of oral presentations by students on specially chosen topics.
Content	The course consists of an introduction of the basic langage and certain fundamental results of knot theory to study its interactions with other subjects, like the topology of 3-manifolds, physics and molecular biology. The following contents are to be addressed in the course. - Basic notions. Definition of knot, knot projection and knot diagram, the Reidemeister moves, the 3-coloring, the linking number. - The Jones polynomial. The kauffman bracket, the jones polynomial, the relation between the number of crossings of an alternating knot and the Jones polynomial. -The topology of surfaces applied to knot theory. Seifert surfaces, the classification of surfaces with boundary, surgery on surfaces, the linking number as the intersection number with a Seifert surface. -The knot genus. The additivity of the knot genus, prime knots and the unique decomposition of knots in terms of prime knots. -The Alexander polynomial. Presentation matrices for modules, the Seifert matrix and the homology of the knot complement, the infinite cyclic cover of a link, the Alexander module, the Alexander polynomial and the genus. -The fundamental group. The topology of the knot complement, the Wirtinger presentation, the p-coloring via the fundamental group. -Special topics. Examples : - The 2-variable polynomials (HOMFLY-PT and Kauffman), - The Jones polynomial as the Witten invariant - Knots and 3-manifolds, - 3- and 4-manifolds (Kirby calculus), - Braids, tangles and the Temperley-Lieb algebra, - Khovanov link homology, - Applied knot theory (biology, chemistry, cryptography').
Inline resources	Page web du cours sur moodle
Bibliography	W. Lickorish: An Introduction to Knot Theory, GTM 175, Springer 1997. J. Roberts: Knot knotes (disponible sur iCampus) K. Murasugi: Knot theory and its applications, Birkhäuser, Springer 1996.
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Aims
Master [120] in Mathematics	MATH2M	5		🔍
Master [60] in Mathematics	MATH2M1	5		🔍
Master [120] in Physics	PHYS2M	5		🔍