lmat1331

2019

UCLouvain

Commutative algebra

In view of the health context linked to the spread of the coronavirus, the methods of organisation and evaluation of the learning units could be adapted in different situations; these possible new methods have been - or will be - communicated by the teachers to the students.

4 credits	30.0 h + 15.0 h	Q2

Teacher(s)	Gran Marino ;
Language :	French
Place of the course	Louvain-la-Neuve
Prerequisites	Cours LMAT1131. Topics discussed in the course : solution of systems of algebraic equations. Arithmetic of polynomial rings and elimination theory. Structure of modules over a principal ideal domain, and application to the classification of linear operators on finite-dimensional vector spaces. *The prerequisite(s) for this Teaching Unit (Unité d'enseignement – UE) for the programmes/courses that offer this Teaching Unit are specified at the end of this sheet.*
Main themes	Solution of systems of algebraic equations. Arithmetic of polynomial rings and elimination theory. Structure of modules over a principal ideal domain, and application to the classification of linear operators on finite-dimensional vector spaces.
Aims	1 Contribution of the course to learning outcomes in the Bachelor in Mathematics programme. By the end of this activity, students will have made progress in: - Recognising and understanding a basic foundation of mathematics. -- Choosing and using the basic tools of calculation to solve mathematical problems. -- Recognising the fundamental concepts of important current mathematical theories. -- Establishing the main connections between these theories, analysing them and explaining them through the use of examples. - Identifying, by use of the abstract and experimental approach specific to the exact sciences, the unifying features of different situations and experiments in mathematics or in closely related fields (probability and statistics, physics, computing). - Showing evidence of abstract thinking and of a critical spirit : -- Arguing within the context of the axiomatic method, recognising the key arguments and the structure of a proof, constructing and drawing up a proof independently. -- Evaluating the rigour of a mathematical or logical argument and identifying any possible flaws in it. -- Distinguishing between the intuition and the validity of a result and the different levels of rigorous understanding of this same result. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Factor multivariate polynomials into irreducible factors. - Analyse systems of algebraic equations to determine whether they admit solutions and to represent them geometrically. - Find equations with a given parametrised set of solutions. - Analyse the structure of modules over a principal ideal domain. - Reduce linear operators over a finite-dimensional vector space to canonical forms. - - - - *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".*
Evaluation methods	**Due to the COVID-19 crisis, the information in this section is particularly likely to change.** Assessment is based on a written examination that focuses on theory and on exercises. The examination tests knowledge and understanding of fundamental concepts and results, ability to construct and write a coherent argument, and competence in calculation techniques.

Teaching methods	**Due to the COVID-19 crisis, the information in this section is particularly likely to change.**
	Learning activities consist of lectures including supervised exercise sessions The lectures aim to introduce fundamental concepts, to explain them by showing examples and by delineating their use, to show their reciprocal connections and their connections with other courses in the programme for the Bachelor in Mathematics. The supervised exercise sessions aim to teach how to select the appropriate method in the resolution of exercices. The two activities are given in presential sessions.
Content	This course introduces abstract algebraic notions related to divisibility, which play an important role throughout the cursus of Bachelor and Master in Mathematics : ideals and factorization in commutative rings, and modules over principal ideal domains. The emphasis will be on polynomial rings.
	The following topics are discussed :
	- Commutative rings and ideals
	- Local rings
	- Modules, exact sequences, tensor products
	- Noethérian rings.
	- Polynomials and affine algebraic varieties.
	- Groebner bases of ideals in polynomial rings.
	- Unique factorisation, resultants, and elimination in polynomial rings.
	- Existence of solutions for systems of algebraic equations (Hilbert's Nullstellensatz).
	- Localisation
	- Structure of modules of finite type over principal ideal domains
Inline resources	Some notes of the course will be available on the website of the course on MoodleUCLouvain.
Bibliography	M. Atiyah & I. MacDonald, Introduction to Commutative Algebra, Add. Wesley 1969 (13-01/ATI/ex. 2).
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)				
Program title	Acronym	Credits	Prerequisite	Aims
Bachelor in Mathematics	MATH1BA	4	LMAT1131	🔍
Additionnal module in Mathematics	LMATH100P	4		🔍
Minor in Mathematics	LMATH100I	4		🔍