lmat2260

2018

6 credits	45.0 h	Q2

Teacher(s)	Claeys Tom ;Haine Luc ;
Language :	French
Place of the course	Louvain-la-Neuve
Main themes	Complex analysis: orthogonal polynomials, Toeplitz and Hankel determinants, Riemann-Hilbert problems. Complex geometry: compact Riemann surfaces theory with applications to integrable systems.
Aims	1 Contribution of the course to learning outcomes in the Master in Mathematics programme. By the end of this activity, students will have made progress in: - Recognise and understand a basic foundation of mathematics. In particular: -- Recognise the fundamental concepts of important current mathematical theories. -- Establish the main connections between these theories. - Show evidence of abstract thinking and of a critical spirit. In particular: -- Identify the unifying features of different situations and experiments in mathematics or in closely related fields (probabilty and statistics, physics). -- Argue within the context of the axiomatic method. -- Construct and draw up a proof independently, with clarity and rigour. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Complex analysis: -- Understand the general theory of orthogonal polynomials on the real line and on the unit circle in the complex plane. -- Use orthogonal polynomials to compute some Toeplitz and Hankel determinants. -- Characterize orthogonal polynomials via Riemann-Hilbert problems. -- Understand the basic ideas of the non-linear steepest descent method. - Complex geometry: -- Understand the origin and the use of the notion of a sheaf in the study of the problem of analytic continuation and the construction of the Riemann surface of an algebraic function. -- Use the first cohomology group with coefficients in a sheaf to approach the classical problems of compact Riemann surfaces theory, like the Riemann-Roch problem and the Mittag-Leffler problem. -- Compute on specific examples the genus of a compact Riemann surface as well as a basis of holomorphic differentials. -- Understand the role of the Jacobi inversion problem and of Riemann's theta function in the modern theory of integrable systems, as well as the notion of a tau function as a generalization of Riemann's theta function. - - - - *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".*
Evaluation methods	In complex analysis, evaluation is based on an oral presentation by each student during the semester, on a selection of exercises solved by the student and by an oral examination. For the oral presentation, each student chooses a project from a list of projects offered by the teacher and presents this project to the other students. The examination tests knowledge and understanding of the concepts, methods and results seen during the course. In complex geometry, evaluation is based on an oral presentation by groups of two or three students during the semester and by an oral examination on subjects seen during the course. The subject of the oral presentation during the semester is a chapter of the course reference work or a research article that opens up new perspectives. The oral examination tests knowledge and understanding of fundamental concepts, examples and results, as well as mastery of the techniques of proof introduced during the course.
Teaching methods	The course is taught through lectures. During the sessions, students are asked to give suggestions and formulate ideas on the basis of their previous knowledge in analysis and complex geometry in order to further the course.
Content	The course deals alternatively with topics in complex analysis or geometry. The following subjects are treated in the course. - Complex analysis: -- Toeplitz and Hankel determinants. -- Orthogonal polynomials on the real line and on the unit circle in the complex plane. -- Riemann-Hilbert problems.

	-- Asymptotic analysis of Riemann-Hilbert problems. - Complex geometry: -- Notion of a Riemann surface, Riemann surface of an algebraic function. -- Riemann-Roch theorem, Riemann-Hurwitz formula, Abel's theorem. -- Jacobi inversion problem and link with Riemann's theta function. -- Modern theory of integrable systems: Kadomtsev-Petviashvili equation, Korteweg-de Vries equation, Toda lattices, Segal-Wilson tau-functions.
Inline resources	Site iCampus (http://icampus.uclouvain.be/).
Bibliography	En analyse complexe: - syllabus disponible sur iCampus. - B. Simon, Orthogonal polynomials on the unit circle part I, AMS Colloquium Publications 54, Providence, RI, 2005. En géométrie complexe: - B. Dubrovin: Integrable Systems and Riemann Surfaces Lecture Notes, SISSA, 2009. - O. Forster: Lectures on Riemann Surfaces, Graduate Texts in Mathematics 81, Springer-Verlag. - D. Mumford: Tata Lectures on Theta I, Birkhäuser.
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Aims
Master [120] in Mathematics	MATH2M	6		
Master [120] in Physics	PHYS2M	6		