UCLouvain

lmat2150

2018

Category theory

5 credits	45.0 h	Q2

Teacher(s)	Vitale Enrico ;
Language :	English
Place of the course	Louvain-la-Neuve
Main themes	Categories, functors, natural transformations. Adjoint functors and equivalences of categories. Limits and colimits. Regular, exact and abelian categories. Exact sequences and homological lemmas.
Aims	Contribution of the course to learning outcomes in the Master in Mathematics programme. By the end of this activity, students will have made progress in: - Recognise and understand a basic foundation of mathematics.He will have made progress in: -- Recognise the fundamental concepts of some important current mathematical theories. -- Establish the main connections between these theories. - Show evidence of abstract thinking and of a critical spirit. He will have made progress in: -- Identify the unifying aspects of different situations and experiences. -- Argue within the context of the axiomatic method. 1 -- Construct and draw up a proof independently, clearly and rigorously. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Identify, in his mathematical knowledge, several meaningful examples of categories, functors and natural transformations. - Establish the adjointness of some pairs of functors and the equivalence of some categories. - Construct limits and colimits, eventually using adjoint functors and equivalences of categories. - Recognise and prove some important exactness properties of regular, exact and abelian categories. - Concretely explain different notions and results in the categories of sets, groups, abelian groups and topological groups. - - - - *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".*
Evaluation methods	Assessment is by oral examination. This tests knowledge and understanding of concepts, examples and fundamental results, ability to construct a coherent argument, and ability to master the techniques of proof introduced in the course. The examination is usually based on six questions, two of which are chosen by the student. Students may also choose the examination language (English or French).
Teaching methods	The course is taught through lectures. During the sessions, students are asked to give suggestions and formulate ideas on the basis of their previous knowledge in order to further the course. Special attention is paid to the analysis of connections between the new concepts introduced in the course and the other courses in the Bachelor and Master in Mathematics.
Content	In this course we introduce the basic language and some fundamental results in category theory, in order to explain some mathematical situations encountered in other courses during the bachelor program and the first year of the master program in mathematics. The following subjects are studied: - Definition and examples of categories, functors, natural transformations. - Isomorphisms, monomorphisms and epimorphisms in a category. - Adjoint functors (unit, counit, triangular identities) and their fundamental properties. Reflective subcategories and equivalences of categories. - Special limits and colimits. Existence and construction of limits and colimits. - Limits and adjoint functors. Freyd's adjoint functor theorem. - Definition of regular and of exact category, main properties and examples. Barr-Kock theorem. Mal'tsev categories. - Exact sequences, five lemma, nine lemma, snake lemma.
Inline resources	Website iCampus (http://icampus.uclouvain.be/).

Bibliography	Syllabus pour la partie sur les catégories exactes, additives, et abéliennes (disponible sur iCampus). F. Borceux : Handbook of categorical algebra, Vol. 1-2 (Cambridge University Press). D. Bourn et M. Gran, Regular, Protomodular and Abelian Categories, (Cambridge University Press) (disponible sur iCampus). P. Freyd : Abelian categories (disponible sur iCampus). S. Mac Lane : Categories for the Working Mathematician (Springer).
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)				
Program title	Acronym	Credits	Prerequisite	Aims
Master [120] in Mathematics	MATH2M	5		🔍
Master [60] in Mathematics	MATH2M1	5		🔍