UCLouvain

lmat1235

2018

Some notions of mathematical logic

5 credits	30.0 h + 15.0 h	Q2

This learning unit is not being organized during this academic year.

Language :	French
Place of the course	Louvain-la-Neuve
Main themes	We start with a naive viewpoint on the concept of a set. Within this framework we introduce ordinals and cardinals. While developing this theory we see clearly that our naive approach is not tenable. We then start studying Zermelo and Fraenkel's axiomatic set theory. We give particular attention to problems of independence and (in)coherence, taking as particular examples the axiom of choice and the continuum hypothesis. In parallel we sketch the basics of propositional calculus and predicate calculus, that is to say, the structure of first-order languages, which we need to fully understand and solve the problems that appear within set theory.
Aims	1 Contribution of the course to learning outcomes in the Bachelor in Mathematics programme. By the end of this activity, students will have made progress in: - Recognising and understanding a basic foundation of mathematics. He will in particular have developed his capacity to: 'Choose and use the basic tools of calculation to solve mathematical problems. ' Recognise the fundamental concepts of important current mathematical theories. ' Establish the main connections between these theories, analyse them and explain them through the use of examples. - Identify, by use of the abstract and experimental approach specific to the exact sciences, the unifying features of different situations and experiments in mathematics or in closely related fields. - Show evidence of abstract thinking and of a critical spirit. He will in particular have developed his capacity to: ' Argue within the context of the axiomatic method 'Recognise the key arguments and the structure of a proof. ' Construct and draw up a proof independently. ' Evaluate the rigour of a mathematical or logical argument and identify any possible flaws in it. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Reason within the framework of propositional and predicate calculus, make a natural deduction. - Recognise whether a given collection of objects forms a set. - Use the theory of ordinals and cardinals to determine the size of a set, and to compare the sizes of two given sets. - Use transfinite induction and Zorn's lemma. - Understand the status of the axiom of choice and the continuum hypothesis within the axiomatic settings of Zermelo--Fraenkel and von Neumann--Bernays--Gödel. - - - - *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled “Programmes/courses offering this Teaching Unit”.*
Evaluation methods	The assessment is based on a written examination (exercises, 40%) and an oral examination (theory, 60%). These examinations test knowledge and understanding of fundamental concepts and results, the ability to construct and write a coherent argument, and mastery of the techniques of logic.
Teaching methods	The learning activities consist of lectures and exercise sessions. The lectures aim to introduce fundamental concepts, to motivate them by giving examples and proving results, to show their reciprocal connections and their connections with other courses in the programme for the Bachelor in Mathematics. The exercise sessions aim to teach the basic calculation techniques of propositions and predicates, that is to say, of first-order structures and languages.
Content	The aim of this activity is to make the laws that govern mathematical reasoning when it is presented as a formal theory explicit. We examine which particular languages to use, which properties to take as a starting point, which deduction rules are commonly admitted. As an example we consider naive set theory and its formalisations (ZF) and (NBG) and arithmetic and its formalisations (PA, RA).

	We focus on the limits of the formalisation process itself, in particular on the question whether it is possible at all to be strictly rigourous. The spirit and the presentation are the same as in any other course in Mathematics: we give definitions, build chains of propositions, prove theorems. The following themes are touched upon in the framework of this course: - Naive set theory: ordinals and cardinals - Axiomatic set theory: (ZF) and (NBG), the axiom of choice, coherence - Propositional calculus, predicate calculus
Inline resources	Website iCampus (http://icampus.uclouvain.be /). The site contains the lecture notes, the problems for the exercise sessions and a detailed overview of the course.
Bibliography	Syllabus disponible sur iCampus. Références de base: K.J. Devlin, Fundamentals of Contemporary Set Theory, Springer, 1979 K. Hrbacek, K.T. Jech, Introduction to Set Theory, 3rd Edition, Marcel Dekker, 1999
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Aims
Additionnal module in Mathematics	LMATH100P	5		