UCL Université catholique de Louvain

LMAT2150

2015-2016

Category theory

5.0 credits	45.0 h	2q

Teacher(s) :	Vitale Enrico ; Gran Marino ;
Language :	Anglais
Place of the course	Louvain-la-Neuve
Inline resources:	<!--{cke_protected}{C}%3C!%2D%2D%0A%20%2F*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F*%20Style%20Definitions%20*%2F%0Ap.MsoNormal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-update%3Aauto%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0Ap.CorpsA%2C%20li.CorpsA%2C%20div.CorpsA%0A%09%7Bmso-style-name%3A%22Corps%20A%22%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--> Website iCampus (> http://icampus.uclouvain.be/).

Prerequisites :	<!--{cke_protected}{C}%3C!%2D%2D%0A%20%2F*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F*%20Style%20Definitions%20*%2F%0Ap.MsoNormal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-update%3Aauto%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0Ap.CorpsA%2C%20li.CorpsA%2C%20div.CorpsA%0A%09%7Bmso-style-name%3A%22Corps%20A%22%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--> LMAT1131 - linear algebra (first year of the bachelor program in mathematics) or any equivalent cours. LMAT 1231 - multilinear algebra and group theory (second year of the bachelor program in mathematics) or any equivalent cours.
Main themes :	<!--{cke_protected}{C}%3C!%2D%2D%0A%20%2F*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F*%20Style%20Definitions%20*%2F%0Ap.MsoNormal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-update%3Aauto%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0Ap.CorpsA%2C%20li.CorpsA%2C%20div.CorpsA%0A%09%7Bmso-style-name%3A%22Corps%20A%22%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--> Categories, functors, natural transformations. Adjoint functors and equivalences of categories. Limits and colimits. Regular, exact and abelian categories. Exact sequences and homological lemmas.
Aims :	Contribution of the course to learning outcomes in the Master in Mathematics programme. By the end of this activity, students will have made progress in: - Recognise and understand a basic foundation of mathematics.He will have made progress in: -- Recognise the fundamental concepts of some important current mathematical theories. -- Establish the main connections between these theories. - Show evidence of abstract thinking and of a critical spirit. He will have made progress in: -- Identify the unifying aspects of different situations and experiences. -- Argue within the context of the axiomatic method. -- Construct and draw up a proof independently, clearly and rigorously. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Identify, in his mathematical knowledge, several meaningful examples of categories, functors and natural transformations. - Establish the adjointness of some pairs of functors and the equivalence of some categories. - Construct limits and colimits, eventually using adjoint functors and equivalences of categories. - Recognise and prove some important exactness properties of regular, exact and abelian categories. - Concretely explain different notions and results in the categories of sets, groups, abelian groups and topological groups. *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".*

Evaluation methods :	Assessment is by oral examination. This tests knowledge and understanding of concepts, examples and fundamental results, ability to construct a coherent argument, and ability to master the techniques of proof introduced in the course. The examination is usually based on six questions, two of which are chosen by the student. Students may also choose the examination language (English or French).
Teaching methods :	The course is taught through lectures. During the sessions, students are asked to give suggestions and formulate ideas on the basis of their previous knowledge in order to further the course. Special attention is paid to the analysis of connections between the new concepts introduced in the course and the other courses in the Bachelor and Master in Mathematics.
Content :	In this course we introduce the basic language and some fundamental results in category theory, in order to explain some mathematical situations encountered in other courses during the bachelor program and the first year of the master program in mathematics. The following subjects are studied: - Definition and examples of categories, functors, natural transformations. - Isomorphisms, monomorphisms and epimorphisms in a category. - Adjoint functors (unit, counit, triangular identities) and their fundamental properties. Reflective subcategories and equivalences of categories. - Special limits and colimits. Existence and construction of limits and colimits. - Limits and adjoint functors. Freyd's adjoint functor theorem. - Definition of regular and of exact category, main properties and examples. Barr-Kock theorem. Mal'tsev categories. - Exact sequences, five lemma, nine lemma, snake lemma.
Bibliography :	& mp;amp;lt;!--{cke_protected}{C}%3C!%2D%2D%0A%20%2F*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%3B%7D%0A%20%2F*%20Style%20Definitions%20*%2F%0Ap.MsoNormal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-update%3Aauto%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0Ap.CorpsA%2C%20li.CorpsA%2C%20div.CorpsA%0A%09%7Bmso-style-name%3A%22Corps%20A%22%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--& mp;amp;gt; Lecture notes for exact, additive and abelian categories (available on the iCampus website). F. Borceux : Handbook of categorical algebra, Vol. 1-2 (Cambridge University Press). D. Bourn et M. Gran, Regular, Protomodular and Abelian Categories, (Cambridge University Press) (available on the iCampus website). P. Freyd : Abelian categories (available on the iCampus website). S. Mac Lane : Categories for the Working Mathematician (Springer).
Faculty or entity in charge:	MATH

Programmes / formations proposant cette unité d'enseignement (UE)				
Intitulé du programme	Sigle	Credits	Prerequis	Acquis d'apprentissage
Master [120] in Mathematics	MATH2M	5	-	🔍
Master [60] in Mathematics	MATH2M1	5	-	🔍