UCL Université catholique de Louvain

LMAT2110

2014-2015

Eléments de géométrie différentielle

5.0 credits	30.0 h + 30.0 h	1q

Teacher(s) :	Haine Luc ;
Language :	Français
Place of the course	Louvain-la-Neuve
Inline resources:	The iCampus website (> http://icampus.uclouvain.be/) provides the syllabus of the course with bibliographical references as well as all problem sets for the exercise sessions.
Prerequisites :	Geometry LMAT1141 and LMAT1241. Language skills: French (written and spoken) at high school level.
Main themes :	Differentiable manifolds, vector fields and Lie bracket of vector fields, differential forms.
Aims :	Contribution of the course to learning outcomes in the Bachelor in Mathematics programme. By the end of this activity, students will have made progress in: - Recognise and understand a basic foundation of mathematics. In particular: -- Choose and use the basic tools of calculation to solve mathematical problems. -- Recognise the fundamental concepts of important current mathematical theories. -- Establish the main connections between these theories, analyse them and explain them through the use of examples. - Identify, by use of the abstract and experimental approach specific to the exact sciences, the unifying features of different situations and experiments in mathematics or in closely related fields. - Show evidence of abstract thinking and of critical spirit. In particular: -- Argue within the context of the axiomatic method. -- Recognise the key arguments and the structure of a proof. -- Construct and draw up a proof independently. -- Distinguish between the intuition and the validity of a result and the different levels of rigorous understanding of this same result. - Be clear, precise and rigorous in communicating. In particular: -- Write a mathematical text in French according to the conventions of the discipline. -- Structure an oral presentation in French, highlight key elements, identify techniques and concepts and adapt the presentation to the listeners' level of understanding. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Define a differentiable manifold by charts, by equations or by a parametrization. - Study a vector field on a variety, link its singular points to the topology of the variety, visualize the flow on simple examples. - Master the computation and the geometric meaning of the Lie bracket of vector fields and some of its applications. - Master the fundamental tool of differential forms and some of its applications (Stokes-Cartan theorem and degree theory). *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".*
Evaluation methods :	Evaluation is on the basis of a written examination relating in equal parts to theory and to practical exercises. The examination tests knowledge and understanding of fundamental concepts and results, ability to construct and to write a coherent argument, ability to supply examples and counter-examples, and mastery of the techniques of calculation. Oral presentations given during exercise sessions account for 4 out of 20 marks in the final grade.
Teaching methods :	Learning activities consist of lectures and practical exercises. The lectures aim to introduce the basic concepts, to explain them through showing examples and producing results, to show their reciprocal connections and their relationship to other courses in the Bachelor in Mathematics programme. The aim of the practical exercises is to construct proofs, to study numerous examples and counter-examples, and to master the methods of calculation. In each session a number of students are asked to present at the board exercises or additional theoretical material that have previously been assigned to them.
Content :	The course is a follow up to LMAT1141 and LMAT1241 which deal principally with the study of curves and surfaces in R^2 and R^3. One studies abstract differentiable manifolds, vector fields and differential forms with form the basis of differential geometry, in particular to study Riemannian geometry and Lie groups. The following subjects are treated in the course: - Differentiable manifolds: notion, examples, topology of a manifold, submanifolds, morphisms between manifolds. - Vector fields : tangent space and point derivations, differential of a morphism (immersions, submersions and embeddings), tangent bundle, vector fields and derivations, integral curves and flows, Poincaré-Hopf theorem, critical points and topology of manifolds (gradient vector fields). - Lie bracket of vector fields: definition, Lie bracket and Poisson bracket, Hamiltonian vector fields and Arnold-Liouville theorem, Frobenius theorem.

	- Differential forms: exterior forms, differential forms (exterior bundles, exterior differentiation, Cartan formula), Poincaré Lemma, integration on orientable manifolds, Sokes-Cartan formula and elements of degree theory.
Bibliography :	Syllabus available on iCampus. Other references : - M. Berger et R. Gostiaux, Géométrie différentielle: variétés, courbes et surfaces, P.U.F., Paris 1992. J. Lafontaine, Introduction aux variétés différentielles, Presses Universitaires de Grenoble 1996. V.I. Arnold, Méthodes mathématiques de la mécanique classique, Editions Mir, Moscou 1976.
Cycle and year of study :	> Bachelor in Mathematics > Bachelor in Economics and Management > Bachelor in Engineering > Bachelor in Physics > Master [120] in Mathematical Engineering
Faculty or entity in charge:	MATH