UCL Université catholique de Louvain

LMAT1341

2014-2015

Courbes elliptiques

3.0 credits	30.0 h	1q

Teacher(s) :	Haine Luc ;
Language :	Français
Place of the course	Louvain-la-Neuve
Inline resources:	The iCampus website (> http://icampus.uclouvain.be/) provides a course outline, bibliographical references, as well as the problem sets to be done during the semester.
Prerequisites :	Complex analysis LMAT1222. Language skills: French (written and spoken) at high school level.
Main themes :	Elliptic functions of Weierstrass and Jacobi, associated elliptic curves, Abel's theorem, addition theorem, selected applications in geometry, mechanics and number theory.
Aims :	Contribution of the course to learning outcomes in the Bachelor in Mathematics programme. By the end of this activity, students will have made progress in: - Recognise and undertsand a basic foundation of mathematics. In particular: -- Choose and use the basic tools of calculation to solve mathematical problems. -- Recognise the fundamental concepts of important current mathematical theories. -- Establish the main connections between these theories, analyse them and explain them through the use of examples. - Identify, by use of the abstract and experimental approach specific to the exact sciences, the unifying features of different situations and experiments in mathematics or in closely related fields. - Show evidence of abstract thinking and of critical spirit. In particular; -- Argue within the context of the axiomatic method. -- Recognise the key arguments and the structure of a proof. -- Construct and draw a proof independently. -- Evaluate the rigour of a mathematical or logical argument and identify any possible flaws in it. - Be clear, precise and rigorous in communicating. -- Write a mathematical text in French according to the conventions of the discipline. -- Structure an oral presentation in French, highlight key elements, identify techniques and concepts and adapt the presentation to the listeners' level of understanding. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Construct holomorphic and meromorphic functions in terms of infinite series or products. - Apply Abel's theorem and the addition theorem of elliptic functions theory in various contexts. - Solve problems which use elliptic functions and elliptic curves. *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".*
Evaluation methods :	Assessment is based on an oral examination relating to theory and on the individual assignment carried out during the term, in equal parts. The examination tests knowledge and understanding of fundamental concepts and results, ability to solve problems and ability to draft the solutions with rigour and clarity.
Teaching methods :	Learning activities consist of lectures which aim to introduce fundamental concepts, to explain them by showing examples and by determining their results, to show their reciprocal connections and their connections with other courses in the programme for the Bachelor in Mathematics. At the beginning of the semester students receive a list of problems which constitute an individual assignment to be performed, on which they will deliver a report at the end of the semester. Some lectures are devoted to informal discussion of these problems.
Content :	The course aims at introducing to the theory of algebraic curves via the study of elliptic curves and to study some of their applications. It tries to show how the subject combines in an attractive way three important themes of mathematics: complex analysis, geometry and number theory. The following subjects are treated in the course. - The Riemann sphere: definition, compactness, automorphisms, behaviour of functions at infinity, meromorphic functions on the Riemann sphere. - Construction of holomorphic and meromorphic functions: series of holomorphic and meromorphic functions, infinite products of holomorphic functions, examples. - Elliptic functions and elliptic curves: complex tori, Abel's theorem, Weierstrass theory (Pfunction, zeta and sigma functions), Jacobi's theory (sn, cn and dn functions), associated elliptic curves and addition theorems, inversion problem of elliptic integrals. - Applications: pendulum equation, Poncelet problem, Mordell-Weil theorem.

<table>
<tr><td>Bibliography :</td><td>& t;!--{cke_protected}{C}%3C!%2D%2D%0A%20%2F*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F*%20Style%20Definitions%20*%2F%0Ap.MsoNormal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-update%3Aauto%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0Ap.Corps%2C%20li.Corps%2C%20div.Corps%0A%09%7Bmso-style-name%3ACorps%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22%E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--& t;
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G.A. Jones, D. Singerman, Complex functions, an algebraic and geometric viewpoint, Cambridge University Press, Cambridge, 1987.
H. McKean, V. Moll, Elliptic curves, Function theory, geometry, arithmetic, Cambridge University Press, Cambridge, 1997.</td></tr>
<tr><td>Cycle and year of study :</td><td>> Bachelor in Mathematics</td></tr>
<tr><td>Faculty or entity in charge:</td><td>MATH</td></tr>
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