LMAT1322

2014-2015

Measure theory

3.0 credits	22.5 h + 15.0 h	1q

Teacher(s) :	Roselli Paolo ;
Language :	Français
Place of the course	Louvain-la-Neuve
Inline resources:	Website iCampus (> http://icampus.uclouvain.be/). The website stores the lecture notes, the texts of the exercise lectures, and other documents and links allowing to get a deeper insight in the subject and feed his/her own curiosity.
Prerequisites :	Mathematical skills acquired in the courses LMAT1121, LMAT1122 et LMAT1221. Master French language at the level corresponding to the last year of the secondary school education at least.
Main themes :	The outer Lebesgue measure on the real line R. Carathéodory outer abstract measures. Measures on a sigma algebra of subsets. Lebesgue-Stiltjes measures on R. Measurable functions. Integration of measurable functions with respect to a measure. Convergence theorems (Lebesgue's dominated Theorem, Levi's monotone convergence Theorem and Fatou's Lemma). The L^p spaces. Product measures. Fubini's and Tonelli's Theorems. Convergence of sequences of measures. Connections with Probability Theory.
Aims :	Contribution of the course to learning outcomes in the Bachelor in Mathematics programme. By the end of this activity, students will have made progress in: - knowing and understanding a fundamental subject of the mathematics. He/her will have developed in particular his/her capacity in: -- choosing and using methods and fundamental tools of calculation to solve mathematical problems ; -- recognizing the fundamental concepts of certain current mathematical theories ; -- establishing the main links between those theories, explaining them and motivating them by examples ; --identifying the unifying aspects of different situations and phenomena in the mathematical science, thanks to the abstract and experimental approach peculiar to the exact sciences. ' showing abstraction and critical mind. He/her will have developed in particular his/her capacity in: -- reasoning within the framework of the axiomatic method. -- Recognizing the key arguments and the structure of a mathematical proof. -- Structuring and drafting a mathematical proof in a autonomous way. -- Assessing the rigor of a mathematical reasoning and revealing its possible weaknesses. -- making the distinction between the intuition of the validity of a result and the various levels of rigorous understanding of the same result. Learning outcomes specific to the course. By the end of this activity, students will be able to: - retrace the historic evolution of the mathematical concepts introduced during the course and the their associated problems. - master, in an abstract context, the fundamental theorems of integration learned during the previous courses of analysis for the case of the Euclidean spaces, harmonizing the latter with the example of the outer Lebesgue measure. - Master the techniques of proof peculiar to measure theory. - Recognize the presence of certain algebraic structures in class of subsets. - Build a measure starting from a countable additive set function defined on a semi-algebra of subsets or starting from a sequence of suitably chosen measures. - Integrate a measurable function with respect to a measure. - Use the Convergence Theorems. - Know the main connections between the notions of measure and probability. *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled “Programmes/courses offering this Teaching Unit”.*
Evaluation methods :	Assessment is based on an oral examination covering both theory and exercises. The oral test will have two parts: one is theoretical and will include three questions from the course. The other part will include two exercise topics; students will be asked to choose and deal with one of them. The oral examination tests knowledge and understanding of fundamental concepts and results, ability to construct and write a coherent argument, and mastery of the techniques of calculation.

Teaching methods :	Learning activities consist of lectures and exercise sessions. The lectures aim to introduce fundamental concepts, to explain them by showing examples and by determining their results, to show their reciprocal connections and their connections with other courses in the program for the Bachelor in Mathematics. The exercise sessions aim to teach how to select and use calculation methods and how to construct proofs. The two activities are given in presential sessions.
Content :	This activity consists in introducing the notion of measure and integration in an abstract frame, starting from the notion of length of an interval of real numbers. Such subject plays an essential role in all the Bachelor and Master degrees in mathematical sciences and in physical sciences. The study of the notion of length on the real line R, the area on the plane R^2 and the volume in the space R^3 is at the same time an objective of the course and the paradigm which motivates the introduction of the notion of an abstract measure. Within the framework of the course, the following contents are approached. - A starting extension of the length of an interval. - The Lebesgue's measure on R. - The Carathéodory criterion of measurability. - A measure on a sigma algebra of subsets. - The Stieltjes-Lebesgue-Radon measures. - Measurable functions. - The integral of a measurable function with respect to a measure. - Convergence theorems. - L^p spaces. - Product measures. - Fubini's and Tonelli's Theorems. - Convergence of a sequence of measures. - Connections with Probability Theory.
Bibliography :	Lecture notes available on the iCampus website.
Cycle and year of study :	> Master [120] in Statistics: General > Bachelor in Mathematics > Bachelor in Information and Communication > Bachelor in Philosophy > Bachelor in Pharmacy > Bachelor in Computer Science > Bachelor in Economics and Management > Bachelor in Motor skills : General > Bachelor in Human and Social Sciences > Bachelor in Sociology and Anthropology > Bachelor in Political Sciences: General > Bachelor in Biomedicine > Bachelor in Engineering > Bachelor in religious studies
Faculty or entity in charge:	MATH