Multiple-Criteria Decision-Making Techniques and Their Role in Livestock Ration Formulation

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SUMMARY

The livestock ration formulation problem is postulated within the framework of multiple-criteria decision-making techniques. This exercise is motivated by the fact that the ordinary least-cost approach can, and does, generate solutions that either cannot be implemented or supply nutritionally undesirable levels of various nutrients. This originates from using cost as the criterion for selecting the ingredients of the diet. But in fact, diet formulation problems involve several criteria and should, therefore, be solved using techniques designed for modelling such problems. This paper is an attempt at introducing these techniques to agricultural systems modellers and then demonstrating their use in livestock ration formulation. The multiple-criteria decision-making techniques covered include goal programming and its variants such as weighted and lexicographic approaches and multiple-objective programming.

INTRODUCTION

Of various mathematical optimisation techniques, linear programming (LP) is probably the most commonly used technique for establishing the
least cost combination of foods that will meet a specified level of nutritional requirements for livestock. Waugh (1951) was perhaps first in applying LP to problems of animal nutrition. Subsequently, several successful attempts have been made to explain the underlying methodology and its practical applicability (for an excellent treatment see Dent & Casey, 1967).

Despite its widespread use, the theoretical development of LP as a means of formulating livestock rations seems to have remained static over the years. The original analytical framework has been extended somewhat by such attempts as: (a) the introduction of probabilistic restraints (Rahman & Bender, 1971; Chen, 1973); (b) the investigation of the relationship between bulk and cost (Mohr, 1972) and (c) the use of the technique within a practical environment (Chappell, 1974; Crabtree, 1982).

The main weakness of the LP paradigm when applied to ration formulation lies in the exclusive reliance on cost as the only decision criterion. This is a very rigid assumption, as more often than not the decision-maker is interested in an economically optimal ration that achieves a compromise amongst several conflicting objectives such as minimisation of cost, imbalances of nutrient supplies and food bulk, and the satisfaction of certain conditions like the calcium/phosphorus ratio. A further weakness of LP is the rigidity with which given nutritional requirements have to be met. In many real life situations these requirements do not impose nutritional constraints that are absolutely binding.

We believe that these methodological weaknesses of LP can be overcome, and its application to diet formulation can be substantially improved if the problem is analysed within the relatively new framework of multiple-criteria decision-making techniques. Such an approach offers the prospect of important practical advantages.

This paper introduces the applicability of multiple-criteria decision-making techniques to agricultural systems modellers. The techniques to be explored in this context are: goal programming (GP) and its variants: weighted goal programming (WGP) and lexicographic goal programming (LGP), and multi-objective programming (MOP).

To our knowledge no previous attempt exists that analyses livestock ration formulation as a problem involving multiple objectives and goals; except, perhaps, the works of Arthur & Lawrence (1980) where the blending problems—closely related to the diet problem—is analysed

It should be remembered, however, that our main purpose is to illustrate the applicability of a new method; therefore, it is quite possible that some of the rations obtained may be unimplementable, and/or the problems postulated in this paper are not particularly realistic or interesting from a nutritional point of view. At this stage we wish to explain and illustrate a new method rather than solve specific animal feeding problems.

LIVESTOCK RATION FORMULATION EXAMPLE INVOLVING MULTIPLE-CRITERIA DECISIONS

This section presents a ration formulation problem published in this journal (Crabtree, 1982) which is used as a basis for discussion in this paper. Crabtree's example has been chosen as it has been derived from the most widely accepted sources of information on animal nutrition and also because it seems to have many of the problems that lend themselves to analysis by multiple-criteria decision-making techniques.

Crabtree's LP model analyses the ration formulation of a 600 kg dairy cow with a milk yield of 30 kg/day and no liveweight change. The available ingredients, their compositions, and nutritional requirements of the animal are given in Table 1. However, for our own purposes we have relaxed somewhat the upper limits on total dry matter intake and the amount of swedes and distillers' wet grain in the ration as well as the proportion of dry matter derived from silage and straw.

Most of Table 1 is self-explanatory but the constraint (14) means that at least 20% of the dry matter intake must be silage and straw, while the constraint (16) implies that the dry matter intake from distillers' wet grains must not exceed 20% of the total dry matter.

The least-cost diet for this problem is comprised of the following:

Silage ($x_1$) = 4.981 kg. Straw ($x_2$) = 0.0 kg.
Distillers' wet grains ($x_3$) = 3.436 kg. Swedes ($x_4$) = 2.0 kg.
Barley ($x_5$) = 2.778 kg. Dairy compound ($x_6$) = 5.069 kg.

The diet reported in the above solution and the subsequent ones discussed in this paper are all given as units of kg DM/day.
TABLE 1
Crabtree's Modified Matrix

<table>
<thead>
<tr>
<th>Silage (kg DM)</th>
<th>Straw (kg DM)</th>
<th>Distillers' wet grains (kg DM)</th>
<th>Swedes (kg DM)</th>
<th>Barley (kg DM)</th>
<th>Dairy compound (kg DM)</th>
<th>Transfer activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>48.2</td>
<td>26.8</td>
<td>69.8</td>
<td>92.6</td>
<td>105.6</td>
<td>162.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

10.3          6.5          10.3          12.8          13.0          12.8          0.0          $\geq$ 213.3 Metabolisable energy—ME (MJ) (1)
108.0         32.0         138.6         97.2          105.3         107.2         0.0          $\geq$ 1663.3 Rumen Degradable Protein—RDP (g) (2)
19.0          8.0          59.4          10.8          11.7          52.8          0.0          $\geq$ 620.5 Undegradable Protein—UDP (g) (3)
3.9           2.7          1.7           3.6           0.5           11.4          0.0          $\geq$ 73.6 Calcium—Ca (g) (4)
3.3           0.9          3.7           3.2           4.0           8.5           0.0          $\geq$ 67.8 Phosphorus—P (g) (5)
1.6           0.7          1.4           1.2           1.3           3.5           0.0          $\geq$ 24.0 Magnesium—Mg (g) (6)
3.8           1.1          0.9           2.6           0.2           5.4           0.0          $\geq$ 26.9 Sodium—Na (g) (7)
5.7           3.2          10.0          3.8           4.8           19.0          0.0          $\geq$ 180.0 Copper—Cu (mg) (8)
0.09          0.04         0.2           0.04          0.04          1.50          0.0          $\geq$ 2.0 Cobalt—Co (mg) (9)
1.0           1.0          1.0           1.0           1.0           1.0           0.0          $\leq$ 19.0 Dry matter intake—DM (kg) (10)
1.0           1.0          1.0           1.0           1.0           1.0           0.0          $\leq$ 0.0 Tie line (11)
0.56          0.35         0.56          0.70          0.71          0.70          $\leq$ 0.0637  $\leq$ 0.0 Metabolisability minimum $q$ (12)
0.56          0.35         0.56          0.70          0.71          0.70          $\leq$ 0.0642  $\leq$ 0.0 Metabolisability maximum $q + r$ (13)
-0.8          -0.8         0.2           0.2           0.2           0.2           0.0          $\leq$ 0.0 Long roughage (%) (14)
0             0            0             0             0             0             0.0          $\leq$ 2.0 Swedes (kg) (15)
-0.2          -0.2         0.8           -0.2          -0.2          -0.2          0.0          $\leq$ 0.0 Distillers' grain (%) (16)
1.0           0            0             0             0             0             0.0          $\leq$ 6.5 Silage (kg) (17)

Source: Crabtree (1982, p. 301) with modifications to right-hand side parameters for dry matter and swedes and the proportionate amount of distillers' wet grain that should be included in the ration.
This ration costs £1.782/day and Table 2 compares its composition with the specified requirements. The nutrient requirements are binding restraints only for metabolisable energy, undegradable protein and copper. Several nutrients are in surplus supply—particularly magnesium, sodium and cobalt—which may be an undesirable nutritional balance in the diet. This situation could be redeemed by setting upper, as well as lower, limits for each nutrient. This approach, however, produces over-constrained problems with empty feasible sets in many cases. For instance, if, in this example, the supplies of sodium, copper and cobalt are bound to upper limits of 50 g, 400 mg and 6 mg, respectively, there is no solution that satisfies all the restrictions.

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Requirements</th>
<th>Ration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metabolisable energy (MJ)</td>
<td>213.3</td>
<td>213.3</td>
</tr>
<tr>
<td>Rumen-degradable protein (g)</td>
<td>1663.9</td>
<td>2044.6</td>
</tr>
<tr>
<td>Undegradable protein (g)</td>
<td>620.5</td>
<td>620.5</td>
</tr>
<tr>
<td>Calcium (g)</td>
<td>73.6</td>
<td>91.6</td>
</tr>
<tr>
<td>Phosphorus (g)</td>
<td>67.8</td>
<td>89.8</td>
</tr>
<tr>
<td>Magnesium (g)</td>
<td>24.0</td>
<td>36.5</td>
</tr>
<tr>
<td>Sodium (g)</td>
<td>26.9</td>
<td>55.1</td>
</tr>
<tr>
<td>Copper (mg)</td>
<td>180.0</td>
<td>180</td>
</tr>
<tr>
<td>Cobalt (mg)</td>
<td>2.0</td>
<td>8.9</td>
</tr>
<tr>
<td>Dry matter (kg)</td>
<td>19.0</td>
<td>18.30</td>
</tr>
</tbody>
</table>

The reduction of surplus supplies of various nutrients in least-cost diets ought to be one of the goals in formulating rations using mathematical techniques. Anderson & Earle (1983) have argued this point and used minimisation of nutritional imbalance as the objective function in formulating human diets using goal-programming techniques. However, their particular purpose could have been equally well served by using a traditional LP approach, as demonstrated by Romero & Rehman (1984b). Nonetheless, the emphasis on cost as the single criterion within the framework of LP does nothing to reduce the chances of producing nutritionally undesirable rations.

Besides cost and nutritional imbalances, another important consideration in ration formulation is how bulky the diet is. Obviously, rations
which are bulky, either because of low physical density or because of high moisture content, imply considering extra costs of storage and handling, and the limitations imposed by storage capacity. Therefore, reducing the bulk of the ration becomes a desirable goal. With this in mind, Crabtree’s example can now be reformulated as a problem with three different objectives: (a) minimise the cost of the ration; (b) minimise the imbalanced supply of copper, sodium and cobalt and (c) minimise the bulk (i.e. the wet weight) of the diet.

Obviously, such a formulation of the diet selection problem leads to a situation where the achievement of one objective is in conflict with the attainment of another. But such a problem cannot be solved by optimising each objective function independently. However, problems with multiple goals and objectives can be solved using goal-programming and multi-objective programming techniques. This is what we set out to do in the next sections.

RATION FORMULATION AS A WEIGHTED GOAL-
PROGRAMMING PROBLEM

The theory behind weighted goal-programming techniques (WGP) was first introduced to the management science literature by Charnes & Cooper (1961). Any problem involving multiple goals or objectives is solved in such a way that the solution ensures their satisfaction. This procedure is in contrast to LP where only one objective can be optimised while the rest has to be treated as restraints. However, the concept of a goal is fundamental to these techniques which is obtained on combining an objective with a target. The targets are desired levels of achievement for any one of a decision-maker’s objectives (Romero & Rehman, 1983). In the context of ration formulation minimising the bulk of a ration is an objective, while the specific target of obtaining it at a level less than 40 kg is a goal reflecting the aspirations of the ration-formulator.

WGP minimises the deviations among the desired levels of goals and the actual results. This is included in the model by converting inequalities into equalities through the addition of positive and negative deviation variables that permit either under- or over-achievement of each goal. WGP considers all goals simultaneously in a composite objective function. This function minimises the sum of all the deviations among goals and their aspiration levels. The deviations are weighted according to
the relative importance of each goal. For a detailed explanation of this technique in the context of farm planning see Romero & Rehman (1984a).

Now the three objectives proposed in the previous section can be converted into goals as follows.

**Goal g₁**

The ration should not have a bulk larger than 40 kg. Using the dry matter content of various ingredients from Table 3, the expression for goal $g_1$ is given by:

$$3.704x_1 + 1.219x_2 + 3.876x_3 + 9.259x_4$$

$$+ 1.200x_5 + 1.137x_6 + n_1 - p_1 = 40 \quad (1)$$

where the deviational variable $n_1$ measures the under-achievement of goal $g_1$ while $p_1$ plays the opposite role. As the desired level of ration bulk should not be greater than 40 kg, the deviational variable $p_1$ must be minimised.

**Goal g₂**

The supply of sodium in the diet should not exceed 100% of its specified requirements. This imbalance goal, $I_{Na}$, is obtained from constraint (7) in Table 1:

$$I_{Na} = 3.8x_1 + 1.1x_2 + 0.9x_3 + 2.6x_4 + 0.2x_5 + 5.4x_6 - 26.9 \quad (2)$$

Considering eqn. (2) as percentages instead of absolute values, we have:

$$\frac{(3.8x_1 + 1.1x_2 + 0.9x_3 + 2.6x_4 + 0.2x_5 + 5.4x_6 - 26.9) \times 100}{26.9} \quad (3)$$

Therefore the expression for $g_2$ is:

$$14.13x_1 + 4.09x_2 + 3.35x_3 + 9.67x_4$$

$$+ 0.74x_5 + 20.07x_6 + n_2 - p_2 = 200 \quad (4)$$

To achieve the desired level of this goal $p_2$ must be minimised.

**Goals g₃ and g₄**

Using the same procedure as for sodium, the goals for copper and cobalt are derived from constraints (8) and (9) in Table 1, so that their supplies,
TABLE 3
Dry Matter Content of Ingredients (g/kg)

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Silage</th>
<th>Straw</th>
<th>Distillers' wet grains</th>
<th>Swedes</th>
<th>Barley</th>
<th>Dairy compound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry matter content</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silage Straw</td>
<td>270</td>
<td>820</td>
<td>258</td>
<td>108</td>
<td>833</td>
<td>880</td>
</tr>
<tr>
<td>Distillers' Swedes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barley</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dairy compound</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


too, should not exceed 100 % of the specified requirements. This is stated by eqns (5) and (6).

\[
3 \cdot 17x_1 + 1 \cdot 78x_2 + 5 \cdot 55x_3 + 2 \cdot 11x_4 + 2 \cdot 67x_5 + 10 \cdot 55x_6 + n_3 - p_3 = 200 \tag{5}
\]

\[
4 \cdot 5x_1 + 2x_2 + 10x_3 + 2x_4 + 2x_5 + 75x_6 + n_4 - p_4 = 200 \tag{6}
\]

For achieving \( g_3 \) and \( g_4 \), \( p_3 \) and \( p_4 \) must be minimised.

**Goal \( g_5 \)**

Lastly, the objective of minimising the cost can be converted into a goal by setting a target of £1.782 as cost, which corresponds to the minimum cost associated with the ration recommended by the ordinary LP approach. Therefore, the expression for \( g_5 \) using the unit costs of ingredients from Table 1, is:

\[
0 \cdot 0482x_1 + 0 \cdot 0268x_2 + 0 \cdot 0698x_3 + 0 \cdot 0926x_4 + 0 \cdot 1056x_5 + 0 \cdot 1625x_6 + n_5 - p_5 = 1 \cdot 782 \tag{7}
\]

To achieve \( g_5 \), \( p_5 \) is minimised.

The variables of the objective function must represent percentage deviations from the targets. Minimisation of absolute deviations does not make sense when each goal is measured in different units. Hence, the elements of the objective function are standardised for the WPG model to give:

\[
W_1 \frac{p_1}{40} \frac{100}{1} + W_2 \frac{p_2}{200} \frac{100}{1} + W_3 \frac{p_3}{200} \frac{100}{1} + W_4 \frac{p_4}{200} \frac{100}{1} + W_5 \frac{p_5}{1.782} \frac{100}{1} \tag{8}
\]
TABLE 4
Weighted Goal-Programming Model for Ration Formulation

Objective function
Minimise: $2.5 W_1 p_1 + 0.50 W_2 p_2 + 0.50 W_3 p_3 + 0.50 W_4 p_4 + 56.12 W_5 p_5$

Subject to:
- $3.704 x_1 + 1.219 x_2 + 3.876 x_3 + 9.259 x_4 + 1.200 x_5 + 1.137 x_6 + n_1 - p_1 = 40$ (bulk)
- $14.13 x_1 + 4.090 x_2 + 3.350 x_3 + 9.670 x_4 + 0.740 x_5 + 20.070 x_6 + n_2 - p_2 = 200$ (imbalance in sodium)
- $3.17 x_1 + 1.78 x_2 + 5.55 x_3 + 2.11 x_4 + 2.67 x_5 + 10.55 x_6 + n_3 - p_3 = 200$ (imbalance in copper)
- $4.5 x_1 + 2 x_2 + 10 x_3 + 2 x_4 + 2 x_5 + 75 x_6 + n_4 - p_4 = 200$ (imbalance in cobalt)
- $0.048 x_1 + 0.026 8 x_2 + 0.069 8 x_3 + 0.092 6 x_4 + 0.105 6 x_5 + 0.162 5 x_6 + n_5 - p_5 = 1.782$ (cost)

and

$A x \leq b$ (technical constraints from Table 1)

$x \geq 0$  \hspace{1cm} $n \geq 0$  \hspace{1cm} $p \geq 0$

$x$ is a vector of ration ingredients. In this case $x$ has six elements or possible ingredients.

$n$ is the vector of deviational variables representing under-achievement.

$p$ is the vector of deviational variables representing over-achievement.

As there are five goals being considered in the model; therefore, both $n$ and $p$ have five elements each.
Or:

$$2.5W_1p_1 + 0.50W_2p_2 + 0.50W_3p_3 + 0.50W_4p_4 + 56.12W_5p_5 \quad (9)$$

Where $W_1, \ldots, W_5$ are the weights attached to the deviational variables representing the respective importance given to the achievement of various goals. The structure of this WGP model is given in Table 4. Mathematically, it is a conventional LP problem and can be solved by the Simplex algorithm. Different solutions can be obtained by attaching different values to the $W$ parameters. For instance, if $W_1 = W_2 = \cdots W_5 = 1$, the Simplex method provides the following optimum solution:

- $x_1$ (silage) = 3.511 kg
- $x_2$ (straw) = 0.675 kg
- $x_3$ (distillers' wet grains) = 3.80 kg
- $x_4$ (swedes) = 0
- $x_5$ (barley) = 6.298 kg
- $x_6$ (dairy compound) = 4.716 kg

While the optimum values of the deviational variables are:

- $n_1 = 0$  
- $p_1 = 1.477$ kg
- $n_2 = 35.59\%$  
- $p_2 = 0$
- $n_3 = 100\%$  
- $p_3 = 0$
- $n_4 = 0$  
- $p_4 = 221.41\%$
- $n_5 = 0$  
- $p_5 = 0.10$

This solution permits complete achievement of the goals $g_2$ (sodium

<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td>(3)</td>
<td>(4)</td>
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<td>1</td>
<td>1</td>
<td>5</td>
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</table>
TABLE 6
Sensitivity Analysis of the WGP Solution

<table>
<thead>
<tr>
<th>Run</th>
<th>(x_1) (silage) (kg)</th>
<th>(x_2) (straw) (kg)</th>
<th>(x_3) (distillers' wet grams) (kg)</th>
<th>(x_4) (barley) (kg)</th>
<th>(x_5) (dairy compound) (kg)</th>
<th>Fresh weight (kg/day)</th>
<th>Surplus of Na as per cent of requirement</th>
<th>Deviation from the target</th>
<th>Surplus of Co as per cent of requirement</th>
<th>Deviation from the target</th>
<th>Surplus of Co as per cent of requirement</th>
<th>Deviation from the target</th>
<th>Cost (£)</th>
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<tr>
<td>1</td>
<td>3.511</td>
<td>0.675</td>
<td>3.800</td>
<td>0.675</td>
<td>4.716</td>
<td>41.477</td>
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<td>321.410</td>
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<tr>
<td>2</td>
<td>4.506</td>
<td>0</td>
<td>3.800</td>
<td>0</td>
<td>4.576</td>
<td>43.963</td>
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<td>0.917</td>
<td>3.800</td>
<td>0.917</td>
<td>4.842</td>
<td>41.477</td>
<td>1.477</td>
<td>64.410</td>
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<td>0</td>
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<td>3.800</td>
<td>0.675</td>
<td>4.716</td>
<td>41.477</td>
<td>1.477</td>
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<tr>
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<td>0.917</td>
<td>3.800</td>
<td>0.917</td>
<td>4.842</td>
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supply imbalance) and \( g_3 \) (copper supply imbalance) by virtue of the fact that \( p_1 = p_2 = 0 \). The value of 1.477 kg for \( p_1 \) implies that goal \( g_1 \) has exceeded its target by 1.477 kg or the bulk of the ration is 41.477 kg. Similarly, \( p_4 = 221.41 \% \) means that the imbalance in cobalt has surpassed its target by that proportion; that is, the supply of this nutrient is 321.41 \% of the specified requirements. Finally, \( p_5 = 0.102 \) means that the cost of the ration is £0.102 over the set target of £1.782/day.

The sensitivity analysis of a WGP solution can provide very useful information for ration formulation. To demonstrate the generation and use of this information we have used eleven sets of weights, as shown in Table 5 and corresponding solutions are presented in Table 6. From this information it is possible to obtain something like a measure of the trade-offs between goals. Thus, if, for instance, the decision-maker is undecided between solutions 1 and 2, the former will be preferred if the over-achievement of bulk by 2.486 kg can be traded off with a reduction in cost of £0.012 and for a drop in cobalt imbalance of 10.15 \%. Such an array of information giving trade-offs among various goals should enable the decision-maker to choose the ration that best suits his aims. It must, however, be pointed out that if the weight attached to the cost of the ration is given a value higher than 10, then the LP solution given above is obtained again.

**RATION FORMULATION AS A LEXICOGRAPHIC GOAL PROGRAMMING (LGP) PROBLEM**

The LGP approach was introduced by Charnes & Cooper (1961) and later developed by Ijiri (1965), Lee (1972) and Ignizio (1976). The LGP approach assumes that it is possible to divide goals into priorities. Further pre-emptive weights can be attached to sets of goals situated in different priorities. In other words, the fulfilment of the goals in a given set, \( Q_i \), is immeasurably preferable to the achievement of any other set situated in a lower priority, \( Q_j \). In LGP higher priority goals are satisfied first, and only then are lower priorities considered. For a non-technical exposition of LGP, see Romero & Rehman (1984a).

To illustrate the use of LGP in ration formulation, we have to establish the priority structure of goals in this problem. Let us assume, for example, the goals of obtaining a ration costing less than £2 and weighing below 50 kg are situated in the first priority, \( Q_1 \); that is, they are to be satisfied in
Multiple-criteria decision-making techniques and livestock ration formulation

a pre-emptive way. These targets can be considered as minimum levels of achievement for the cost and bulk of the ration. Using eqns (7) and (1) and on modifying the subscripts of the deviational variables, the goals in priority $Q_1$ are:

Cost: $0.0482x_1 + 0.0268x_2 + 0.0698x_3 + 0.0926x_4$
\[+ 0.1056x_5 + 0.1625x_6 + n_1 - p_1 = 2 \quad (10)\]

Bulk: $3.704x_1 + 1.219x_2 + 3.876x_3 + 9.259x_4$
\[+ 1.20x_5 + 1.137x_6 + n_2 - p_2 = 50 \quad (11)\]

As regards weights attached to the achievement of goals, assume that formulating a ration costing less than £2 is twice as important as the attainment of required bulk. Now using the same procedure as for deriving the objective function of WGP in eqn. (10), the first component to be minimised in the lexicographic process is:

\[\frac{2 \cdot 100}{2} p_1 + \frac{100}{50} p_2 = 100p_1 + 2p_2\]

The next priority $Q_2$, is made up of goals $g_2, g_3$ and $g_4$ described above; that is, the supply–requirement imbalance for sodium, copper and cobalt should not be greater than 100%. This can be derived from eqns (4), (7) and (8), with the appropriate subscripts for the deviational variables; thus the goals in priority $Q_2$ are:

Sodium: $14.13x_1 + 4.09x_2 + 3.35x_3 + 9.67x_4$
\[+ 0.74x_5 + 20.07x_6 + n_3 - p_3 = 200 \quad (12)\]

Copper: $3.17x_1 + 1.78x_2 + 5.55x_3 + 2.11x_4$
\[+ 2.67x_5 + 10.55x_6 + n_4 - p_4 = 200 \quad (13)\]

Cobalt: $4.5x_1 + 2x_2 + 10x_3 + 2x_4 + 2x_5$
\[+ 75x_6 + n_5 - p_5 = 200 \quad (14)\]

If the three goals in $Q_2$ are of equal importance, and their targets and measurements are homogeneous, then the second component to be minimised in the lexicographic process is given by $p_3 + p_4 + p_5$.

Finally, the last priority, $Q_3$, is made up of goals of finding a ration costing less than £1.782 with minimum bulk (which is 22.34kg, see the following section). It is interesting to note the similarity that appears to exist between goals in priorities $Q_1$ and $Q_3$. In fact, in $Q_1$ goals represent
something like minimum levels of achievement while $Q_3$ represents desired levels of achievement. When goals are specified at both a minimum and a desired level in a pre-emptive way, some authors call this approach two-stage LGP (Keown & Martin, 1977).

The algebraic expression for the goals in $Q_3$ is derived from eqns (1) and (7) as given below:

Cost: $0.0482x_1 + 0.0268x_2 + 0.0698x_3 + 0.0926x_4 + 0.1056x_5 + 0.1625x_6 + n_6 - p_6 = 1.782$ (15)

Bulk: $3.704x_1 + 1.219x_2 + 3.876x_3 + 9.259x_4 + 1.20x_5 + 1.137x_6 + n_7 - p_7 = 22.34$ (16)

Once again, assuming that the cost goal is twice as important as the ration bulk, the last component of the lexicographic minimisation process is given by:

$\frac{(2) 100}{1.782} p_6 + \frac{(2) 100}{22.34} p_7 = 112.2p_6 + 4.47p_7$

The whole lexicographic minimisation process would be given by the following vector:

$\text{Min } a = [(100p_1 + 2p_2), (p_3 + p_4 + p_5), (112.2p_1 + 4.47p_2)]$ (17)

The vector $a$ is called the achievement function where each component represents the deviational variables that must be minimised to make sure that the goals ranked in this priority come closest to their targets. The structure of our LGP model is given in Table 7.

This problem cannot be solved by a straightforward application of the Simplex method, because we are now seeking a minimum value ordered vector rather than minimising the scalar product of two vectors. There are several algorithmic approaches that can be adopted to solve LGP problems. Perhaps the most efficient algorithms for LGP problems are the Sequential Linear Method (Dauer & Krueger, 1977; Ignizio & Perlis, 1979) and the Partitioning Algorithm (Arthur & Ravindran, 1978, 1980). On using the Sequential Linear Method we obtain the following optimum solution:

$x_1$ (silage) = 5.145 kg  
$x_2$ (straw) = 0  
$x_3$ (distillers' wet grains) = 3.80 kg  
$x_4$ (swedes) = 0  
$x_5$ (barley) = 5.519 kg  
$x_6$ (dairy compound) = 4.536 kg
TABLE 7
Lexicographic Goal-Programming Model for Ration Formulation

Achievement function:

Min \( a = (100p_1 + 2p_2), (p_3 + p_4 + p_5), (112, 2p_6 + 4.47p_7) \)

Subject to:

\( Q_1: \)
\[
\begin{align*}
0.048x_1 + 0.0268x_2 + 0.0698x_3 + 0.0926x_4 + 0.1056x_5 + 0.1625x_6 + n_1 - p_1 &= 2 \text{ (cost)} \\
3.704x_1 + 1.219x_2 + 3.976x_3 + 9.259x_4 + 1.200x_5 + 1.137x_6 + n_2 - p_2 &= 50 \text{ (bulk)} \\
14.13x_1 + 4.09x_2 + 3.35x_3 + 9.67x_4 + 0.74x_5 + 20.07x_6 + n_3 - p_3 &= 200 \text{ (imbalance in sodium)} \\
3.17x_1 + 1.78x_2 + 5.55x_3 + 2.11x_4 + 2.67x_5 + 10.55x_6 + n_4 - p_4 &= 200 \text{ (imbalance in copper)} \\
4.5x_1 + 2x_2 + 10x_3 + 3x_4 + 2x_5 + 75x_6 + n_5 - p_5 &= 200 \text{ (imbalance in cobalt)}
\end{align*}
\]

\( Q_2: \)
\[
\begin{align*}
0.048x_1 + 0.0268x_2 + 0.0698x_3 + 0.0926x_4 + 0.1056x_5 + 0.1625x_6 + n_6 - p_6 &= 1.782 \text{ i.e. cost} \\
3.704x_1 + 1.219x_2 + 3.876x_3 + 9.259x_4 + 1.200x_5 + 1.137x_6 + n_7 - p_7 &= 22.34 \text{ i.e. bulk}
\end{align*}
\]

and:

\( Ax \leq b \) (technical constraints from Table 1)
\( x \geq 0 \quad n \geq 0 \quad p \geq 0 \)
While the optimum values of the deviational variables are:

\[
\begin{align*}
 n_1 &= 0.167 \text{ Pl} = 0 \\
 n_2 &= 4.434 \text{ kg} \quad p_2 = 0 \\
 n_3 &= 19.458 \% \\n n_4 &= 100 \% \\n n_5 &= 0 \\n n_6 &= 0 \\n n_7 &= 0
\end{align*}
\]

This solution permits complete achievement of the goals in \( Q_1 \); that is, the minimum levels set for cost and bulk are achieved. For \( Q_2 \) the targets for imbalances in sodium and copper supplies are also achieved but not for cobalt which has a positive deviation of 212.384 \% which means that more than three times the requirement is supplied. Finally, in \( Q_3 \) the least cost and minimum bulk goals have positive deviations of £0.051 and 23.227 kg, respectively; that is, their actual values are £1.833 and 45.517 kg.

Both sensitivity and parametric analyses of the LGP solutions can produce useful information, particularly if the analyst wishes to study the effects of rearranging priorities, changing targets and attaching different weights to the goals in a given priority. These analyses can be implemented easily using the optimal tableau of the LGP solution. (For details see Ignizio, 1976, Chapter 4 and Ignizio, 1982, Chapter 9).

**RATION FORMULATION AS A MULTI-OBJECTIVE PROGRAMMING (MOP) PROBLEM**

When several objectives, which are subject to a set of restraints (usually linear), are to be optimised simultaneously, then such problems can be solved using MOP (vector optimisation) techniques. Thus, while GP works with several goals, MOP deals with multiple objectives. However, when several objectives are involved simultaneously, it is impossible to define an optimum solution; therefore, MOP seeks to find the set of efficient solutions (also called non-dominated or Pareto optimal solutions).

The elements of an efficient set must be a feasible solution. Further, they have to be such that there are no other feasible solutions that will yield an improvement in any one of the objectives without reducing the
achievement of at least another one. To illustrate this definition, assume that in our ration formulation problem there are two objectives, cost and bulk, with the following three feasible solutions:

\[
\begin{align*}
a &= [2, 40] \quad \text{Efficient} \\
b &= [2.5, 40] \quad \text{Inefficient} \\
c &= [3, 20] \quad \text{Efficient}
\end{align*}
\]

The first component of each vector is cost (£) and the second is bulk (kg). The solution in \( b \) is inefficient or inferior and should never be chosen by a rational decision-maker as it is dominated by the solution in \( a \). The bulk of the ration is the same in both solutions, but solution \( a \) is cheaper than \( b \). Whereas solution \( c \) is efficient and is not dominated by \( a \), because, although more expensive, it is less bulky. The purpose of MOP, therefore, is to generate the \textit{efficient} set of solutions. The problem is formulated in the following general framework:

\[
\textbf{Eff } Z(x) = [Z_1(x), Z_2(x), \ldots, Z_q(x)]
\]

subject to:

\[ x \in F \]

Where \textbf{Eff} means to search for the efficient solutions and \( F \) represents the feasible set of solutions.

To demonstrate the use of MOP in ration formulation, assume that in Crabtree's problem the three objectives to be identified are: (a) minimise the cost; (b) minimise the bulk and (c) minimise the aggregated oversupply in sodium, copper and cobalt. The formal structure of the multi-objective model is thus given by:

\[
\textbf{Eff } Z(x) = [Z_1(x), Z_2(x), Z_3(x)]
\]

Where

\[
\begin{align*}
Z_1(x) &= 0.048x_1 + 0.0268x_2 + 0.0698x_3 + 0.0926x_4 \\
&\quad + 0.1056x_5 + 0.1625x_6 \quad \text{cost (£)} \\
Z_2(x) &= 3.704x_1 + 1.219x_2 + 3.876x_3 + 0.259x_4 \\
&\quad + 1.20x_5 + 1.137x_6 \quad \text{bulk (kg)} \\
Z_3(x) &= 21.80x_1 + 7.87x_2 + 18.09x_3 + 13.78x_4 + 5.41x_5 \\
&\quad + 105.62x_6 - 300 \quad \text{imbalance in sodium, copper and cobalt}
\end{align*}
\]
Subject to:

\[ \mathbf{A} \mathbf{x} \leq \mathbf{b} \text{ (technical constraints from Table 1)} \]

and

\[ \mathbf{x} \geq \mathbf{0} \]

The expression for \( Z_3(x) \) has been derived from the addition of the imbalances of the three nutrients as defined above.

There are basically three approaches to generate, or at least to approximate, the efficient set: (a) weighting method; (b) constraint method and (c) multi-criterion Simplex method. A detailed explanation of these approaches can be seen in Cohon (1978) Chapter 6, and Goicochea et al. (1982) Chapter 3.

For our problem we have used the weighting method. The basic idea of this method is to combine all the objectives into a single one. A weight is attached to each objective function before they are all added into a single one. Subsequently, the efficient set is generated through a parametric variation of the weights. Therefore, the MOP problem defined by eqn. (18) becomes the following parametric programming problem:

\[
\text{Maximise or Minimise } \sum_{i=1}^{q} W_i Z_i(x) \tag{20}
\]

subject to:

\[ \mathbf{x} \in \mathbf{F} \]

This method was first proposed by Zadeh (1963) and Geoffrion (1968) provided the formal proof that if the weights are strictly greater than zero \( (W_i > 0) \), the solutions generated from eqn. (20) are efficient. These points are efficient only in a mathematical sense and can result in technically unacceptable solutions, particularly when the restraint set has not been properly defined. However, one practical drawback of this method is the possibility that some efficient points may be skipped. To avoid this possibility it is necessary to reduce, considerably, the scale of weights which can result in a large number of calculations. The weighting method is useful only as a tool to approximate the efficient set.

As our problem in eqn. (19) has only three objective functions, the weighting method generates all the efficient points. The six efficient rations of our problem are shown in Table 8. It is interesting to note that solution 1 corresponds to the minimum cost ration, solution 3 to the
minimum imbalance ration and solution 4 to the minimum bulk ration. These three solutions establish what in MOP literature is called the ideal point or utopian point; that is, the point where all the objectives achieve the optimum value. The ideal or utopian point of our ration formulation problem is: [£1.782; 382.399 kg; 22.344 kg].

The optimum ration is one that is chosen by the decision-maker from the set of efficient solutions. But, of course, the choice depends on the preferences of the decision-maker, depending upon his subjective values attached to the trade-offs among cost, imbalance and bulk. For instance, if solution 1 (least cost ration) is preferred to solution 2, in that situation a reduction of 15.370 kg in bulk and 33.234% in imbalance does not compensate for an increase of £0.010 in cost.

Basically there are two general approaches to choosing the optimal solution from the efficient ones: compromise programming and interactive techniques. The first approach defines as optimum the efficient solution that is closest to the ideal point. Depending on the particular measure of distance used, a set of compromise solutions can be established (Zeleny, 1973, 1974b, 1982; Chapters 5, 6 and 10). The interactive techniques rely on the progressive definition of the decision-maker’s preferences; that is, there is an interaction between him and the model. The interaction proceeds by asking the decision-maker his preferences or trade-offs that he is prepared to accept and then solving the model in response to each question. The interactive techniques proposed by Masud & Hwang (1981) and Zionts & Wallenius (1976) are perhaps the most promising. However, the explanation of either interactive or compromise techniques is beyond the scope of this paper.
To conclude this section it should be mentioned that theoretically the efficient points calculated here are actually *extreme points* or *corner solutions*; therefore, the efficient points generated by MOP represent only a subset of the whole set of efficient points. In fact, there are two types of efficient points: (i) *extreme points* (generated by the methods used in this section); and (ii) *interior points* (points connecting efficient extreme points). The feasible set of solutions for the MOP model is derived from a convex and continuous region and, as such, it contains an infinite number of efficient interior points. But, from a practical point of view, only the subset of efficient extreme points is of interest; so, to find the whole efficient set is unnecessary. There are, however, some algorithms which search for efficient faces; that is, the convex combinations of efficient extreme points (Zeleny 1974b; Yu & Zeleny, 1975; Iserman 1977; Ecker et al., 1980). Thus, the whole efficient set is defined as the union of efficient faces. On applying the Yu and Zeleny algorithm to our example we observe that all the faces are efficient. This result is very common when there is a restraint that is binding for all the efficient extreme points, as is the case for undegradable protein constraint in our example.

**PENALTY FUNCTIONS IN RATION FORMULATION**

Some nutritionists argue that in formulating least-cost diets for livestock the ratios between some key ingredients such as forage and concentrate, calcium and phosphorus, etc., must be kept within nutritionally desirable limits. If these ratios are violated then a penalty system should operate within the diet formulation model to automatically penalise the cost of the ration. In contrast to the ordinary LP approach to diet formulation, such a penalty system can be built relatively easily into the GP model, which, in fact, provides a new perspective for dealing with the problem of choosing livestock rations.

Stranks (1983) suggests that in Crabtree's example the optimum forage/concentrate ratio should be set at 40% or more forage, using the following penalties:

- 40–35%—incur a 1 unit penalty per 1% fall
- 35–30%—incur a 2 unit penalty per 1% fall
- 30–25%—incur a 10 unit penalty per 1% fall
- Below 25%—incur an infinite penalty
This suggestion implies that a penalty function similar to the one in Fig. 1 is hooked to our goal programming model by introducing one constraint and three additional goals as follows:

\[
x_1 + x_2 - 0.25x_3 + x_4 - 0.25x_5 - 0.25x_6 \geq 0 \tag{21}
\]

\[
\frac{x_1 + x_2 + x_4}{x_3 + x_5 + x_6} + n_1 - p_1 = 30 \tag{22}
\]

\[
\frac{x_1 + x_2 + x_4}{x_3 + x_5 + x_6} + n_1 + n_2 - p_2 = 35 \tag{23}
\]

\[
\frac{x_1 + x_2 + x_4}{x_3 + x_5 + x_6} + n_1 + n_2 + n_3 - p_3 = 40 \tag{24}
\]

The contribution to the objective function of the WGP model or to the achievement function of the LGP model will be given by \(10n_1 + 2n_2 + n_3\) if we work with absolute deviations.

The constraint (21) means that rations with a forage/concentrate ratio below 0.25 are not feasible (i.e. incur an infinite penalty). The negative deviation \(n_1\) of eqn. (22) measures the percentage points by which forage/concentrate ratio is below 30%. In the same way the negative deviational variables \(n_1 + n_2\) and \(n_1 + n_2 + n_3\) measure the percentage points by which forage/concentrate ratio is below 35% and 40%, respectively.

It must be pointed out that if variable \(n_1\) in eqn. (23) and variables \(n_1 + n_2\) in eqn. (24) are omitted the total penalty is overestimated. For example, if the forage/concentrate optimum ratio is 0.32 and variables \(n_1\) and \(n_1 + n_2\) are omitted from eqns (23) and (24) then the total penalty (absolute deviations) will be 14, when the actual total penalty is only 11. This drawback is associated with some goal programming formulations using penalty functions (see Kvanli, 1980 and later modifications by Romero, 1984).

The equalities (22) to (24) are not linear, as in each expression cross multiplication by the denominator produces a non-linear constraint. Thus, the GP algorithms devised only for linear structures cannot be used. But for problems with such fractional goals the algorithmic approaches suggested by Kornbluth (1973) (pp. 201-3) and Kornbluth & Steuer (1981) can be used.

A natural progression of this penalty system approach to respect the nutritionally desirable qualities of a ration is to treat most nutrient
Fig. 1. Penalty function for forage/concentrate ratio.
Fig. 2. Penalty function for cobalt.
requirements in a similar way. We demonstrate this by dealing with the particular case of cobalt requirements in the ration; but the approach outlined is a general one and could be used for all nutrients. For Crabtree's matrix assume that the penalty function for cobalt is characterised by:

(a) The intake in the ration is permitted to fall below 2 mg but never lower than 1.5 mg—that is, there is an infinite penalty for it to go beyond that point.
(b) If the intake is between 1.5 and 4 mg no penalty is incurred.
(c) If the intake is more than 4 mg and less than 6 mg, then there is a penalty of one unit per milligram of excess supply.
(d) If the intake is more than 6 mg and less than 8 mg, the penalty is five units per milligram of excess supply.
(e) The intake must never be more than 8 mg—that is, there is an infinite penalty for this situation to occur.

Figure 2 shows the above penalty function while the equations below specify its structure within the GP model:

\[0.09x_1 + 0.04x_2 + 0.2x_3 + 0.04x_4 + 0.04x_5 + 1.5x_6 \geq 1.5 \quad (25)\]
\[0.09x_1 + 0.04x_2 + 0.2x_3 + 0.04x_4 + 0.04x_5 + 1.5x_6 + n_1 - p_1 - p_2 = 4 \quad (26)\]
\[0.09x_1 + 0.04x_2 + 0.2x_3 + 0.04x_4 + 0.04x_5 + 1.5x_6 + n_2 - p_2 = 6 \quad (27)\]
\[0.09x_1 + 0.04x_2 + 0.2x_3 + 0.04x_4 + 0.04x_5 + 1.5x_6 \leq 8 \quad (28)\]

The contribution to the objective function of the WGP model or to the achievement function of the LGP model will be given by \(lp_1 + 5p_2\) in terms of absolute deviations.

**SOME CONCLUDING REMARKS**

The main methodological objections to the use of linear programming for generating livestock rations are its inability to tackle more than one ‘ingredient choice’ criterion and the rigidity with which it treats the nutrient requirements specified for a diet. This approach could generate either unimplementable rations or a nutritionally undesirable mix of ingredients. But these problems can be overcome if the ration formulation is postulated within the multiple-criteria decision-making framework by using techniques such as lexicographic goal-programming, weighted
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goal-programming and multiple-objective programming. Not only that, the information generated by weighted goal-programming permits an exploration of the trade-offs that exist among different criteria such as cost, ration bulk and nutrient over-supply and how they influence the quality of the ration.

If the aims of a diet formulation exercise can be arranged into a priority structure, then the lexicographic approach explores the effects of re-arranging these priorities, setting different nutrient requirement targets and attaching different weights to nutritional goals before the most desirable ration can be chosen. The multiple-objective programming presents the nutritionist with an array of nutritionally efficient solutions from which the most suitable one can be picked and implemented. Similarly, the ability to incorporate penalty functions for over- or under-achieving the nutritional specifications underlines the superiority of the goal-programming approach over simple least-cost diet formulation. If the nutritionist and/or his client can 'interact' with the model on a computer terminal within an advisory environment such as the one reported by Crabtree (1982) then the advantages that multiple-criteria decision techniques offer can be actually realised.

We have been mainly concerned to demonstrate the applicability of multiple-criteria techniques to livestock ration formulation problems using a simple example, but their use by nutritionists either in place or in tandem with linear programming requires research. Hopefully this paper provides the conceptual transition that is needed and can trigger work to test the application of multiple-criteria techniques in developing ration formulation models for more realistic situations.

REFERENCES


