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A Unifying Approach to Efficient (Near)-Gathering of Disoriented Robots with Limited Visibility

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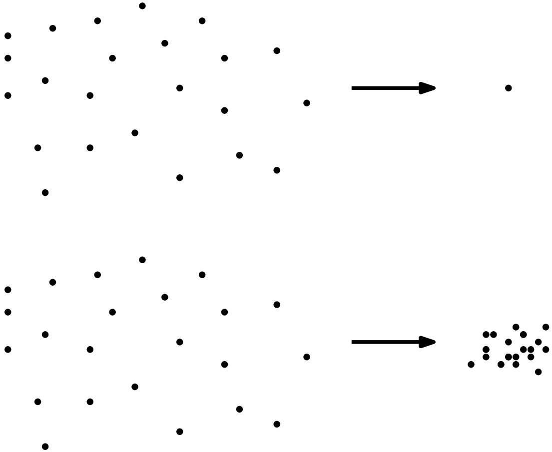
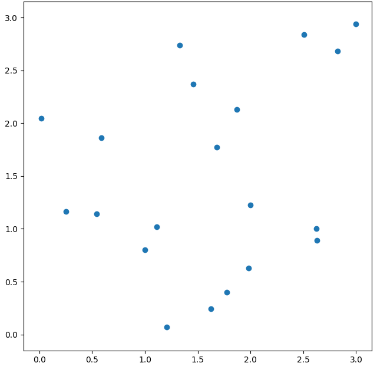
Friedhelm Meyer auf der Heide

Gathering and Near-Gathering

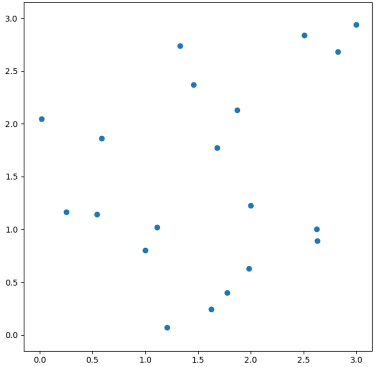


Simulation

Gathering



Near-Gathering



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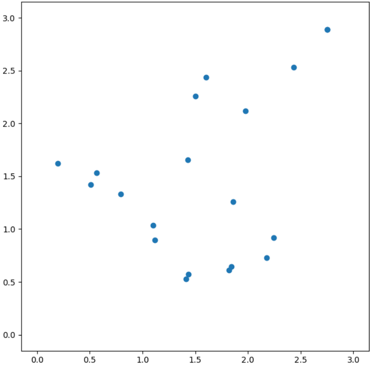

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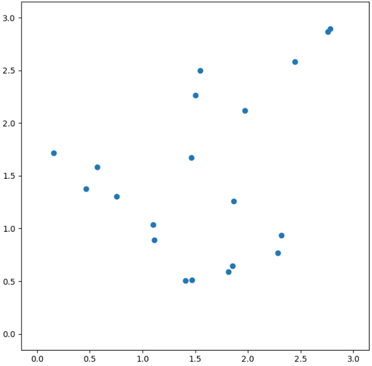

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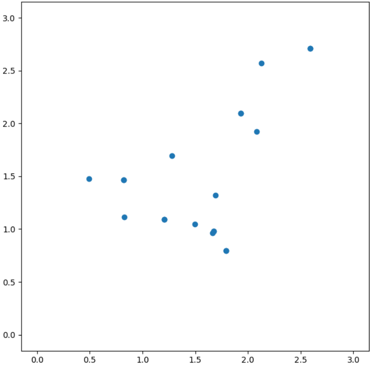

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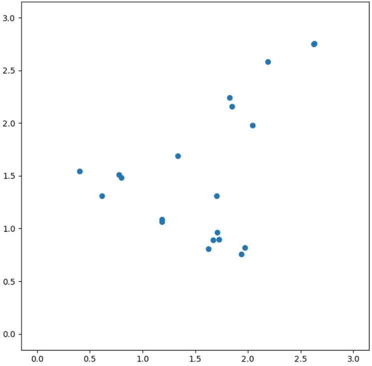

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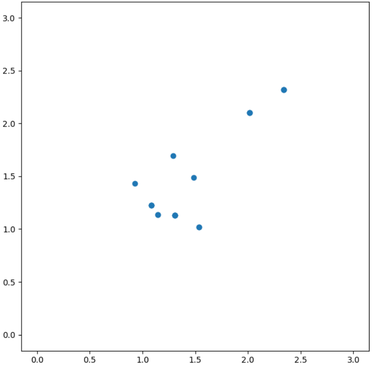

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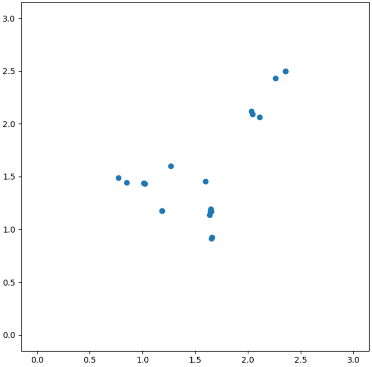

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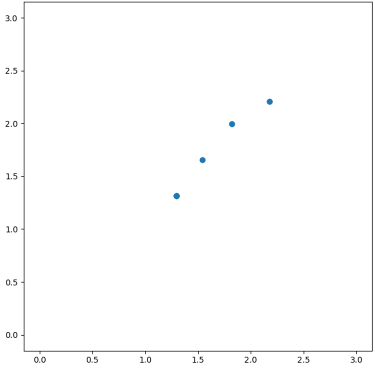

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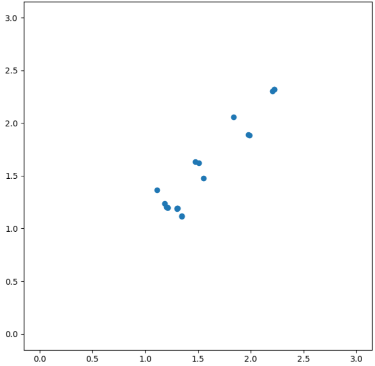

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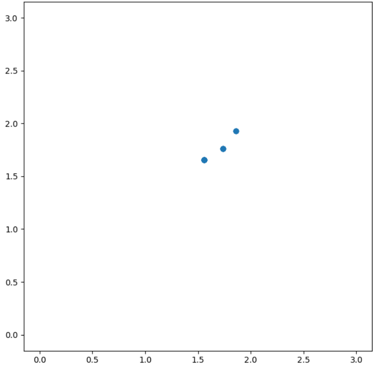

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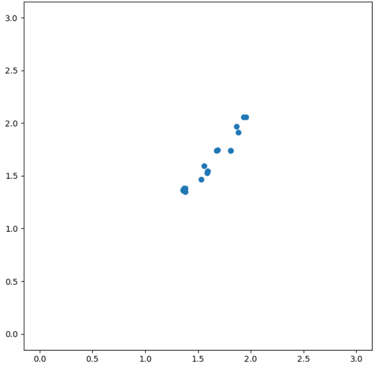

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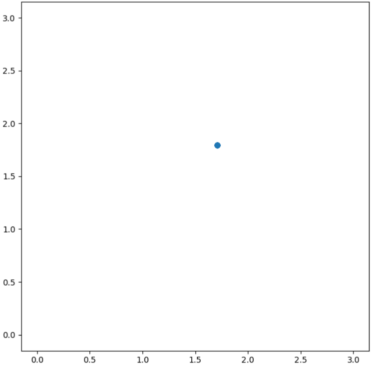

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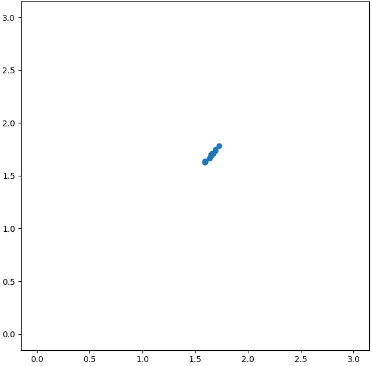

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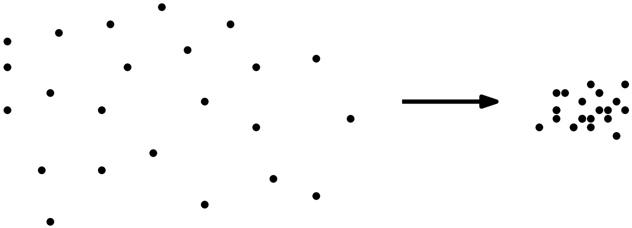
Near-Gathering

Definition 1 (Near-Gathering)

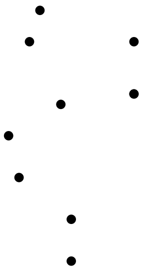
Gather all robots in close proximity

- constant diameter
- collisionless
- simultaneous termination



Model

- autonomous mobile robots ($\mathcal{OBLOT}$)
  - identical, anonymous, homogeneous
  - oblivious & silent
- euclidean space (k dimensional)



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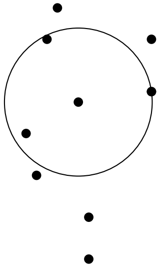
  - connectivity range: 1
  - viewing range: ≥ 1



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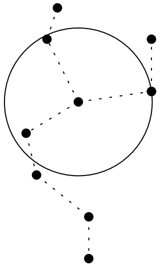

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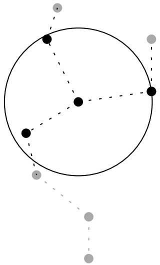

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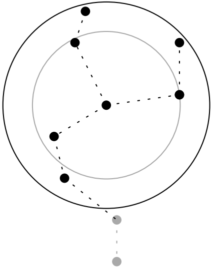

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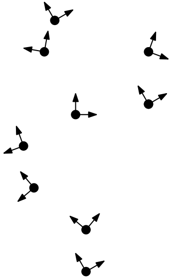
  - disoriented



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  - identical, anonymous, homogeneous
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  - connectivity range: 1
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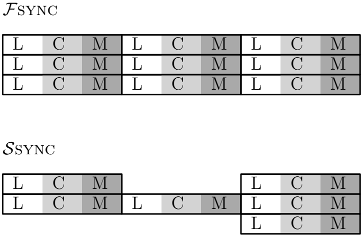
  - disoriented
- $\mathcal{F}$SYNC and $\mathcal{S}$SYNC



Related Work

Gathering

- Go-To-The-Center
  - Ando et al. 1999, Kempkes et al. 2011
  - gathering in $\mathcal{O}(n + \Delta^2)$
  - 3 dimensional: Braun et al. 2020
- class of continuous gathering strategies in $\mathcal{O}(n \cdot \Delta)$, Podlipyan et al. 2021
- gathering on the grid in $\mathcal{O}(n^2)$, Jung et al. 2020
- gathering with compass in $\mathcal{O}(\Delta)$, Poudel et al. 2021

Related Work

Near-Gathering

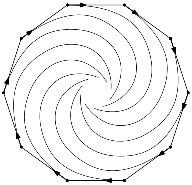
- NEAR-GATHERING introduced by Pagli et al.
  - 2012, "Getting close without touching."
  - both axis agreement (later one axis)
  - no running time
- Near-Gathering with unlimited memory used for pattern formation under limited visibility, Yamashita et al. 2013

Our Results

- Class of Discrete Gathering Strategies in $\mathcal{O}(\Delta^2)$
  - k dimensional
  - improved running time (previously $\mathcal{O}(n + \Delta^2)$)
- Class of Discrete Near-Gathering Strategies in $\mathcal{O}(\Delta^2)$
  - first result under the disoriented $\mathcal{OBLOT}$ model

Discrete Gathering Strategies

- Motivated by Continuous Gathering Strategies (Podlipyan at al.)
  - there: run with constant speed inside convex hull



Discrete Gathering Strategies

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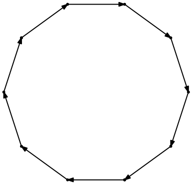
  - there: run with constant speed inside convex hull
- Discrete Gathering Strategies
  - target points must lie *sufficiently* inside convex hull

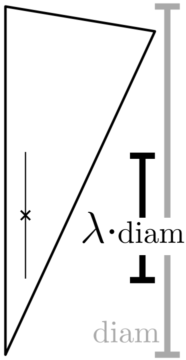


Discrete Gathering Strategies

Definition of a point "*sufficiently inside*" local convex hull

Definition 2 (λ-centered Point)

- $1 \geq \lambda > 0$
- line segment of length $\mathrm{diam} \cdot \lambda$
  - completely inside convex hull
- point in its middle is "λ-centered"



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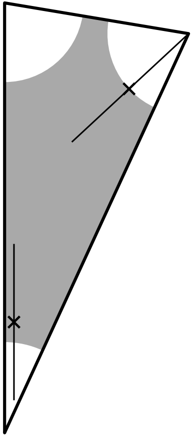
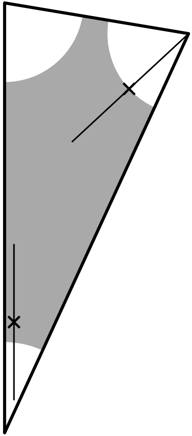


Discrete Gathering Strategies

Definition 3 (λ-contracting gathering protocol)

- target point is λ-centered
- connectivity preserving
- collapsing



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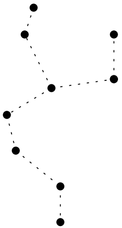

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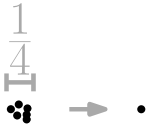
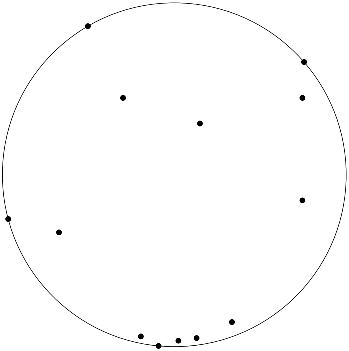


Running Time Proof

- idea from Go-To-The-Center proof (Kempkes et al. 2011)
- we analyze diameter of global SEC



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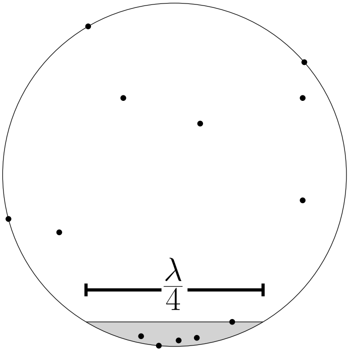

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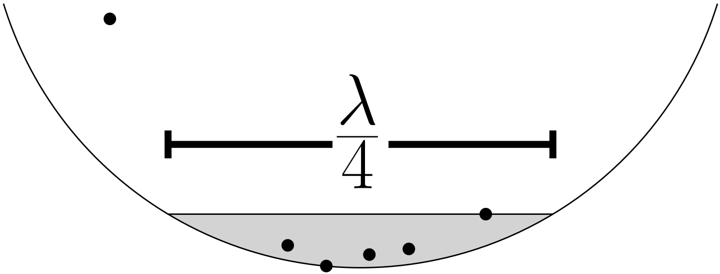

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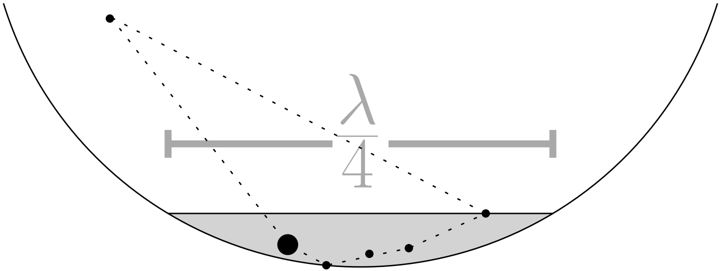

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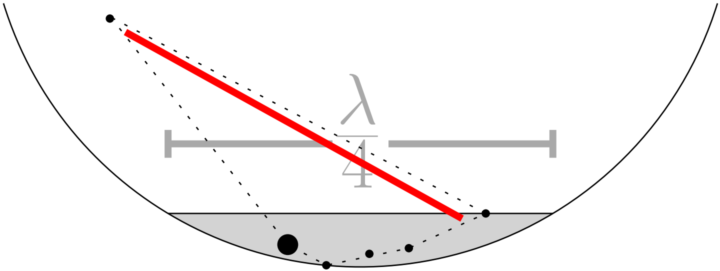

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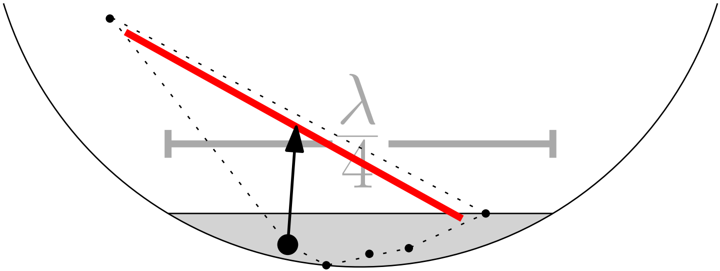

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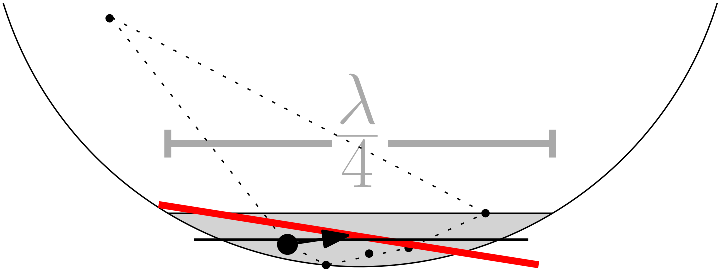

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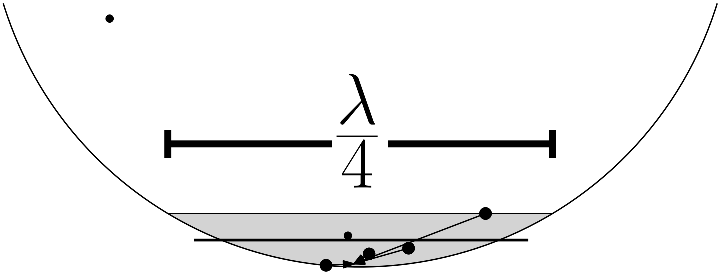

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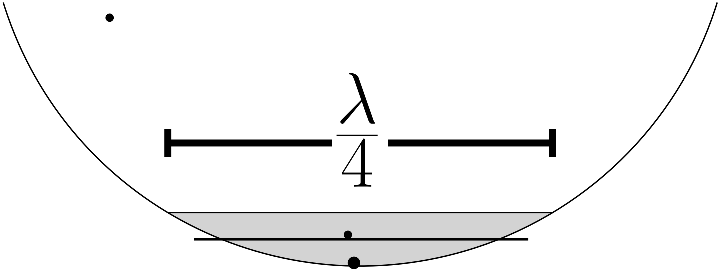

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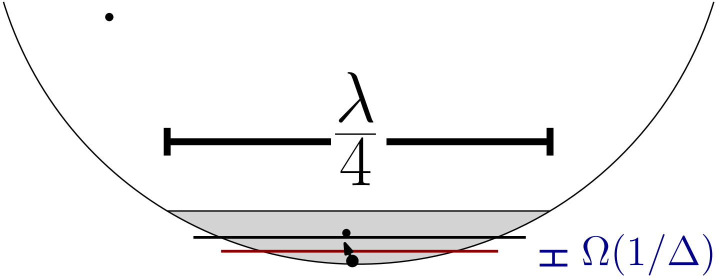

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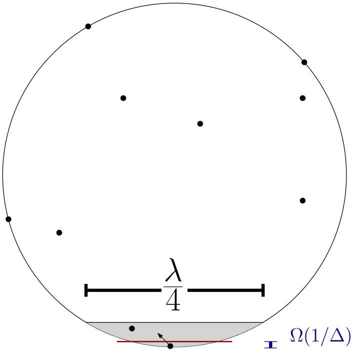

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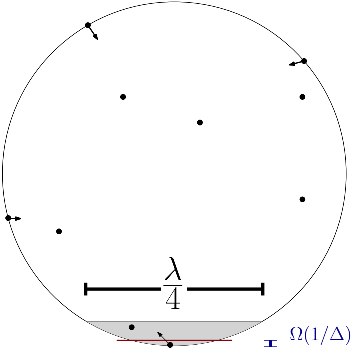
- idea from GO-TO-THE-CENTER proof (Kempkes et al. 2011)
- we analyze diameter of global SEC
  - decreases by $\Omega(1/\Delta)$ after 2 rounds



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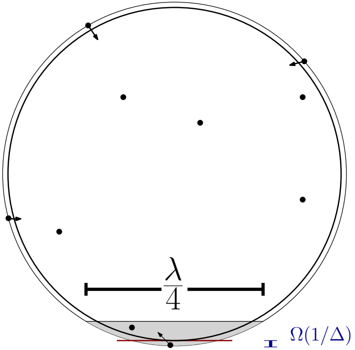

λ-contracting gathering protocol

Theorem 4

After $\mathcal{O}(\Delta^2)$ rounds ($\mathcal{F}$SYNC), a λ-contracting gathering protocol reaches a GATHERING.

Near-Gathering

Properties in general

- collisionless
- robots converge to one point

Near-Gathering

Properties in general

- collisionless
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Our Approach:

- take λ-contracting gathering protocol
- implement collision avoidance

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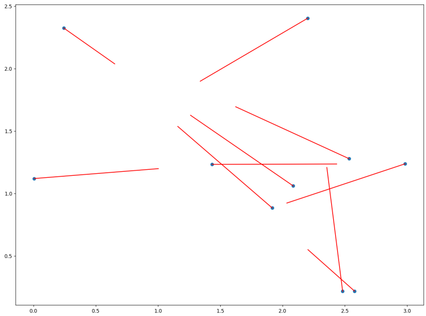
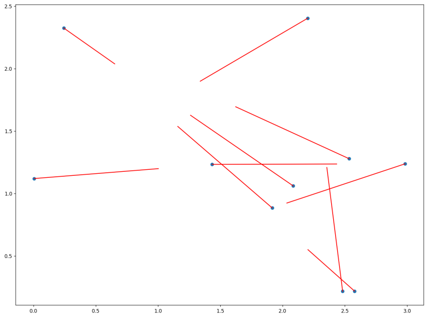
Collision Avoidance

- robots move not full to target point but stop in between
  - far enough (at least constant fraction)
  - stop only on points without possible collisions



Collision Avoidance

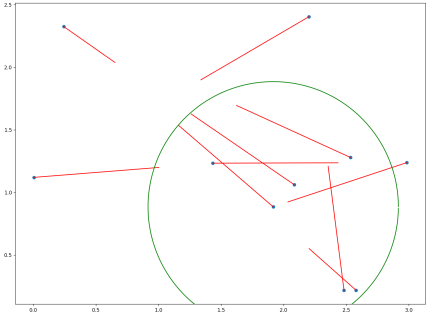
- robots move not full to target point but stop in between
  - far enough (at least constant fraction)
  - stop only on points without possible collisions
- Where are possible collisions?
  - on intersections between trajectories



Collision Avoidance

Avoiding intersections of trajectories

- need to compute trajectory of neighbors
  - larger viewing range (still constant)
  - trajectory must be invariant of local rotation and translation



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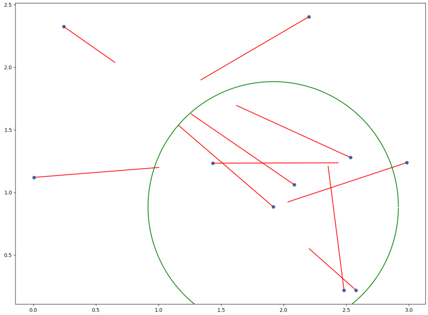

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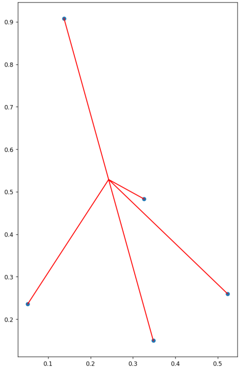

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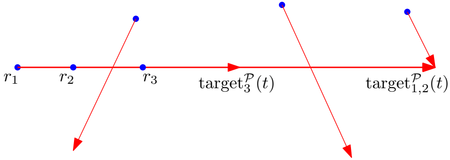
- collinear trajectories: compute unique target points



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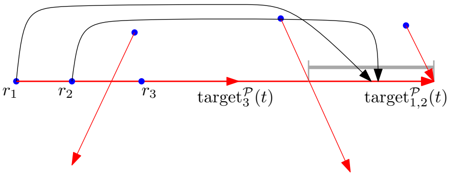

Viewing Range

Do we really need a
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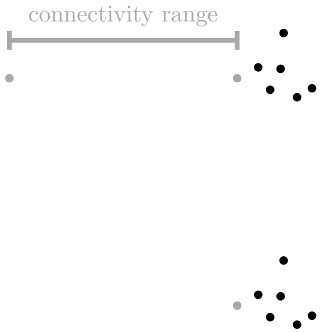
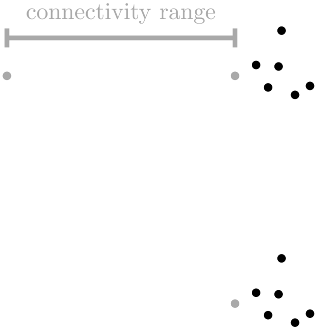


Viewing Range

Do we really need a viewing range > connectivity range?

- Remember:
  Near-Gathering needs simultaneous termination



Viewing Range

Do we really need a
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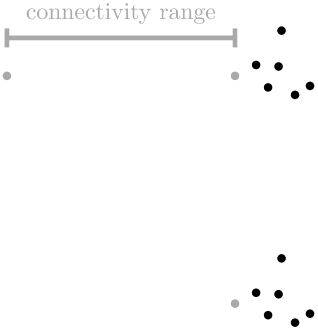
- Remember:
  Near-Gathering needs simultaneous termination
- both configurations right are locally indistinguishable



Class of Near-Gathering Strategies

Theorem 5

After $\mathcal{O}(\Delta^2)$ epochs ($\mathcal{S}$SYNC), λ-contracting gathering protocol with collision avoidance reaches a NEAR-GATHERING.

Summary

- Class of Discrete Gathering Strategies in $\mathcal{O}(\Delta^2)$
  - high dimensional
  - improved running time (previously $\mathcal{O}(n + \Delta^2)$)
- Class of Discrete Near-Gathering Strategies in $\mathcal{O}(\Delta^2)$
  - first result under the disoriented $\mathcal{OBLOT}$ model