USC Viterbi
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ANRG

LINE SEARCH FOR AN OBLIVIOUS MOVING TARGET

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INTRODUCTION

- Is my friend moving **toward** or **away** from me?
- What else do I know about my friend?
  - Nothing
  - Their speed v?
  - Their initial distance from me d?
  - Both v and d?

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  - Nothing — **NoKnowledge**
  - Their speed v? — **NoDistance**
  - Their initial distance from me d? — **NoSpeed**
  - Both v and d? — **FullKnowledge**

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- Solved if $v = 0$ (friend is standing still)
  - Competitive ratio 9
- Solved if friend is moving away from me with a known speed (NoDistance/Away, [2])
  - Competitive ratio $1 + 8\frac{1+v}{(1-v)^2}$

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- Competitive ratio of an algorithm A: How many times worse algorithm A is than if you knew where your friend were to begin with

$$CR_A = \sup_{v,d} \frac{T_A(v,d)}{T_{OPT}(v,d)}$$

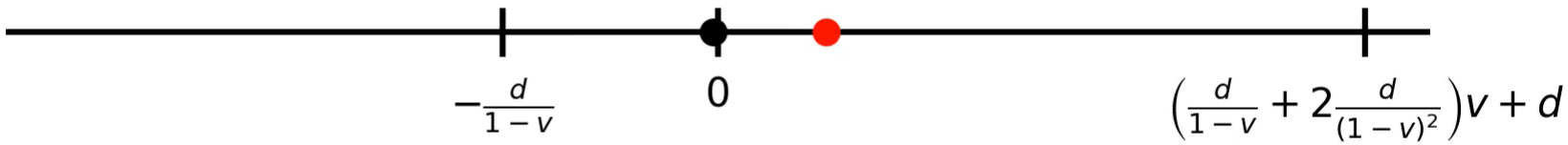
INTRODUCTION

Knowledge	Movement	Competitive Ratio		Section
v, d	Away	$CR = 1 + \frac{2}{1-v}$		2.1
	Toward	$CR = 1 + \frac{2}{1+v}$	if $v < 1$	2.2
		$CR = 1 + \frac{1}{v}$	otherwise	
v	Away	$CR = 1 + 8\frac{1+v}{(1-v)^2}$		3.1 [2]
	Toward	$CR = 1 + \frac{1}{v}$	if $v \geq \frac{1}{3}$	3.2
		$CR = 1 + 8\frac{1-v}{(1+v)^2}$	otherwise	
d	Away	$CR \leq 5$	if $v \leq \frac{1}{2}$	4.1
		$CR \leq 1 + 16\frac{\left(\log \frac{1}{1-v}\right)^2}{(1-v)^4}$	otherwise	
	Toward	$CR = 3$		4.2
$\emptyset$	Away	$CR \leq 1 + \frac{16}{d}\left[\log\log\left(\max\left(d, \frac{1}{1-v}\right)\right) + 3\right] \cdot \max\left(d, \frac{1}{1-v}\right)^8 \cdot \log^2\left[\max\left(d, \frac{1}{1-v}\right)\right]$		5.1
	Toward	$CR = 1 + \frac{1}{v}$		5.2

FULL KNOWLEDGE / AWAY

Algorithm 1 Online Algorithm for `FullKnowledge/Away` Model

1: **input:** target speed v and initial distance d
2: choose any direction and go for time $\frac{d}{1-v}$
3: **if** target not found **then**
4: change direction and go until target is found

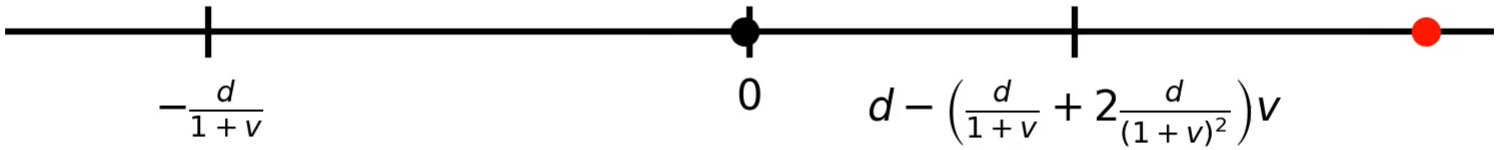


- Observation: $\frac{d}{1-v}$ is the earliest time you can decide location of target
- Worst case: you choose wrong direction to try first
- Competitive Ratio: $1+\frac{2}{1-v}$

FULL KNOWLEDGE / TOWARD

Algorithm 2 Online Algorithm for the `FullKnowledge/Toward` Model

1: **input:** target speed v and initial distance d
2: choose any direction and go for time $\frac{d}{1+v}$
3: **if** target not found **then**
4: change direction and go until target is found



- Observation: $\frac{d}{1+v}$ is the earliest time you can decide location of target
- Worst case: you choose wrong direction to try first
- Competitive Ratio: $1+\frac{2}{1+v}$

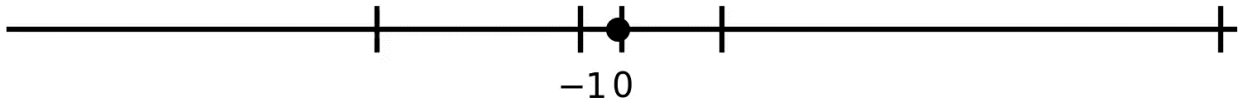
NO DISTANCE / AWAY

- Must execute a zig-zag algorithm
  - Search to left and right in alternating rounds
  - Expand search distance by a factor of α in each round

Algorithm 3 Online Algorithm for `NoDistance/Away` and `NoDistance/Toward` Models

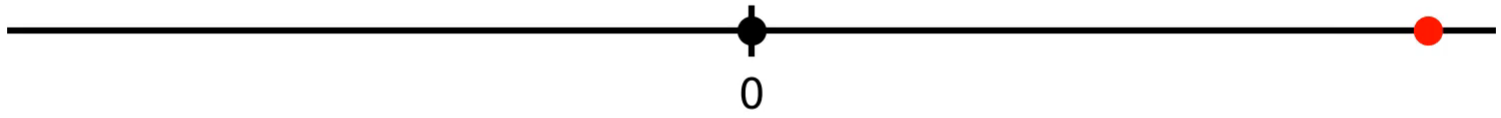
```
input: target speed v and expansion ratio a
i ← 0
while target not found do
    if at origin then
        d ← (−a)^i
        i ← i + 1
    else if at d then
        d ← 0
    move toward d
```

- $\alpha = 2\frac{1+v}{1-v}$ proven to minimize the competitive ratio by Alpern and Gal [2]
- Competitive Ratio: $1 + 8\frac{1+v}{(1-v)^2}$



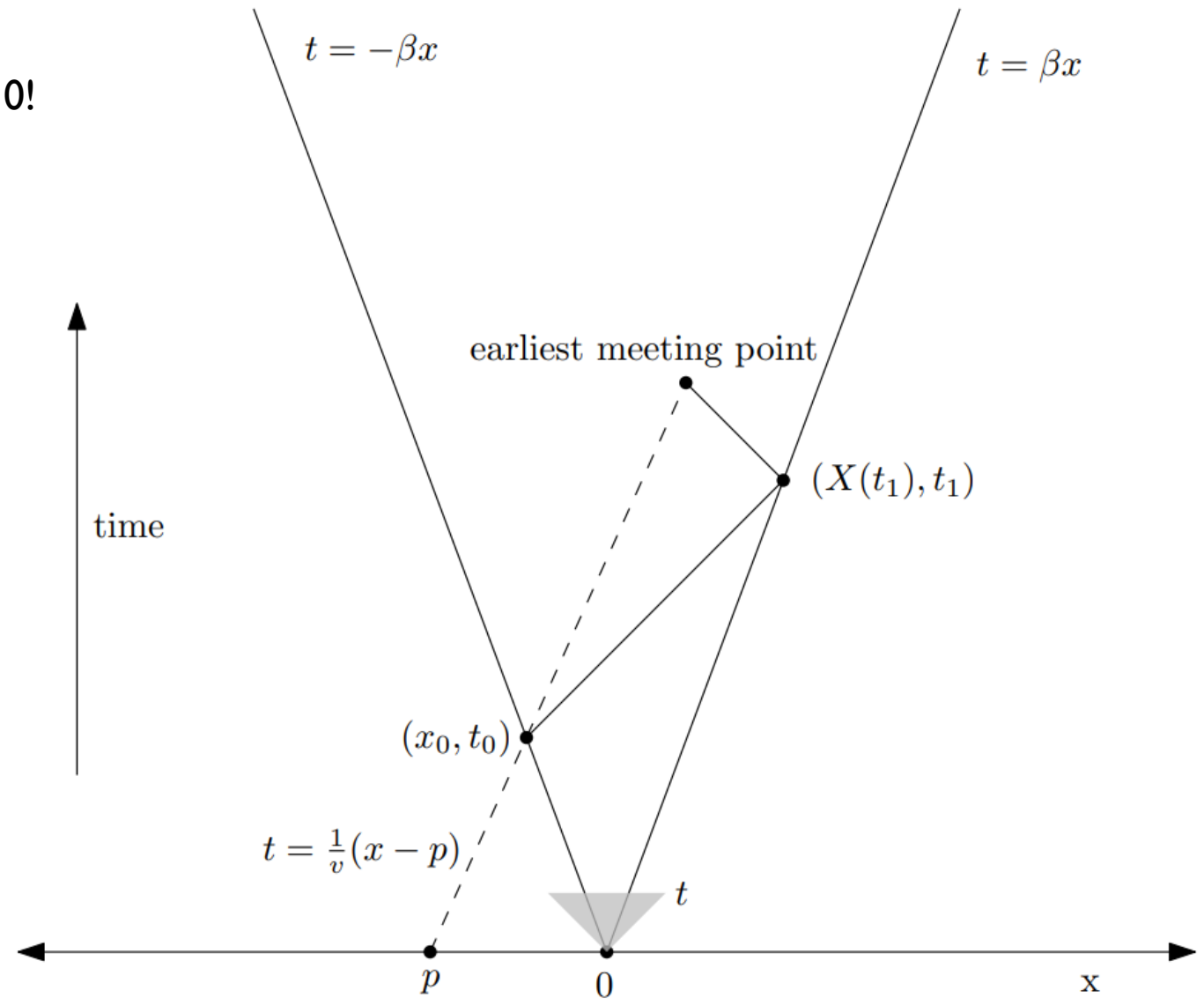
NO DISTANCE / TOWARD

- Must the agent zig-zag? It could just wait at 0!



NO DISTANCE / TOWARD

- Must the agent zig-zag? It could just wait at 0!
- Expansion ratio
  - $\alpha = 2\frac{1-v}{1+v}$ is optimal when $v < \frac{1}{3}$
  - 0 otherwise
- Competitive Ratio
  - $1 + \frac{1}{v}$ if $v < \frac{1}{3}$
  - $1 + 8\frac{1-v}{(1+v)^2}$ otherwise



NO SPEED / AWAY

- Must execute a zig-zag algorithm
  - Problem: constant expansion factors don't work – you may never find the target!
  - Solution: on each round, guess the target speed and travel as far as necessary to catch it

Algorithm 5 Online Algorithm for `NoSpeed/Away` Model

input: target initial distance d
integer sequence $\{f_i : i \geq 0\}$ such that $f_i < f_{i+1}$, for $i \geq 0$ and $f_0 = 1$;
$t \leftarrow 0$
for $i \leftarrow 0, 1, 2, \ldots$ until target found **do**
  $v_i \leftarrow 1 - 2^{-f_i}$
  $x_i \leftarrow (-1)^i \cdot \frac{d+tv_i}{1-v_i}$
  move to x_i and back to the origin
  $t \leftarrow t + |x_i|$

- Question: how should we search over possible target speeds?
  - How to select integer sequence f_i
- Note: if $v < \frac{1}{2}$ then the competitive ratio has an upper bound of 5 for any sequence

NO SPEED / AWAY

- First prove competitive ratio

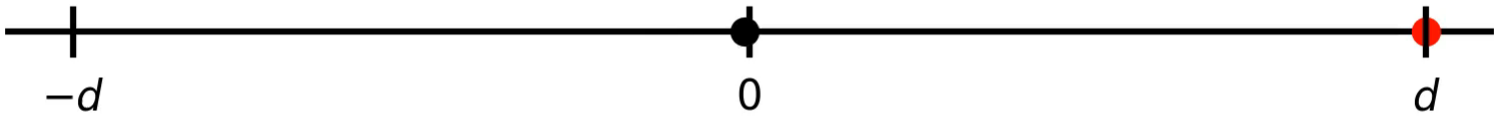
$$1 + 2^{1+\sum_{j=0}^{k} f_j} \cdot 4^{k+1}$$

- Where k is the first round that the agent guesses a speed equal to or greater than the actual speed of the target
  - Target must be found on either round k or $k+1$
- The sequence $f_j = 2^j$ yields an upper bound on the competitive ratio of
  - 5 if $v < \frac{1}{2}$
  - $1 + 16\frac{\log\left(\frac{1}{1-v}\right)^2}{(1-v)^4}$ otherwise
- Lower bound from Full Knowledge model, $1 + \frac{2}{1-v}$, still applies

NO SPEED / TOWARD

Algorithm 6 Online Algorithm for `NoSpeed/Toward` Model

1: **input:** target initial distance d
2: choose any direction and go for time d
3: **if** target not found **then**
4: change direction and go until target is found



- Lower Bound: any algorithm must eventually visit both $-d$ and d in case $v \to 0$
- Upper Bound: worst case is when agent chooses wrong direction to explore first
- Competitive Ratio: 3

NO KNOWLEDGE / AWAY

- Must execute a zig-zag algorithm
  - Problem: constant expansion factors yield unbounded competitive ratios
  - Solution: on each round, guess the target speed and initial distance and travel as far as necessary to catch it

Algorithm 7 Online Algorithm for `NoKnowledge/Away` Model

Inputs; Integer sequences $\{f_i, g_i : i \geq 0\}$ such that $f_i < f_{i+1}$ and $g_i < g_{i+1}$, for $i \geq 0$ and $f_0 = 1$ and $g_0 = 0$;
$t \leftarrow 0$
for $i \leftarrow 0, 1, 2, \ldots$ until target found **do**
    $d_i \leftarrow 2^{g_i}$
    $v_i \leftarrow 1 - 2^{-f_i}$
    $x_i \leftarrow (-1)^i \cdot \frac{d_i + t v_i}{1 - v_i}$
    move to x_i and back to the origin
    $t \leftarrow t + |x_i|$

- Question: how should we search over possible target speeds and initial distances?
  - How to select integer sequence f_i and g_i

NO KNOWLEDGE / AWAY

- First prove competitive ratio

$$1 + \frac{2(i+1)}{d} \cdot 2^{g_{i+1}} \cdot 2^{\sum_{j=0}^{i} f_i} \cdot 4^{i+1}$$

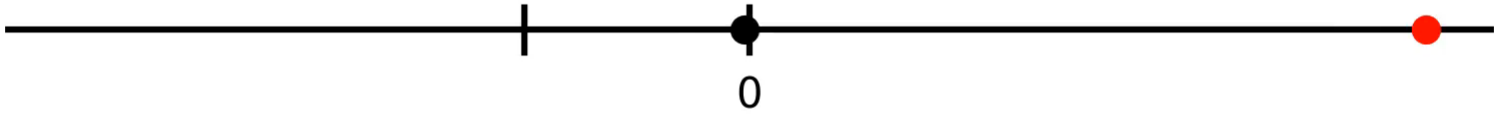
- Where the algorithm terminates in round $i + 1$
- The sequence $f_i = g_i = 2^i$ yields an upper bound on the competitive ratio of

$$1 + \frac{16}{d}\left[\log\log\max\left(d, \frac{1}{1-v}\right) + 3\right] \cdot \max\left(d, \frac{1}{1-v}\right)^3 \cdot \log^2 \max\left(d, \frac{1}{1-v}\right)$$

- Lower bound from Full Knowledge model, $1 + 8\frac{1+v}{(1-v)^2}$, still applies

NO KNOWLEDGE / TOWARD

- Waiting algorithm is the best you can do



CONCLUSION

- We provide algorithms and bounds for remaining cases of the Linear Search Problem for Moving Targets
- All bounds are tight if
  - Target is moving toward the origin
  - Speed of the target is known
- Can we prove tight bounds when speed is unknown, and target is moving away?
- What about if the direction the target is moving (toward or away) is unknown?
- Accelerating targets? Other topologies?