Bayesian Graduation of Mortality Rates: 
an application to mathematical reserve evaluation.

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Abstract:

This paper presents Bayesian graduation models of mortality rates, using Markov chain Monte Carlo (MCMC) techniques. Graduated annual death probabilities are estimated through the predictive distribution of the number of deaths, which is assumed to follow a Poisson process, considering that all individuals in the same age class die independently and with the same probability. The resulting mortality tables are formulated through dynamic Bayesian models. The best such mortality table is then compared with known tables from abroad used by the market in Brazil. Calculation of adequate reserve levels is exemplified, via MCMC, making use of the value at risk concept, demonstrating the importance of using “true” observed mortality figures for the population exposed to risk in determining the survival coverage rate.

Key Words: Bayesian graduation; dynamic models; predictive distribution; MCMC; Bayesian mortality table; mathematical reserve; value at risk.
1. Introduction

Complementary open pension and life insurance plans that have survival coverage (retirement benefits) require prior approval for commercialization in Brazil. Open pension entities and life insurance companies must set the mortality tables and interest rates that will be applied to their plans at the time of obtaining this approval. However, the mortality tables must be valid from the adhesion of the first participant to the plan until the death of the last one, a very long interval. In this paper, we refer generically to the plans as pension plans and to the companies that sell them as insurers.

Among the problems faced by insurers, one of the chief ones is deciding which mortality table to establish for survival coverage of their plans, since this must be used to calculate contributions, benefits and reserves. This problem is aggravated by the trend toward greater longevity in the population at large and the use in Brazil of mortality tables constructed from the experiences of other countries, particularly the United States.

Insurers constitute mathematical reserves for future liabilities based on the values of the life annuities, as we shall see in Section 4. These are calculated as a function of the interest rate and the probability of death. Indeed, they are inversely proportional to these two rates. Hence, to avoid constituting mathematical reserves that are insufficient to cover their future commitments, and consequent insolvency, due to the lack of knowledge about the true mortality of the exposed population, the companies’ actuaries have been putting aside statistical and actuarial considerations and trying to prevent future losses by setting reduced real interest rates, generally equal to zero. With this artifice, they hope to compensate for future technical losses arising from poorly formulated mortality tables, i.e., by using higher death probabilities than are really the case for the population covered by survival benefits, with financial gains from the guarantee of a low real interest rate. In this form, the Brazilian insurance market is becoming less technically grounded.

Based on data collected from all Brazilian insurers for the four years 1998 to 2001, we use a Bayesian statistical model to prepare mortality tables for both sexes that reflect the mortality rates of those exposed to risk who have survival coverage under pension plans in Brazil. To obtain the probabilities of death contained in these tables, we use the Bayesian graduation process. The distribution of these numbers is obtained from the predictive distribution of the number of deaths, which is modeled as being Poisson-distributed, assuming that all individuals of the same age die independently and with the same probability. Graduation is fundamental to smooth out the raw mortality rates so that the death probabilities are monotonically increasing with respect to age, since it is well known that human mortality behaves this way from a certain age onward.

The main objective of this paper is to propose dynamic models for formulating Bayesian mortality tables. Besides this, we compare the mortality table resulting from our best model thus found against some tables used in practice by the market. We also show the importance of better fitting the mortality tables used in calculating survival coverage to the reality of the exposed population, in order to maintain the solvency of insurers. We do this by calculating the mathematical reserve, comparing the deterministic method adopted by the Brazilian insurance market with our proposed Bayesian method, where we employ value at risk (VaR) concepts.

This paper is organized as follows. The data and concepts are presented in Section 2. In Section 3 we propose dynamic models for preparing mortality tables and compare the table obtained by the best-fitting model with some existing mortality tables. In Section 4 we calculate the mathematical reserve by means of a MCMC and value at risk concepts to demonstrate the importance of adjusting mortality tables to the real mortality of the exposed population so that the
insurer can remain solvent. Finally, in Section 5 we present our conclusions and suggestions for extensions of this work.

2. Graduation of Mortality Rates

The data used refer to participants in pension plans in Brazil with survival coverage for the years 1998 to 2001. We consider these data with the following structure: \( D_t = (x, e_{x,t}, d_{x,t}) \), where \( x \) is the age of the individuals in years, \( e_{x,t} \) is the central number of those exposed to risk observed at age \( x \) in year \( t \) and \( d_{x,t} \) is the number of deaths observed at age \( x \) in year \( t \), for \( t = 1, \ldots, 4 \), already corrected for any notification delays.

In the period under study, there were nearly 5.6 million men at risk and 2.9 million women. Of these 94% were between 20 and 60 years of age. In the same period, nearly 7.6 thousand men died and 2.8 thousand women, with the great majority, approximately 97%, doing so between the ages of 25 and 90 (see Annex I).

One would expect the mortality rate to behave regularly, with older individuals having higher probabilities of dying than younger people. In practice, however, the crude rates do not behave this way in relation to age, as shown in Figure 1. To be used in pension plans, the rates must be smoothed out. This process of smoothing out crude mortality rates is called graduation.

![Figure 1: Crude mortality rates by age, on a logarithmic scale, for survival coverage for male (solid line) and female (dotted line).](image)

Graduation, besides correcting the problem mentioned above, serves to overcome the lack of information for some ages studied. The crude rates are smoothed so that the annual probabilities of death \( q_x \) are monotonically increasing with advancing age. The use of mortality rates without graduation would cause inconveniences because, for example, contributions calculated according to age would be unpredictable and difficult to justify to participants.
We graduated the crude mortality rates as a function of the force of mortality, i.e., the instantaneous variation in the intensity of death. Assuming that all individuals with the same age die independently and with the same probability, then the number of deaths observed - \( d_{x,t} \), for each year and age, is Poisson-distributed with mean \( e_{x,t} \cdot \mu_{x,t} \), where \( \mu_{x,t} \) is the force of mortality at age \( x \) at time \( t \) and \( e_{x,t} \) denotes the exposed population, which is assumed known.

Since the aim of the graduation process is to obtain the annual probabilities of deaths for each age class and we are graduating as a function of the force of mortality, we consider these to be constant in each age bracket and work with the following relation (Gerber, 1997):

\[
q_x = 1 - \exp(-\mu_x)
\]

(1)

To estimate the smoothed mortality rates, the literature suggests parametric and non-parametric graduation, in the nomenclature of Haberman and Renshaw (1996), or global and local models, respectively. Graduations are usually performed through parametric models, which we call global models, by adjusting the probabilities of death or of mortality forces to a mathematical model. The most common mathematical models represent survival functions based on laws of mortality, such as those of de Moivre, Gompertz, Makeham and Weibull (Bowers et al., 1986).

In Brazil, the studies conducted on graduation of the mortality rate with data from the insurance market use classic statistics and non-predictive and static global models. In this paper, however, we propose dynamic and predictive Bayesian models, with two prior modeling alternatives: local and global. The Bayesian approach to the graduation process involves statistical estimation of the unknown parameters, where initial knowledge of the parameters of interest (prior distribution) is aggregated to the data. Some works have already appeared on Bayesian graduation, among them: Kimeldorf and Jones (1967), Hickman and Miller (1977), Broffit (1988), Carlin (1992), Gordon (1998) and Mendonza et al. (2001).

3. Proposed Models

A mortality table is defined as a set of annual probabilities of death graduated by age. Its construction must consider a period of study greater than one year. Haberman and Renshaw (1996) and Renshaw and Haberman (2000) use four years, and some tables available in the market use up to six years of data.

We propose a broad class of dynamic models for preparing Bayesian mortality tables to apply to the exposed population having survival coverage in Brazil. We extend the model presented by Broffit (1988) to include the temporal dimension. Hence, to describe the relation between the number of observed deaths and the corresponding ages, we use the following model:

\[
d_{x,t} \mid \mu_{x,t} \sim \text{Poisson}(e_{x,t} \cdot \mu_{x,t}) , \quad x = x_{\text{inf}}, ..., x_{\text{sup}}
\]

where: \( t = 1, ..., T \), the number or periods of observation, \( \mu_{x,t} > 0 \) and \( e_{x,t} \) are known constants. Two alternatives are used in the modeling: local and global.

3.1 Local Dynamic Model

To model the evolution over time of the mortality forces, we use generalized dynamic models (West et al., 1985), considering \( \mu_{x,t} \) related through multiplicative perturbations:

\[
\log(\mu_{x,t}) = \log(\mu_{x,t-1}) + \omega_t ,
\]

(2)

\[
\omega_t \sim \text{Normal}(0, W_t) , \quad x = x_{\text{inf}}, ..., x_{\text{sup}} \quad \text{and} \quad t = 2, ..., T
\]
where: $W_t$ is modeled through non-informative Inverse Gamma distribution. We stress that if $W_t = 0$, we revert to a static model. In this case, the data over the years are considered with the same relevance, as if it were a regression with replications in each age bracket. The model is completed by specifying $v_{t,x}$ prior Gamma distributions for the parameters $\mu_{s,t}$. So, we have:

$$\mu_{s,t} \sim \text{Gama}(\alpha, \beta) I_{(\mu_{s-1,t}, \mu_{s+1,t})}(\mu_{s,t}) , \ x = x_{\inf},...,x_{\sup}$$

where: $t = 1, I_{A}(y)$ is the indicator function, assuming the values $I_{A}(y) = 1$, if $y \in A$ and 0 otherwise, $\alpha = 0.001$ and $\beta = 0.001$. These hyperparameters are assumed known, unlike Carlin (1992), only because of the computational limitations of WinBUGS (Spiegelhalter et al., 1996).

It should be observed that by graduating $\mu_{s,t}, \forall t > 1$, all $\mu_{s,t}$ will also be graduated, due to the evolution of the parameters (2). Besides this, Bayes’ Theorem assures that the restrictions on the prior will also be valid for the posterior (Carlin, 1992), in this way producing graduated Bayesian estimators for all classes in time. A more recent application of these concepts, in the context of estimating demographic models with restrictions in the parameters, can be found in McDonald and Prevost (1997).

### 3.2 Global Dynamic Model

The graduation in this case is done by adjusting the mortality rates to Makeham’s Law (Bowers et al., 1986), permitting the parameters to evolve smoothly over time:

$$\mu_{s,t} = \alpha_t + \beta_t \cdot \delta_t^x , \ x = x_{\inf},...,x_{\sup} \text{ and } t = 1,...,T.$$

Starting from the canonic link function ($\eta_{s,t}$) of the Poisson distribution, as in generalized dynamic linear models, we obtain the structure:

$$\eta_{s,t} = \log(e_{s,t} \cdot \mu_{s,t}) = \log(e_{s,t}) + \log(\alpha_t + \beta_t \cdot \delta_t^x) , \ x = x_{\inf},...,x_{\sup} \text{ and } t = 1,...,T.$$

where: $\mu_{s,t} > 0$ and $e_{s,t}$ are known constants.

The time evolution of the parameters that constitute Makeham’s Law are described by:

$$\begin{align*}
\log(\alpha_t) &= \log(\alpha_{t-1}) + \omega_a \\
\log(\beta_t) &= \log(\beta_{t-1}) + \omega_b \\
\log(\delta_t) &= \log(\delta_{t-1}) + \omega_c , \ t = 2,...,T.
\end{align*}$$

where: $\omega_z \sim \text{Normal}(0, W_z) , z = a, b, c$ and $W_z$ are modeled through non-informative Inverse Gamma distributions. Once again, static models are a particular case corresponding to $Wa_t = 0$, $Wb_t = 0$ and $WC_t = 0$. The model is completed by attributing non-informative prior normal distributions to the parameters: $\alpha_t, \beta_t \in \delta_t$, satisfying the restrictions imposed by Makeham’s Law.

### 3.3 Predictive Distribution of the Number of Deaths

The predictive distribution of the number of deaths a time $T+1$ ($d_{x,T+1}$) is given by:

$$p(d_{x,T+1} \mid D_T) = \int p(d_{x,T+1} \mid \mu_{x,T+1}, D_T) \cdot p(\mu_{x,T+1} \mid D_T) \ d\mu_{x,T+1} , \ x = x_{\inf},...,x_{\sup}$$

where: $\mu_{x,t} > 0$, $T$ is the number of observation periods and $D_T = (x, e_{x,T}, d_{x,T})$ represents the available information. It is worth pointing out that the uncertainties associated with the non-observable parameters $\mu_{x,T+1}$ is incorporated in the above calculation.
Based on the concept of predictive distribution described in equation (3), we obtain the predictive distributions for the probabilities of death of each age class, through the simplification described in equation (1), assuming that $e_{x,T+1}$ are known constants:

$$q_{x,T+1} = 1 - \exp(-\mu_{x,T+1}), \ x = x_{\text{inf}} , \ldots , x_{\text{sup}}$$

Hence, the means of the predicted sample values for this quantity are considered the Bayesian estimators of the probabilities of future deaths.

### 3.4 Implementation and Analysis of Convergence

We implemented the proposed models through MCMC, particularly Gibbs sampling, using WinBUGS version 1.4 (Spiegelhalter *et al.*, 2003). Due to the characteristics of the data, we estimated the probabilities of death for ages between 25 and 90 years ($x_{\text{inf}} = 25$ and $x_{\text{sup}} = 90$), considering an observable horizon of four years ($T = 4$), between 1998 and 2001. We should mention that there were no recorded deaths for some of these ages. These are dealt with easily in the inference process used in this paper. In implementing Gibbs sampling, we used three parallel chains with different initial values, seeking to keep the samples generated from clustering in regions around a local mode, in the case of posterior multimodality, as described in Gamerman (1997).

Verification of convergence is an important step in applying these methods. Here we use three informal graphical analysis techniques (density, autocorrelation function and traces) and the Gelman-Rubin statistic, as modified by Brooks and Gelman (1998), and as applied by Migon and Moura (2002).

For each parameter we generated a sample of 120,000 observations, through three parallel chains, of which the first 60,000 were discarded. It took around 15 minutes to implement the local dynamic model, on a PC with 1.8GHz and 512MB of RAM. The global dynamic model took roughly three times as long.

### 3.5 Selection of Models and Analysis of Results

Selection of models is fundamental in modern statistics (Gelfand and Ghosh, 1998) due to computational advances that allow fine-tuning multiple complex and alternative models.

We apply three model selection methods to our data to identify the best model for preparing mortality tables. Besides the Bayes factor, calculated from the predictive log likelihood (LS) of each model estimated by the harmonic mean of the likelihood values (Newton and Raftery, 1994), we use two other methods. These criteria simultaneously consider the fitting of the observed data and the prediction of replicated data, and are: EPD (expected predictive deviance), proposed by Gelfand and Ghosh (1998); and DIC (deviance information criterion), developed by Spiegelhalter *et al.* (2002). Table 1 shows the results obtained by the methods of comparison of the proposed models.
Table 1: Results of the comparison methods of the proposed models, for both sexes.

<table>
<thead>
<tr>
<th>Models</th>
<th>EPD</th>
<th>DIC</th>
<th>LS</th>
<th>G(m)</th>
<th>P(m)</th>
<th>D(m)</th>
<th>D</th>
<th>pD</th>
<th>DIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>local dynamic</td>
<td>30,500.00</td>
<td>8,123.47</td>
<td>38,623.47</td>
<td>2,170.69</td>
<td>24.22</td>
<td>2,194.92</td>
<td>-1,130.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>global dynamic</td>
<td>20,940.00</td>
<td>7,672.98</td>
<td>28,612.98</td>
<td>1,782.85</td>
<td>7.38</td>
<td>1,790.23</td>
<td>-905.33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We conclude that regardless of gender and the comparison method employed, the global model is the best for preparing Bayesian mortality tables. The estimated probabilities of death, as well as their 95% probability intervals obtained by this model, for males and females, can be requested to the authors of this paper. Besides this, we can see that the estimated probabilities of death by this model are smoother than those obtained with the local model (Figure 2).

![Figure 2](image)

Figure 2: Estimated probabilities of death (logarithmic scale) based on the proposed models for male, from the global dynamic model (solid line) and local dynamic model (dotted line).

We compare the mortality table obtained through this model, for male, with tables widely accepted in the market. Figure 3 depicts the point estimates of the probabilities of death and their 95% probability interval, along with the probabilities of death given by three commercial tables: AT-83 Male – established as a maximum limit of probability of death for survival coverage in Brazil; AT-2000 Basic Male – currently the most-used table; and VBT-2001 Male, with a
selection period of one year – a more recent table described in American Academy of Actuaries (2002), not yet used in Brazil.

Figure 3: Comparison of the Bayesian mortality table for male (logarithmic scale): estimated probabilities of death (solid black line), 95% probability interval (dotted black line), AT-83 Male (green), AT-2000 Basic Male (red) and VBT-2001 Male (blue).

Figure 3 shows that the table used as a maximum probability limit (AT-83 Male) has greater probabilities of death than those obtained for the population that contracts survival coverage in Brazil. The same occurs with the AT-2000 Basic Male. This means that most insurers are considering that their policyholders will survive for fewer years than estimated. This can cause them to become insolvent due to constitution of insufficient mathematical reserves. The probabilities of death estimated for 2002 for the population acquiring survival coverage in Brazil are similar to those projected for 2001 for American policyholders (VBT-2001 Male). The same comments are in order for female.

As one would expect, the probabilities of death for female are lower than those for male (see Figure 4). This is usually observed in all families of tables, since women generally have longer life expectancies than do men.
4. Effect of Bayesian Mortality Tables in the Reserve Evaluation

In this section we show the importance of adjusting the mortality table used in survival coverage to the real mortality rates of the exposed population in order for insurers to remain solvent. We do this by calculating the mathematical reserve by means of MCMC, using value at risk (VaR) concepts. These reserves reflect the future commitment of the insurer less that of participants, and is divided into: mathematical reserve for benefits to be granted (MRBG), constituted before granting the benefit, and mathematical reserve for benefits already granted (MRBAG), constituted while the beneficiary is receiving benefit. The MRBAG reflects only the insurer’s total future obligations, since the beneficiaries are no longer contributing to the plan, and is calculated by multiplying the value of the benefit by the value of the life annuity. In Brazil, payments are usually made monthly and the MRBAG is calculated and accounted for monthly. Hence, the monthly calculation of this value consists of multiplying the value of the monthly benefit by the value of the monthly life annuity.

A life annuity is defined as the present actuarial value of a series of unit-value payments. There are various types of life annuities, which can be classified as follows:
1. Regarding frequently of payment: monthly; bi-monthly; quarterly; semi-annually or annually.
2. Regarding form of payment: whole life; temporary; deferred; etc.
3. Regarding payment schedule: annuity-due; or immediate annuity, depending on whether the payments will be made at the start of end of each period.

Regardless of their classification, life annuities are calculated based on the mortality tables, interest rate and methodology established in the pension plan, subject to approval in Brazil by the federal regulatory organ.

We now compare the mathematical reserve for benefits already granted, considering benefit is to be paid for life at the start of each month, obtained by means of the deterministic method used by the insurance market, considering the mortality table established in the plan; and
also by calculating the predictive distribution of the life annuity obtained by stochastic simulation. We use the Bayesian mortality table chosen in the previous section and the concept of value at risk (VaR) to determine the amount necessary to guarantee the insurer’s solvency.

Initially, let us succinctly describe the deterministic calculation of the monthly whole life annuity-due to a life aged \( x \) (\( 12a_x \)), which represents the present actuarial value of the unit-value payments made for life at the start of each month to an individual of age \( x \).

We let \( k E_x \) denote the value of the actuarial discount, i.e., the present value of a unit value of benefit owed at time \( k \) to a life aged \( x \). This is called \( k \)-year pure endowment:

\[
k E_x = \frac{1}{(1+i)^k} \cdot k P_x
\]

where: \( E_x = 1 \), \( i = \) real interest rate per year and \( k P_x = \) probability that an individual of age \( x \) will survive until age \( x + k \).

Hence, the annual whole life annuity-due to a life aged \( x \) is given by:

\[
\overline{a}_x = \sum_{k=0}^{\infty} k E_x = \frac{N_x}{D_x}
\]

where: \( N_x = \sum_{k=0}^{\infty} D_{x+k} \), \( D_x = l_x \cdot \frac{1}{(1+i)^x} \), and \( l_x = l_{x+1} \cdot q_{x-1} \), with \( l_x = \) number of survivals to each age \( x \) and \( l_0 = \) root of the table, set at 10,000. On the other hand, monthly whole life annuity-due to a life aged \( x \) will be (Bowers \textit{et al.}, 1986):

\[
12\overline{a}_x = 12 \cdot \left( \sum_{k=0}^{\infty} \frac{k E_x}{(1+i)^{12k}} \right) = 12 \cdot \frac{1}{(1+i)^{12}} \cdot k P_x \approx 12 \cdot (\overline{a}_x - \frac{11}{24})
\]

where: \( \overline{a}_x \) is the present actuarial value of a life annuity-due of 1 payable twelve times a year to a life aged \( x \).

We can easily obtain the predictive distribution of the life annuities, \( p(12a_x \mid D_x) \), since the \( 12a_x \) are functions of the death probability. The value at risk (VaR), percentile of order \( 1 - \alpha \) \( (\alpha = \text{level of risk}) \) of the above predictive distribution will be denoted by \( 12.\overline{a}_x \), i.e.:

\[
p(12.\overline{a}_x > 12a_x \mid D_x) = \alpha
\]

The estimated life annuity for age \( x \) is equal to the value corresponding to the VaR for level \( \alpha \) of the predictive distribution of the monthly whole life annuity-due, with the level of risk’s being a question to be decided by company management. So, the MRBAG value necessary for the insurer to remain solvent is calculated by multiplying the VaR by the monthly lifetime benefit. The difference between this and the MRBAG calculated by means of the mortality table, using the deterministic methodology approved in the plan, must be borne by the insurer. This amount corresponds to the additional reserve for solvency, which is called \textit{reserve for insufficient contribution} (RIC). Constitution of such a reserve is mandatory by force of specific regulation.

Below we exemplify the importance of adjusting the mortality table by calculating the reserve, making use of value at risk concepts.
4.1 Example of Calculating Reserve for a Single Participant

Given a hypothetical pension plan that guarantees 6% real interest per year and uses the AT-83 Male table, let us calculate the reserves for a 60-year old male beneficiary who receives a monthly lifetime benefit at the start of each month of R$1,000.00.

Applying equation (4), we obtain a monthly whole life annuity-due of 141.34. The mathematical reserve for benefits already granted is R$141,340.00. Using the outputs of the MCMC implemented in WinBUGS, we obtain the predictive distribution shown in Figure 5.

![Figure 5: Predictive density distribution of the monthly whole life annuity-due to male, age 60.](image)

Setting the level of risk $\alpha$, we can obtain the value of the MRBAG necessary for the insurer to remain solvent and the reserve to be constituted (RIC). Table 2 shows these quantities for different values of $\alpha$. We can also verify the evolution of the MRBAG calculated by the Bayesian method as a function of the increase in the level of risk by analyzing Figure 6.
Table 2: Calculation of life annuities and reserves, in reais, using the VaR concept, for different levels of risk.

<table>
<thead>
<tr>
<th>level of risk</th>
<th>estimated life annuity</th>
<th>MRBAG necessary to guarantee the insurer's solvency (1)</th>
<th>deterministic MRBAG (2)</th>
<th>RIC (1) - (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0%</td>
<td>159.60</td>
<td>159,600.00</td>
<td>141,340.00</td>
<td>18,260.00</td>
</tr>
<tr>
<td>25.0%</td>
<td>160.00</td>
<td>160,000.00</td>
<td>141,340.00</td>
<td>18,660.00</td>
</tr>
<tr>
<td>10.0%</td>
<td>160.30</td>
<td>160,300.00</td>
<td>141,340.00</td>
<td>18,960.00</td>
</tr>
<tr>
<td>5.0%</td>
<td>160.50</td>
<td>160,500.00</td>
<td>141,340.00</td>
<td>19,160.00</td>
</tr>
<tr>
<td>2.5%</td>
<td>160.60</td>
<td>160,600.00</td>
<td>141,340.00</td>
<td>19,260.00</td>
</tr>
<tr>
<td>1.0%</td>
<td>160.80</td>
<td>160,800.00</td>
<td>141,340.00</td>
<td>19,460.00</td>
</tr>
</tbody>
</table>

Figure 6: MRBAG necessary for an insurer to remain solvent, in reais, as a function of the level of risk.

Analyzing Table 2 and Figure 6, we can see that the lower the risk the insurer wants to incur, the greater the MRBAG necessary for it to remain solvent and the greater the RIC it must fund. Hence, the task of choosing the level of risk is very important to manage the company with an adequate safety margin.
5. Conclusion

In this paper we implement Bayesian graduation models of mortality rates using MCMC techniques, calculated with WinBUGS. The probabilities of future deaths are estimated by means of the predictive distribution of the number of deaths for each age, which is modeled as being Poisson-distributed, considering that all individuals of the same age die independently and with the same probability.

To construct Bayesian mortality tables, we propose two dynamic models – local and global – considering the temporal evolution of mortality occurring in the sample population in the years 1998-2001 to estimate the death probabilities for 2002. After comparing the models, we find that the dynamic global model is best for preparing Bayesian mortality tables for both sexes.

After obtaining the Bayesian mortality tables, we demonstrate that the tables currently used in Brazil to calculate survival coverage under pension plans are not well suited. They have higher probabilities of death than is really the case for the population studied, assuming that the available data are reliable, and can cause problems of insolvency among insurers.

As expected, the Bayesian mortality tables for males have higher death probabilities than those for females, since as is known, women have longer expected life spans.

With the aim of exemplifying the importance of adjusting the mortality table used in survival coverage to the real mortality of the exposed population in order to avoid future insolvency, we compare the calculations of the mathematical reserves for benefits already granted, obtained by the following methods:
1. Deterministic (employed by the insurance market), using a known mortality table; and
2. Bayesian, using MCMC techniques and VaR concepts to construct the Bayesian mortality table presented in this paper.

From this comparison we calculate the additional reserve (RIC) insurers should constitute to remain solvent.

We hope with this to help insurers evaluate the true mortality rates of their exposed populations, by proposing graduated Bayesian models implemented through a statistical package (WinBUGS) that can be obtained at no cost. This will enable them to adopt mortality tables that are better adjusted to reality and hence to adequately constitute their reserves to avoid future insolvency.

A natural extension of this work, in view of the tendency for people to live longer, would be to model the mortality reduction factor, as done by Renshaw and Haberman (2000), but with a Bayesian and predictive focus, unlike the model described by these authors. However, since we have only four years of data for the Brazilian market, at the moment this modeling is impossible.

6. References


Annex I – Data: Central number of exposed to risk observed and number of deaths observed in the period 1998 to 2001.

<table>
<thead>
<tr>
<th>age</th>
<th>men</th>
<th>women</th>
<th>men</th>
<th>women</th>
<th>men</th>
<th>women</th>
<th>men</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>expos. to risk</td>
<td>deaths</td>
<td>expos. to risk</td>
<td>deaths</td>
<td>expos. to risk</td>
<td>deaths</td>
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</tr>
<tr>
<td>0</td>
<td>1,457</td>
<td>0</td>
<td>1,149</td>
<td>0</td>
<td>191,767</td>
<td>131</td>
<td>93,594</td>
<td>33</td>
</tr>
<tr>
<td>1</td>
<td>2,002</td>
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<td>1,594</td>
<td>0</td>
<td>189,534</td>
<td>138</td>
<td>91,644</td>
<td>48</td>
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