The Growing Triangle technique for choosing a model for loss reserves

Jennifer S.K. Chan[1] , Boris S.T. Choy[2] and Udi Makov[3]

[1] *The University of Hong Kong,* [2] *University of Technology, Sydney* and [3] *The University of Haifa*

Summary. The problem of choosing an appropriate reserve method for estimating loss reserves for a given set of loss data has long been an important task for actuaries. A *Growing Triangle* technique is proposed for this purpose. At the root of this technique, sub-triangles of losses, embedded in the full triangle of available data, are used to assess the prediction power of various candidate methods of estimation. The technique is demonstrated using real data.

Keyword: loss reserves, run-off triangle, chain-ladder method, multiplicative models, Schnaus models.

1 Introduction

As a return of paying future claims on specific losses described on insurance contracts, insurer received premiums from policyholders in advance. However, the actual losses are not known in advance. Therefore, a method to estimate the expected liability is needed so that the insurer can calculate the profit of written policies and allocate assets to ensure liquidity.

[1] *Address for correspondence:* Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam Road, Hong Kong

E-mail: jchan@hkustasc.hku.hk

[2] *Address for correspondence:* Department of Mathematical Sciences, University of Technology, Sydney P.O.Box 123, Broadway, NSW 2007, Australia

E-mail: boris.choy@uts.edu.au

[3] *Address for correspondence:* Department of Statistics, The University of Haifa, Haifa, Irsael

E-mail: makov@rstat.haifa.ac.il

For life insurance, the task is easier. As mortality and morbidity of a population can be predicted fairly accurately, reserve can be calculated based on formula established from the plan design. For example, life reserve actuaries use life contingency formula (Bowers *et al.* 1933) for estimating net premium reserve for a whole life policy in life contingency. However, for property and casualty insurers, losses are usually due to events that are highly random in nature. For example, in calculating property losses due to fire, the probability of a forest fire depends on weather condition, location and physical structure of the building and many other factors, all of which hinder precise estimation of losses. This makes the loss projection based solely on the fundamental causes of loss impossible. Instead, property and causality actuaries usually estimate loss reserves based on all reported or known losses and events.

The chain-ladder method is the most popular method of loss reserving. During the past decades, loss-reserving has become a very sophisticated and developed field in insurance industry. A complete list of these methods is non-exhausting, ranging from the simplest one of loss ratio estimation to the complicated Bayesian hierarchical models. For a recent book summarizing many of the existing methods, see *Loss Reserving - An Actuarial Perspective* by G. Taylor (2000).

The chain-ladder method has been extended based on the stochastic nature of the losses. Two broadly used models are the log-linear and the log-normal ANOVA-type models (Renshaw and Verrall 1994, Renshaw 1989, Verrall 1991, 1993, 1996 and Haberman and Renshaw 1996). Others include the dynamic state space model (Verrall 1989 and 1994) as an extension to the log-normal models and the Bayesian models using Markov chain Monte Carlo method (Makov, Smith and Liu 1996).

As for simulation methods, Stanard (1985) and Pentikäinen and Rantala (1995) describe methods of simulating random loss triangles in which data of the upper triangle are simulated and used to estimate the simulated losses, that is, the missing values of the lower triangle. The former is based on a loss severity distribution of individual claim amounts whereas the latter uses an aggregate stochastic claim process. Simulated models are desirably compared in Narayan and Warthen (2000). However the problem with simulation methodology is that the underlying assumptions behind the simulation may not be realistic, namely, that the nature of the simulated losses does not mimic closely that of the actual losses. These new methods differ in their underlying assumptions, degree of sophistication and complexity.

For the practicing actuary, the idea of exchanging the simple-to-use 'chain-ladder' method, with another more complicated one, is quite unattractive. This is reflected by the fact that most actuaries solely use the chain-ladder method and rarely attempt any other methods. For researches onto different methods of loss reserving, selecting an appropriate reserve method thus becomes an important task. At present, there are not much research work on model selection for loss reserves.

A common practice to introduce a new methodology for loss reserves is to offer numerical examples in which the supremacy of one method over the others is demonstrated. However, in most cases, the methods are judged by their ability to predict accurately the values of the existing data, in the form of upper triangle. This is an undesirable practice since the same data is used both for model building and for model assessment.

In this paper, we propose a Growing Triangle (GT) technique for comparing between different models for loss reserves. At the root of the proposed method, lies the assumption that there is no universal 'good' methodology and that for every data set of losses there are methodologies which are better suited for predicting loss reserves. In order to assess a methodology for a particular set of data, we split the data into smaller embedded triangles, in a way that their lower triangle values are known and can be used for prediction assessment. To be specific, the data in the triangle are used to train the proposed model and then the predicted losses outside the run-off triangle will be compared with the observed losses in the triangle. Finally a weighted average of the sum of squared discrepancy for each triangle of size n is obtained which is then used as a measure for model selection.

Section 2 introduces nine simple models for loss reserves and they are compared using the proposed GT technique. We did not consider complicated loss reserve models because the aim of this paper is to demonstrate the use of growing triangles to evaluate the predictive power of different models and hence to facilitate models selection. Section 3 describes the GT technique for comparing between different models for loss reserves. Finally in Section 4, the proposed technique is demonstrated using three data sets.

2 Models for loss reserves

For each triangle of size n, $n = 2, \ldots N-1$, let X_{ij}, $i, j = 1, \ldots, n$, $i+j \leq n+1$ denote the incremental loss of policies written in year i and reported in the j-th year thereafter. Claims from policies written are incurred in year 1 to n. Moreover, for policies written in year i, there will be $n+1-i$ different report lags. Therefore, data for incremental claims are usually summarized in a triangular format as shown in Table 1.

To demonstrate the GT technique for comparing the predictive power between different models for loss reserves, we consider, in this paper, nine models as given below.

1. Chain ladder

   Model 1 uses the chain ladder method which is simple and is widely used in actuarial practice. It indirectly estimates the incremental loss X_{ij} by projecting the cumulative loss S_{ij} given by

   $$S_{ij} = \sum_{k=1}^{j} X_{ij}, \quad i = 1, \ldots, n, \ j = 1, \ldots, n+1-i.$$

   The cumulative loss S_{ij} at lag year $j(> n+1-i)$ can be estimated by multiplying successively the projection factor f_j to the latest known cumulative loss, $S_{i,n+1-r}$ for accident year i using the formula

   $$\widehat{S}_{ij} = \widehat{S}_{i,j-1} \times f_j, \quad i = 2, \ldots n, \qquad j = n+2-i, \ldots, n^* \tag{1}$$

   where $\widehat{S}_{i,n+1-i} = S_{i,n+1-i}$, $n^* = \min\{n, N+1-i\}$ and the projection factors f_j are estimated by

   $$f_j = \frac{\sum\limits_{i=1}^{n+1-j} S_{ij}}{\sum\limits_{i=1}^{n+1-j} S_{i,j-1}} = \sum_{i=1}^{n+1-j} w_{ij} f_{ij}, \qquad j = 2, \ldots, n, \tag{2}$$

   which is a weighed average of the age-to-age development factors

   $$f_{ij} = \frac{S_{i,j}}{S_{i,j-1}}, \qquad i = 2, \ldots, n, \qquad j = 2, \ldots, n+1-i \tag{3}$$

   with weights

   $$w_i = \frac{S_{i,j-1}}{\sum\limits_{i=1}^{n+1-j} S_{i,j-1}}$$

and

$$\sum_{i=1}^{n+1-j} w_{ij} = 1.$$

Note that the predicted S_{ij} which is further away from the latest known loss, is less reliable as more projection factors are required and the projection factors of higher lag year become less reliable because of the availability of less data.

2. Modified chain ladder

Alternatively, in model 2, the projection factors f_j (Brown and Gottlieb 2001) are calculated as a simple average of the age-to-age development factors, that is

$$f_j = \frac{1}{n+1-j} f_{ij}, \qquad j = 2, \ldots, n \tag{4}$$

replaces (2) and the procedures in (1) and (3) remain the same.

3. Modified Bornhuetter-Ferguson

Bornhuetter and Ferguson (1972) developed this reserve method to incorporate both the information from reported claims and also the prior actuarial knowledge from plan design. Firstly, an actuary is required to estimated the expected loss ratio, which is defined as

$$\text{Expected Loss Ratio} = \frac{\text{(Present Value of) Future Loss}}{\text{(Present Value of) Premiums}},$$

either subjectively or based on policy design and past experience. The loss reserves are then estimated using

$$\text{Reserve} = \text{Expected Loss Ratio} \times \text{Premium Received} \times \frac{1}{\prod_j f_j}.$$

Following the idea, Model 3 is defined as

$$\widehat{S}_{ij} = \widehat{S}_{i,j-1} + r_h \times \left(1 - \frac{1}{f_j}\right), \qquad i = 2, \ldots n, \quad j = n+2-i, \ldots n^*$$

where $h = i + j - n, \ h \geq 2$,

$$r_h = \frac{1}{n-h+1} \sum_{k=h}^{n} \widehat{S}_{n+h-k,k-1} \times f_k$$

and $\widehat{S}_{n+2-k,k-1} = S_{n+2-k,k-1}$ for $k = 2, \ldots, n$.

4. Adjustment to total known losses

Model 4, referred as the 'Cape Cod method', is described in Stanard (1980). It consists of averaging the known losses first and then applying an adjustment factor to the sum

$$\widehat{S}_{ij} = \widehat{S}_{i,j-1} + r_i^* \times \left(1 - \frac{1}{f_j}\right), \qquad i = 2, \dots n, \quad j = n+2-i, \dots n^*$$

where

$$r_h^* = \frac{\sum\limits_{k=h}^{n} S_{n+h-k,k-1}}{\sum\limits_{k=h}^{n} \frac{1}{f_k}}.$$

5. Log-normal model

Now we consider a more complicated family of models within the *Generalized Linear models* (GLMs) framework for incremental losses as

$$h(\mathbf{X}) = \mathbf{Z}^{\mathbf{T}}\boldsymbol{\theta} + \boldsymbol{\epsilon}$$

where $\mathbf{X}$ is the matrix with elements X_{ij}, $h(\cdot)$ is a linked function, $\mathbf{Z}$ is the design matrix, $\boldsymbol{\theta}$ is a vector of parameters and $\boldsymbol{\epsilon}$ is a vector of error terms which follows some multivariate distribution say normal with mean zero. Properties of a particular model is determined by the linked function, the design matrix as well as the error distribution.

Ntzoufras and Dellaportas (1998) developed several Bayesian models in actuarial estimation. In model 5, the incremental loss X_{ij} is modeled as

$$\ln(X_{ij}) \sim N(\mu_{ij}, \sigma^2), \tag{5}$$

where

$$\mu_{ij} = \mu + \alpha_i + \beta_j, \qquad i, j = 1, \dots, n$$

such that the mean μ_{ij} relates additively to the policy year i and reported lag j. Models are conveniently implemented using a Bayesian software called 'Bayesian using Gibbs outputs in Window Version, WinBUGS' (Spiegelhalter *et al.* 2000). Prior distributions for the parameters are given by

$$\mu \sim N(0, {\sigma_\mu}^2), \quad \alpha_i \sim N(0, {\sigma_\alpha}^2), \quad \beta_j \sim N(0, {\sigma_\beta}^2), \quad \tau = \sigma^{-2} \sim G(a_\tau, b_\tau). \tag{6}$$

Without any prior information for the parameters, we set $1/\sigma_\mu{}^2$, $1/\sigma_\alpha{}^2$, $1/\sigma_\beta{}^2$, a_τ and b_τ to be very small, say 0.001. Finally, to specify the parameters uniquely, we impose the sum-to-zero constraints on α and β such that

$$\sum_{i=1}^{n} \alpha_i = \sum_{j=1}^{n} \beta_j = 0.$$

6. Log-normal model with linear policy-year effect

   This model 6 is similar to the previous model except that

$$\mu_{ij} = \mu + \alpha \times i + \beta_j, \qquad i, j = 1, \ldots, n$$

   such that the mean relates linearly to the policy-year but the reported-lag effect is still year specific. Prior distributions for the parameters are given by (6) except that $\alpha \sim N(0, \sigma_\alpha{}^2)$ and the sum-to-zero constraint on β is $\sum_{j=1}^{n} \beta_j = 0$.

7. Log-normal model with linear reported-lag effect

   This model 7 is similar to model 5 except that

$$\mu_{ij} = \mu + \alpha_i + \beta \times j, \qquad i, j = 1, \ldots, n$$

   such that the mean relates linearly to reported-lag but the policy-year effect is still year specific. Prior distributions for the parameters are given by (6) except that $\beta \sim N(0, \sigma_\beta{}^2)$ and the sum-to-zero constraint on α is $\sum_{i=1}^{n} \alpha_i = 0$.

8. Log-normal model with linear policy-year and reported-lag effects

   Again, this model 8 is similar to model 5 except that the mean relates linearly to both policy-year and reported-lag as

$$\mu_{ij} = \mu + \alpha \times i + \beta \times j, \qquad i, j = 1, \ldots, n.$$

   Prior distributions for the parameters are given by (6) except that $\alpha \sim N(0, \sigma_\alpha{}^2)$ and $\beta \sim N(0, \sigma_\beta{}^2)$.

9. State Space Model:

   The State space model is an extension of the log-normal such that parameters $\boldsymbol{\beta}$

are allowed to depend on both i and j and α_i and β_{ij} depend recursively on values of the previous written year. The full model is given by (5) where

$$\mu_{ij} = \mu + \alpha_i + \beta_{ij}, \qquad i, j = 1, \ldots, n$$

with

$$\beta_{ij} = \beta_{i-1,j} + v_i, \qquad v_i \sim N(0, \sigma_v^2), \qquad i = 2, \ldots, n, \tag{7}$$

$$\alpha_i = \alpha_{i-1} + h_i, \qquad h_i \sim N(0, \sigma_h^2), \qquad i = 2, \ldots, n, \tag{8}$$

and the corner constraints

$$\alpha_1 = \beta_{i1} = 0, \qquad i = 1, 2, \ldots, n.$$

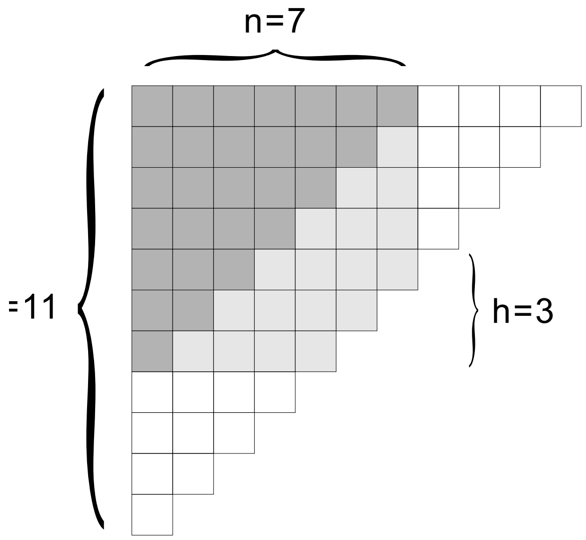
Here the reported-lag effect changes with policy years and the recursive relations in (7) and (8) assert that parameters α_i and β_{ij} evolve through time via known stochastic mechanisms which are determined by the disturbance terms, v_i and h_i. As σ_v^2 approaches to zero, the model degenerates to a log-normal model . The corner constraints imply that μ is the expected log-adjusted claim amount for the first written year paid without delay.

3 The Growing Triangle (GT) technique

The GT technique is developed to compare the efficiency of different loss reserve methods based on the accuracy of their projection on real data. In this technique, out of a full upper triangle of size N for a loss data, run-off triangles of observed losses grow gradually in size n from 5 (or 6) to $N-1$, each divides the full triangle into 3 sub-parts:

1. The dark squares in the upper run-off triangle of size show the data used for model training;
2. The light squares in the lower run-off triangle show the observed data used to compare with the projected data to assess the model predictive power; and
3. The white squares in the full upper triangle of size outside the run-off triangle show the unused data.

This is illustrated with $N = 11$ and $n = 7$ in the figure below:



Experience shows that run-off triangles of size less than 5 may not have stable parameter estimates. Moreover, we define the following quantities:

1. l_{en} : number of data used for model training
2. l_{pn} : number of observed data to be compared with the projected data
3. $l_{tn} = l_{en} + l_{pn}$
4. $l_{rn} = l_{en}/l_{pn}$

where

$$l_{en} = \frac{1}{2}n(n+1)$$

and

$$l_{pn} = \begin{cases} \frac{1}{2}n(n-1) & \text{if } n \leq m \\ \frac{1}{2}n(n-1) - \frac{1}{2}h(h-1) & \text{if } n > m \end{cases},$$

with

$$h = \begin{cases} 2(n-m)+1 & \text{if } N \text{ is odd} \\ 2(n-m) & \text{if } N \text{ is even} \end{cases}$$

and the middle value of N is $m = \frac{1}{2}(N+1)$ if N is odd and $m = \frac{N}{2}$ if N is even. The accuracy of prediction can then be measured using the weighted mean square error ($WMSE$). For a run-off triangle of size n,

$$WMSE = \sum_{n=5}^{N-1} w_n \times MSE_n$$

where

$$MSE_n = \frac{1}{l_{pn}} \sum_{i=5}^{n} \sum_{j=n-i+2}^{n*} (\hat{x}_{ij} - x_{ij})^2$$

and

$$n^* = \begin{cases} n & \text{if } i < m \\ N+1-i & \text{if } i \geq m \end{cases} .$$

There are several ways of defining the importance measures $w_n,\ n = 5, \ldots, N-1$:

1. $w_{en} = \frac{l_{en}}{\sum_{i=5}^{N-1} l_{ei}}$ which is proportional to the number of data used for model training,

2. $w_{tn} = \frac{l_{tn}}{\sum_{i=5}^{N-1} l_{ti}}$ which is proportional to the total number of data, for model training as well as prediction assessment, or

3. $w_{rn} = \frac{l_{rn}}{\sum_{i=5}^{N-1} l_{ri}}$ which is proportional to the number of data used for model training per each data used in prediction assessment.

The first two sets of weights will increase with the size n of the run-off triangle, giving heavier weights to larger run-off triangles which contain more information. The third set of weights will decrease with the size of run-off triangle but will increase again if $n > m$ so that MSE from large run-off triangle will not dominant $WMSE$. Values of l_{pn}, l_{en}, the sum l_{tn} and the ratio l_{rn} for $n = 5, \ldots, N-1$ are given in Table 3 when $N = 18$.

4 Examples and results

The GT technique is illustrated using the following three data set, given in Tables 4-6.

1. Paid losses during 1978-1995

2. Payments per claim during 1981-1995

3. Number of claim notified per 100,000 vehicle years of exposure during 1985-1995

   We consider the square root transformation in (5) for this data set.

MSE for different run-off triangles of size n are shown in Figures 1-3. Then results of $WMSE_k,\ k = e, t, r$ with ranks in brackets using the three weights w_{en} and w_{tn} and w_{rn}

respectively for each data set are shown in Table 3. The log-normal model with linear policy-year effect is the best model for the paid losses data. This reveals the fact that the reported-lag effect on incremental loss is more year specific while the written year effect is generally linear.

On the other hand, the modified chain ladder method provides the best predictions for the other two data sets. The chain ladder method, being one of the most traditional method of reserve estimation used by actuaries, produces relative consistent results in the growing triangle assessment. This is explained by its relative simple and non-parametric nature. Without heavy assumptions on the behaviors of underlying claim amount and frequency, it generates fairly accurate reserve estimation in all cases and out-performs the more complicated models for the other two data sets.

Moreover the modified chain ladder method produces even better estimations of the lower triangles than the chain ladder method. This explains why it is more frequently used and presented in the actuarial context. Another explanation for its popular usage lies in the estimation of projection factors f_j: instead of using the simple arithmetic average, actuaries can also modify f_j in (2) as other types of average of f_{ij}, like average excluding extreme values, median and geometric mean in line with the actuarial expectation of behavior for the loss data. This provides a flexible method for the actuaries.

On the other hand, the log-normal model is a relative straight forward parametric model and out-performs the modified chain ladder method for the paid losses data. Quite out of our expectation, the more advanced state space model fails to produce better estimation in all three data sets. Parameters α_i and β_{ij} in the state space model depend recursively on their values of the previous policy year with the differences being h_i and v_i respectively. Therefore, errors in the estimation of v_i and h_i in (7) and (8) respectively will accumulate and make the prediction of incremental loss X_{ij} less reliable in larger run-off triangles.

5 Conclusion

The GT technique provides a simple and effective method for assessing the projection efficiency of different loss reserve models: the chain ladder methods, the log-normal models and the state space model, as illustrated in this paper. However, there are some limitations of the method. Results show that model predictions are unstable for small run-off triangles probably because of insufficient data. One remedy adopted in this paper is to start the growing of triangles from run-off triangle of size that is suitably large, say from $n = 6$ for the paid losses data and $n = 5$ for the other two data. However when the data set is not large enough, the number of run-off triangles used in assessing the prediction accuracy will be considerably reduced. For example, we have only 6 run-off triangles for the number of claim data when $N = 11$.

Figure 1: MSE for paid losses data

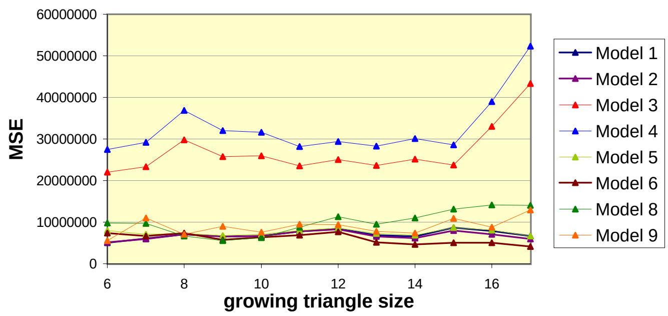


Figure 2: MSE for payments per claim data

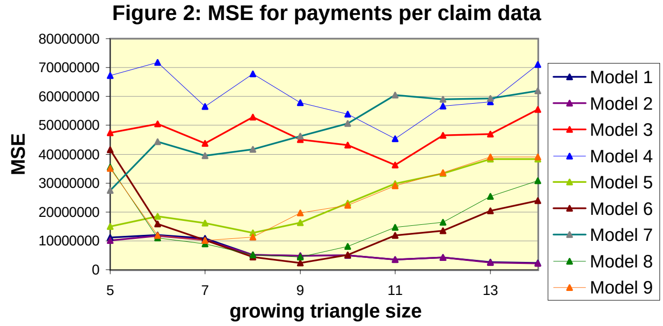


Figure 3: MSE for number of claim data

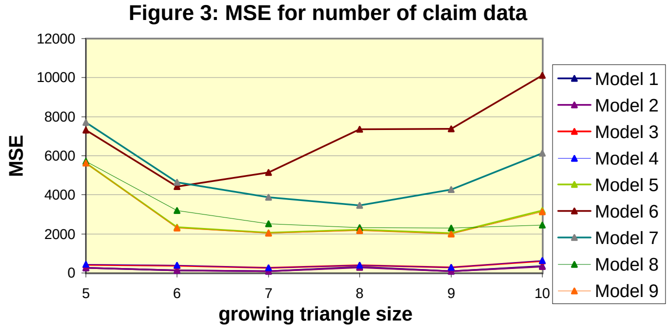


Table 1: Structure of outstanding claim amount data

Year	Development year				
	1	2	$\cdots$	$n-1$	n
1	X_{11}	X_{12}	$\cdots$	$X_{1,n-1}$	X_{1n}
2	X_{21}	X_{22}	$\cdots$	$X_{2,n-1}$	
$\vdots$	$\vdots$	$\vdots$			
$n-1$	$X_{n-1,1}$	$X_{n-1,2}$			
n	X_{n1}				

Table 2: The number of data for model training, for prediction assessment, their totals and their ratios for $N = 18$.

n	5	6	7	8	9	10	11	12	13	14	15	16	17
l_{en}	10	15	21	28	36	44	49	51	50	46	39	29	16
l_{pn}	15	21	28	36	45	55	66	78	91	105	120	136	153
l_{tn}	25	36	49	64	81	99	115	129	141	151	159	165	169
l_{rn}	1.50	1.40	1.33	1.29	1.25	1.25	1.35	1.53	1.82	2.28	3.08	4.69	9.56

Table 3: Weighted mean square errors (WMSE) for the three data sets.

Wt.	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
$WMSE$ (in 1000) for data of the paid losses during 1978-1995									
w_{en}	7,479 (3)	7,052 (2)	28,656 (7)	34,495 (8)	7,713 (4)	*5,583* (1)	154,079 (9)	11,326 (6)	10,032 (5)
w_{tn}	7,484 (3)	7,108 (2)	27,741 (7)	33,435 (8)	7,745 (4)	*5,753* (1)	140,040 (9)	10,918 (6)	9,915 (5)
w_{rn}	7,695 (3)	7,192 (2)	30,700 (7)	37,112 (8)	7,899 (4)	*5,560* (1)	162,016 (9)	12,162 (6)	11,715 (5)
$WMSE$ (in 1000) for data of the payments per claim during 1981-1995									
w_{en}	4,554 (2)	*4,403* (1)	47,081 (7)	59,766 (9)	28,892 (5)	14,825 (3)	54,299 (8)	17,852 (4)	29,237 (6)
w_{tn}	4,856 (2)	*4,702* (1)	46,653 (7)	59,344 (9)	27,446 (5)	13,874 (3)	53,144 (8)	16,492 (4)	27,729 (6)
w_{rn}	4,673 (2)	*4,467* (1)	48,851 (7)	62,607 (9)	29,977 (5)	17,889 (3)	54,264 (8)	21,035 (4)	30,910 (6)
$WMSE$ for data of the number of claim notified per 100,000 vehicle years of exposure during 1985-1995									
w_{en}	215 (2)	*206* (1)	410 (3)	423 (4)	2,686 (6)	7,501 (9)	4,874 (8)	2,728 (7)	2,645 (5)
w_{tn}	207 (2)	*199* (1)	397 (3)	410 (4)	2,682 (6)	7,252 (9)	4,816 (8)	2,788 (7)	2,644 (5)
w_{rn}	238 (2)	*227* (1)	447 (3)	460 (4)	2,885 (7)	7,955 (9)	5,218 (8)	2,795 (5)	2,840 (6)

Table 4: Data of paid losses during 1978-1995

Year	Development year																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1978	3323	8332	9572	10172	7631	3855	3252	4433	2188	333	199	692	311	0.01	405	293	76	14
1979	3785	10342	8330	7849	2839	3577	1404	1721	1065	156	35	259	250	420	6	1	0.01	
1980	4677	9989	8746	10228	8572	5787	3855	1445	1612	626	1172	589	438	473	370	31		
1981	5288	8089	12839	11829	7560	6383	4118	3016	1575	1985	2645	266	38	45	115			
1982	2294	9869	10242	13808	8775	5419	2424	1597	4149	1296	917	295	428	359				
1983	3600	7514	8247	9327	8584	4245	4096	3216	2014	593	1188	691	368					
1984	3642	7394	9838	9733	6377	4884	11920	4188	4492	1760	944	921						
1985	2463	5033	6980	7722	6702	7834	5579	3622	1300	3069	1370							
1986	2267	5959	6175	7051	8102	6339	6978	4396	3107	903								
1987	2009	3700	5298	6885	6477	7570	5855	5751	3871									
1988	1860	5282	3640	7538	5157	5766	6862	2572										
1989	2331	3517	5310	6066	10149	9265	5262											
1990	2314	4487	4112	7000	11163	10057												
1991	2607	3952	8228	7895	9317													
1992	2595	5403	6579	15546														
1993	3155	4974	7961															
1994	2626	5704																
1995	2827																	

Table 5: Data of payments per claim during 1981-1995

Year	Development year														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1981	5686	12823	14072	16648	10035	3196	4184	3103	3364	428	58	185	0	0	0
1982	2256	8698	11228	13258	12488	6000	5021	1962	2544	2543	219	9	25	0	
1983	3735	11039	13805	13129	4795	6310	7886	1603	405	240	975	28	59		
1984	3708	7794	11457	12720	11004	6043	5322	1635	1037	2422	14	1632			
1985	2625	7530	8555	15446	8344	7428	2373	7257	2002	9	1096				
1986	2371	5366	10018	9675	9826	6648	4949	2907	3581	2198					
1987	2349	6234	7442	3312	8904	6050	4428	1856	1799						
1988	2128	4328	6459	8233	6494	4404	2712	3243							
1989	2670	6043	6196	7375	7145	4973	4249								
1990	2833	4028	4177	8053	8475	8352									
1991	2996	8491	6082	8614	7598										
1992	2800	4543	5034	6948											
1993	3561	5997	9485												
1994	2961	5615													
1995	3131														

Table 6: Number of claim notified per 100,000 vehicle years of exposure during 1985-1995

Year	Development year										
	1	2	3	4	5	6	7	8	9	10	11
1985	488	227	53	10.2	4.3	6.0	5.1	4.3	0.9	0.0	0.9
1986	472	228	26	21.9	9.7	10.5	4.9	1.6	0.8	0.0	
1987	434	175	34	14.3	9.6	7.2	4.0	1.6	0.0		
1988	388	203	37	16.8	11.4	3.0	6.1	0.0			
1989	422	150	21	12.2	8.6	2.9	6.4				
1990	369	128	15	7.8	5.9	3.3					
1991	379	127	18	5.6	4.4						
1992	414	104	12	7.4							
1993	364	112	24								
1994	381	93									
1995	375										

References

[1] Bornhuetter, R.L. and Ferguson, R.E. (1972). *The actuary and IBNR*, PCASD LIX, p.181.

[2] Brown, R.L. and Gottieb, L.R. (2001). *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance, (second Edition).*

[3] Haberman, S. and Renshaw, A.E. (1996). Generalized Linear Models and Actuarial Science. *The Statistician*, **45**, 407-436.

[4] Makov, U.E., Smith, A.F.M. and Liu, Y.H. (1996). Bayesian Methods in Actuarial Science. *The Statistician*, **45**, 503-515.

[5] Narayan, P. and Warthen, T. (2000). A comparative study of the performance of loss reserving methods through simulation, *J. Actuarial Practices,* **8**, 63-88.

[6] Ntzoufras, I. and Dellaportas, P. (1998). Bayesian Modelling for Outstanding Liabilities Incorporating Claim count Uncertainty. Unpublished paper.

[7] Pentikäinen, T. and Rantala, J. (1995). A Simulation Procedure for Comparing Different Claims Reserving Methods. *Casualty Actuarial society Forum*, 128-156.

[8] Renshaw A.E. (1989). Chain Ladder and Interactive Modelling. *The Journal of the Institute of Actuaries*, **116**, 559-587.

[9] Renshaw A. and Verrall R. (1994). A Stochastic Model Underlying the Chain-Ladder Technique. *Proceedings XXV ASTIN Colloquium*, Cannes, France.

[10] Spiegelhalter, D., Thomas, A. and Best, N. (2000). Bayesian Inference using Gibbs Sampling for Window Version WinBUGS, a software for Bayesian analysis using MCMC method and Gibbs sampler. It can be downloaded from `http://www.mrc-bsu.cam.ac/bugs`, the website of the BUGS project by the Biostatistics Unit of the Medical Research Council, in the University of Cambridge.

[11] Stanard, J.N. (1980). Experience Rates as Estimators: A Simulation of their Bias and Variance. *Pricing Property and Casualty Insurance Products.* Casualty Actuarial Society Discussion Paper Program and Review by John Robertson, p.485.

[12] Stanard, J.N. (1985). A Simulation Test of Prediction Errors of Loss Reserves Estimation Techniques, *Proceedings of the Casualty Actuarial Society*, **72**, 124-148.

[13] Taylor, G.C. and Ashe, F.R. (1983). Second Moments of of estimates of Outstanding Claims. Journal of Econometrics, 23, 37-61.

[14] Taylor, G.C. (2000). *Loss Reserving - An Actuarial Perspective*, Huebner International series.

[15] Verrall R.J. (1989). State. Space Representation of the Chain Ladder Linear Model. *The Journal of the Institute of Actuaries*, **116**, 589-610.

[16] Verrall R.J. (1991). Chain Ladder and Maximum Likelihood. *The Journal of the Institute of Actuaries*, **118** , 489-499.

[17] Verrall R.J. (1993). Negative Incremental Claims: Chain Ladder and Linear Models. *The Journal of the Institute of Actuaries*, **120**, 171-183.

[18] Verrall R.J. (1994). A Method for Modelling Varying-off Evolutions in Claims Reserving. *Astin Bulletin*, **24**, 325-332.

[19] Verrall, R.J. (1994). *Statistical Methods for Chain Ladder Techniques.* Casualty Actuarial society Forum (spring 1994), 393-446.

[20] Verrall R.J. (1996). Claims Reserving and Generalized additive Models. Insurance *Mathematics and Economics*, **19**, 31-43.