Elliptical distributions and stochastic orders

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Abstract

It is shown that for elliptically distributed vectors $X \sim \mathbb{E}_2(\mu, \Sigma, g_2)$ with the same marginal distributions, an ordering of the Σ matrices' off-diagonal terms implies a respective ordering of their joint probability distributions. When a covariance matrix exists, this implies that the stochastic correlation order (Dhaene and Goovaerts, 1996) translates to just the ordering of correlation coefficients. A similar characterisation of correlation order extends to the case of random vectors with elliptical copulas. Finally, the implications of the obtained results are discussed for capital allocation mechanisms, such as the ones introduced by Wang (2002), Tsanakas (2003) and Landsman (2003).

Keywords: Elliptical distributions, Stochastic order, Copulas, Capital allocation, Risk management.

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