Comments about CIR Model

Wojciech Szatzschneider

School of Actuarial Sciences, Anahuac University, Mexico City
(e-mail: wojciech@anahuac.mx)

We analyze the following problems concerning Cox, Ingersoll & Ross model: Linear risk premia, pricing bonds in the Longstaff double square root model, and negative correlations in a multifactor CIR model.

The Cox, Ingersoll & Ross Model for interest rates was proposed in 1980. Since then it has been the objective of many even recent studies and extensions. However, in spite of many deep theoretical studies, there are many errors, imprecisions or trivialities published in the financial literature. The main goal of this article is to correct some of them, and to show how relevant problems concerning CIR are linked to stochastic analysis.

Usually it is assumed risk premia proportional to $\sqrt{r(t)}$; and that linear risk premia are considered inadmissible; cf. Rogers (1995).

However, if one wants to work with CIR Model in Risk Neutral World (RNW) —the only world that can be observed for interest rates (IR) alone—, then it turns out (in some cases) that linear risk premia are allowed. In this case the IR in the Physical (Real) World follow a different model. We solve this problem in section 1 using elementary application of the Girsanov change of measure.

In section 2, we show how to "solve" the problem of pricing bonds in the double square root Longstaff model. The wrong solution was presented by Longstaff (1989) and a simple version was solved by Beaglehole & Tenney (1992).

In the analysis of the original double square root model the local time should appear (omitted by Longstaff). The appearance of local time makes it impossible to find a closed solution. The best we can do is to price bonds maturing at exponential time independent of the process, and use the decomposition of trajectories from, for example, Yor (1994). In section 3, we offer a short discussion of problems that are essentially equivalent or similar to the Longstaff one, with CIR as a short rate:

i) Pricing of options on assets.

ii) Pricing of default bonds in Merton's Structural Approach.

In both cases we assume that Asset Prices follow a Geometric Brownian Motion, and are correlated with IR. The problem *ii*) was "solved" by Wang (1999) assuming independence and once again in terms of Laplace transforms. In section 4 we will comment how to model negative correlations (observed in some financial markets) in the CIR setting. We cannot agree with the statement by Dai & Singleton (2000) that square–root diffusions are theoretically incapable of generating negative correlations.

Therefore from the point of view of applications sections 1 and 4 offers *positive* results and section 2 and 3 rather *negative* ones. Although section 1 is only of theoretical importance only. The full reproduction of formulas in section 3 would be endless ones, so we will quote the "subroutines" from the Borodin & Salminen handbook.