EFFICIENT ESTIMATION WITHIN PARAMETRIC COPULA MODELS WHEN MARGINS ARE UNKNOWN

JOHN EINMAHL, JOHAN SEGERS, AND BAS WERKER

Tilburg University, P.O.Box 90153, NL-5000 LE Tilburg, the Netherlands

j.h.j.einmahl@uvt.nl, jsegers@uvt.nl, b.j.m.werker@uvt.nl

Abstract. If the copula of an unknown bivariate distribution is modelled by a parametric family while the margins remain unspecified, the resulting model for the whole distribution is semiparametric. Within such a model, we consider the problem of estimating the copula parameter from an independent sample.

Because of the semiparametric nature of the model, the copula parameter cannot by estimated directly by maximum likelihood. To circumvent this problem, Genest *et al.* (1995) propose a pseudo-likelihood procedure in which the margins are estimated by the marginal empirical distribution functions; the copula parameter is then estimated by maximizing the pseudo-likelihood that arises when the margins are treated as known.

However, in Genest and Werker (2002), this method was shown in general not to be efficient in the sense of Bickel *et al.* (1993). The point is that when the copula of a bivariate distribution is known, the marginal empirical distribution functions may not be efficient estimators for the true margins.

We propose semi-parametrically efficient estimators for the margins when the copula is known. Using these instead of the classical marginal empirical distribution functions in the above pseudo-likelihood procedure leads to an efficient estimator of the copula parameter.

Keywords: copula, maximum likelihood, semiparametric efficiency

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