A Non-parametric Dirichlet Process Prior specified for Premiums based on Automobile Insurance Data

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Abstract

One of the techniques that can be considered typically non-life actuarial is experience rating by credibility theory. In particular credibility theory, statistical methods with a Bayesian flavor have long been used in the insurance industry as part of the process of estimating risks and setting premiums. The Bayesian approach can be used to devise prediction procedures that are more sensible – from both a Bayesian and Frequentist perspective – than those in current use. The Bayesian approach is also conceptually more appealing than the Classical approach with many advantages:

Critics of the Bayesian approach have most often cited the following points e.g. the Bayesian practitioner must formally express his prior beliefs about the unknown parameters in the form of a probability distribution or the Bayesian methodology is computer intensive. In many situations, integrations in several dimensions are required to obtain the required posterior distributions. These might have been valid criticisms in the past but by using (a) Non-Informative priors like Jeffreys and Reference priors and (b) Numerical integration techniques like Markov Chain Monte Carlo Methods and more specifically Gibbs Sampling, these problems can be overcome. In this paper the mixed linear model is presented to illustrate the ideas of credibility models where the credibility factors follow as optical coefficients in a Bayesian model incorporating variance components. We provide a general framework for Bayesian analysis of mixed linear models in which a non-parametric Dirichlet process prior is specified for the credibility factors and premiums. Only recently have tools allowing Bayesian analysis to become computationally feasible and opens the way to a fully Bayesian treatment of a range of actuarial problems; here we provide a detailed exploration of the theory based on collective automobile insurance data.

Keywords: Bayesian Analysis, Non-parametrics, Credibility factors, Premium estimation, Dirichlet Process Prior distribution.

References

Antoniak, C.E. (1974). Mixtures of Dirichlet processes with Applications to Non-parametric problems. *Ann. Statist*., 2, 1152 – 74.

Escobar, M.D., and West, M. (1995). Bayesian Density Estimation and Inference using Mixtures. *J. Amer. Statist. Assoc.*, 90, 578 – 588.

Kleinman, K.P., and Ibrahim, J.G. (1998). A Semi-parametric Bayesian Approach to the Random Effect Model. *Biometrics*, 54, 921 – 938.

MacEachern, S.N. and Mueller, P. (1998). Estimating Mixtures of Dirichlet Process Models. *J. Comput. Graph. Statist*., 7, 223 – 238.

Pretorius, A.L., van der Merwe, A.J. (2001). Non-parametric Bayesian Estimation for the Mixed Linear Model. Monographs of Official Statistics. p421–427. ISBA and Eurostat, 2001. Editor Edward George.

Van der Merwe, A.J. and Pretorius, A.L. (2003) Bayesian Estimation in Animal Breeding using Dirichlet Process Prior for Correlated Random Effects. *Genetics Selection Evolution (GSE)*, 35, 137 – 158.